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MCR3U

Key Questions & Concepts

Grade 11 Math

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Unit I: Rational Expressions

Important stuff

Factoring

Common factoring → Take out everything that's the same in it

Decomposition → Break apart the middle by figuring out what adds to the middle (b) and multiplies to the first times the last (a x c)

Sum-Product → If the first number is 1 (a=1), then when you find what adds to the middle and multiplies to the last, that's what goes in the brackets

Difference of Squares → Binomial with 2 perfect squares separated by a negative, square root first, square root last, 2 brackets with a + and a –

Restrictions

State whatever's on the bottom that would make the bottom zero; if it gets cancelled out, it's a hole. If it's there in the end, it's an asymptote!

Other notes

- Factor EVERYTHING – Sometimes you can do it more than once!
- Look for Common factoring
- Take your time!

Questions

1. Simplify: x(2x + 3)(x – 5)(3x + 1)

$x(2x+3)(x-5)(3x+1)$

$(2x^2+3x)(x-5)(3x+1)$

$(2x^3-10x^2+3x^2-15x)(3x+1)$

$(2x^3-7x^2-15x)(3x+1)$

$6x^4+2x^3-21x^3-7x^2-45x^2-15x$

$\boxed{6x^4-19x^3-52x-15}$

2. Factor the following

   a. $8x^3y^2 + 6x^2y - 10x^5y^3$

$8x^3y^2 + 6x^2y - 10x^5y^3$

$\boxed{2x^2y(4xy + 3 - 5x^3y^2)}$

   b. $36x^2 + 78x + 36$

$36x^2 + 78x + 36$ add 13

$6(6x^2 + 13x + 6)$ mult. 36

↳ 9,4

$6(6x^2 + 9x + 4x + 6)$

$6[3x(2x+3) + 2(2x+3)]$

$\boxed{6[(2x+3)(3x+2)}$

   c. $3x^2 + 18x - 480$

$3x^2 + 18x - 480$

$3(x^2 + 6x - 160)$ add 6

mult. -160

$\boxed{3(x+16)(x-10)}$ ↳ 16, -10

   d. $4x^2 - 9$

$4x^2 - 9$

$\boxed{(2x-3)(2x+3)}$

3. Simplify and state restrictions. Indicate asymptotes or holes.

$$\frac{4x^2-9}{6x^2+13x+6}$$

$$\frac{4x^2-9}{6x^2+13x+6}$$

$$\frac{(2x-3)\cancel{(2x+3)}}{(3x+2)\cancel{(2x+3)}}$$

$$\boxed{\frac{2x-3}{3x+2}}$$

Restrictions: $x \neq -\frac{2}{3} \rightarrow$ asymptote

$x \neq -\frac{3}{2} \rightarrow$ hole

4. Simplify and state restrictions

$$\frac{x^2+5x+6}{x^2-16} \div \frac{2x^2+14x+24}{2x^2+3x-20}$$

$$\frac{x^2+5x+6}{x^2-16} \div \frac{2x^2+14x+24}{2x^2+3x-20}$$

$$\frac{x^2+5x+6}{x^2-16} \times \frac{2x^2+3x-20}{2x^2+14x+24}$$

x^2+5x+6
$\hookrightarrow (x+2)(x+3)$

x^2-16
$\hookrightarrow (x-4)(x+4)$

$2x^2+3x-20$
$\hookrightarrow (2x-5)(x+4)$

$2x^2+14x+24$
$\hookrightarrow 2(x+3)(x+4)$

$$\frac{(x+2)\cancel{(x+3)}}{(x-4)\cancel{(x+4)}} \times \frac{(2x-5)\cancel{(x+4)}}{2\cancel{(x+3)}(x+4)}$$

$$\frac{(x+2)(2x-5)}{2(x-4)(x+4)}$$

Restrictions: $x \neq 4 \rightarrow$ asymptote

$x \neq -3 \rightarrow$ hole

$x \neq -4 \rightarrow$ asymptote

5. Simplify and State restrictions

$$\frac{2x+3}{4x^2+2x-6}+\frac{3x^2-7x-6}{2x^2-18}$$

$$\frac{2x+3}{4x^2+2x-6} + \frac{3x^2-7x-6}{2x^2-18}$$

$$\frac{2x+3}{2(2x^2+x+3)} + \frac{3x^2-9x+2x-6}{2(x^2-9)}$$

$$\frac{\cancel{2x+3}}{2\cancel{(2x+3)}(x-1)} + \frac{\cancel{(x-3)}(3x+2)}{2\cancel{(x-3)}(x+3)}$$

Restrictions : $x \neq -\frac{3}{2}, 1, 3, -3$

$$\frac{1}{2(x-1)} \cdot \frac{(x+3)}{(x+3)} + \frac{(3x+2)}{2(x+3)} \cdot \frac{(x-1)}{(x-1)}$$

$$\frac{x+3+(3x+2)(x-1)}{2(x-1)(x+3)}$$

$$\frac{x+3+3x^2-3x+2x-2}{2(x-1)(x+3)}$$

$$\boxed{\frac{3x^2+1}{2(x-1)(x+3)}}$$

Unit II: Functions and Relations

Important Stuff

Functions

Function → It's a function if it doesn't have any repeating x-values

Relation → Relates y to x; can be a function, but isn't necessarily

Vertical Line Test → Draw a vertical line down a graph. Does it hit it twice? If yes, NOT A FUNCTION. If no, FUNCTION!

Domain → All the x values; if x doesn't exist somewhere, SAY THAT!

Range → All the y values; if y doesn't exist somewhere, SAY THAT!

Function Notation → f(x)=, means the function at value x is!

Graphing

Big Five Functions → The functions YOU need to know: line (f(x)=mx+b), quadratic ($f(x)=x^2$), absolute value (f(x)=|x|), square root ($f(x)=\sqrt{x}$) and rational ($f(x)=\frac{1}{x}$)

Transformations → Follow the equation f(x) = a f[k(x-d)] + c; a is vertical stretch/compression and vertical flip, k is horizontal stretch/compression and horizontal flip; d is shift left and right, and c is up or down! Remember k and d are backwards; k means bigger numbers compress and smaller ones stretch, and d goes left when +, and right when –

Mapping → Let's you figure out where points on your graph go; follows

$(\frac{1}{k}x + d, ay + c)$

Inverse

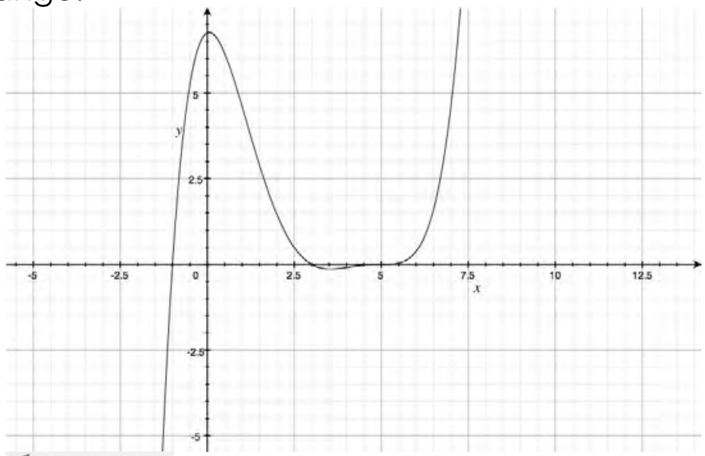
Inverse → what happens if you reflect your equation in the line y=x; to find it, switch your x and y points on your graph (i.e. x becomes y, y becomes x), or switch the place of x and y in your equation and isolate y!

Other Notes

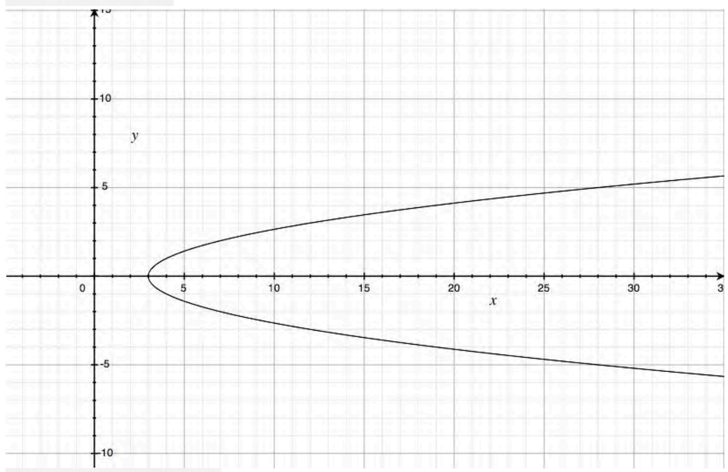
- Remember that k and d ARE OPPOSITE what they seem! a and c work normally!

Questions

6. State which of the following are functions, and give the domain and range.

a. 

Function
D: $\{x \mid x \in \mathbb{R}\}$
R: $\{y \mid y \in \mathbb{R}\}$

b. 

Not a function
D: $\{x \mid x \geq 3, x \in \mathbb{R}\}$
R: $\{y \mid y \in \mathbb{R}\}$

c. {(2,5) , (9, 19) , (-5, 8) , (4, 7)}

Function
D: $\{-5, 2, 4, 9\}$
R: $\{5, 7, 8, 19\}$

d. {(3, 8) , (5, -2) , (3, -7) , (4, -2)}

Not Function
D: $\{3, 4, 5\}$
R: $\{-2, -7, 8\}$

7. Compute the following for $f(x) = 3x^2 + 1$
   a. $f(4)$

$f(4) = 3(4) + 1$

$\boxed{f(4) = 13}$

   b. $f(-3)$

$f(-3) = 3(-3) + 1$

$\boxed{f(-3) = -8}$

   c. $f(2n - 1)$

$f(2n-1) = 3(2n-1)+1$

$f(2n-1) = 6n - 3 + 1$

$\boxed{f(2n-1) = 6n - 2}$

8. Graph the following equation. Find the inverse equation, and graph the inverse on the same axis.

$$f(x) = -2\left[\frac{1}{2}(x-3)\right]^2 - 5$$

Key points

$(x, y) \rightarrow (2x+3, -2y-5)$

$(-2, 4) \rightarrow (-1, -13)$

$(-1, 1) \rightarrow (1, -7)$

$(0, 0) \rightarrow (3, -5)$

$(1, 1) \rightarrow (5, -7)$

$(2, 2) \rightarrow (7, -13)$

$f(x) = -2\left[\frac{1}{2}(x-3)\right]^2 - 5$

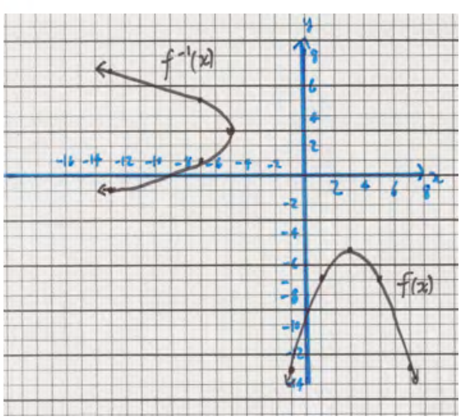
$x = -2\left[\frac{1}{2}(y-3)\right]^2 - 5$

$\frac{x+5}{-2} = \left[\frac{1}{2}(y-3)\right]^2$

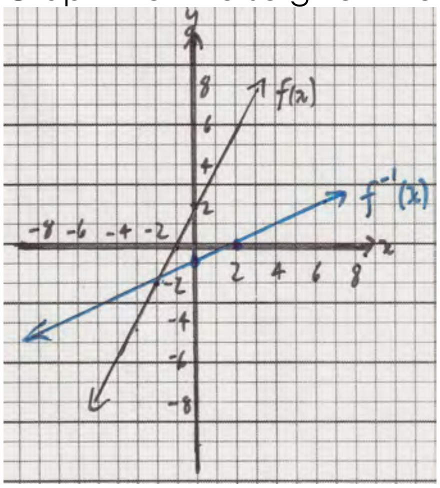
$\pm\sqrt{\frac{x+5}{-2}} = \frac{1}{2}(y-3)$

$2\left(\pm\sqrt{\frac{x+5}{-2}}\right) + 3 = y$

$\boxed{f^{-1}(x) = 2\cdot\left(\pm\sqrt{\frac{x+5}{-2}}\right) + 3}$



9. Graph the inverse given the following graph:



10. Graph the following:

a. $f(x) = -2\sqrt{x+3} + 4$

$f(x) = -2\sqrt{x+3} + 4$

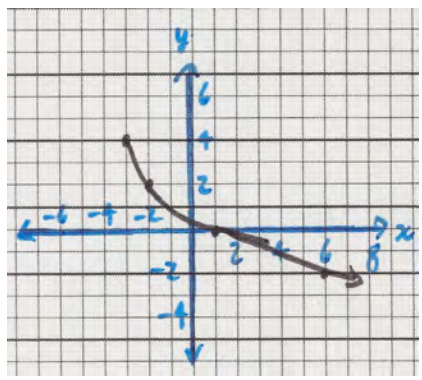
$(x, y) \longrightarrow (x-3, -2y+4)$

$(0, 0) \longrightarrow (-3, 4)$

$(1, 1) \longrightarrow (-2, 2)$

$(4, 2) \longrightarrow (1, 0)$

$(9, 3) \longrightarrow (6, -2)$



b. $f(x) = \frac{1}{2}|4(x-5)| + 5$

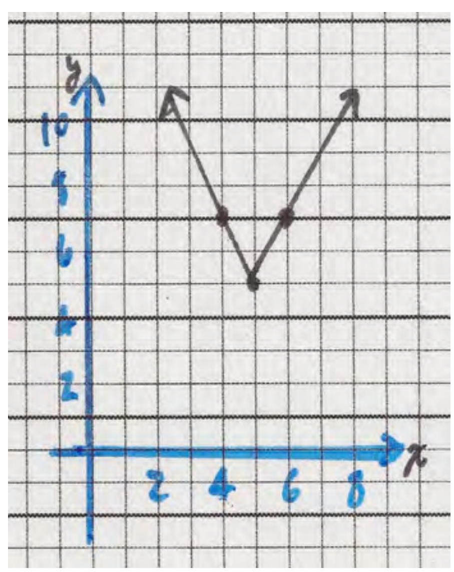
$f(x) = \frac{1}{2}|4(x-5)| + 5$

$(x, y) \longrightarrow (\frac{1}{4}x + 5, \frac{1}{2}y + 5)$

$(-4, 4) \longrightarrow (4, 7)$

$(0, 0) \longrightarrow (5, 5)$

$(4, 4) \longrightarrow (6, 7)$



c. $f(x) = 2\left(\frac{1}{-(x-3)}\right) + 4$

$f(x) = 2\left(\frac{1}{-(x-3)}\right) + 4$

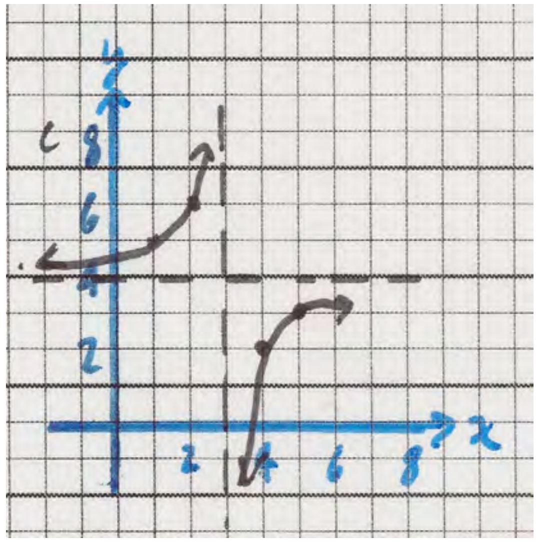
$(x, y) \longrightarrow (-x+3,\ 2y+4)$

$(-2, -\frac{1}{2}) \longrightarrow (5, 3)$

$(-1, -1) \longrightarrow (4, 2)$

$(1, 1) \longrightarrow (2, 6)$

$(2, \frac{1}{2}) \longrightarrow (1, 5)$



Unit III: Quadratic Functions
Important Stuff

Properties
Direction of opening → Opens up if a is positive, down if negative

Vertex → Where the turn happens; top or bottom of the parabola

Max/Min → The top or bottom of the parabola; opens up, it's a min; down, it's a max

Axis of Symmetry → The x value of the vertex; cuts it in half

Inverse → Inverse of the parabola is the square root function; find the equation and graph the same way as last time!

Forms
Standard → $f(x) = ax^2 + bx + c$; tells you stretch/compression and direction of opening

Vertex → $f(x) = a(x-h)^2 + k$; tells you direction of opening, stretch, and (h,k) is your vertex; get to it by Completing the Square

Factored → $f(x) = a(x – s)(x – t)$; direction of opening, stretch, and s and t are the intercepts; get to it by factoring

Solving
Solve → means to find the intercepts; factor and set each bracket equal to zero; if it can't be factored, use quadratic formula

Quadratic formula → It's $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$; plug the numbers in from standard

Discriminant → Part of the quadratic formula under the root; if it's negative, no solutions; 0, one solution; positive, 2 solutions. Find unknown coefficients using it and setting it to zero.

Solving Intersection of line → Isolate y for the line, set it equal to the parabola equation, simplify the polynomial and solve!

Other Notes
- For max/min word problems, make 2 equations, get to one variable and complete the square

Questions

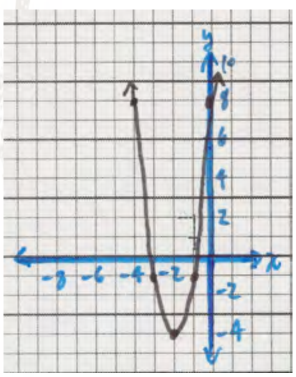
11. Find the inverse of f(x) = 2(x - 4)^2 + 3

$$f(x) = 2(x-4)^2+3$$
$$x = 2(y-4)^2+3$$
$$\frac{x-3}{2} = (y-4)^2$$
$$\pm\sqrt{\frac{x-3}{2}} = y-4$$
$$\boxed{f^{-1}(x) = \pm\sqrt{\frac{x-3}{2}}+4}$$

12. Graph the following parabola: f(x) = -3x^2 +12x + 8

$$f(x) = -3x^2+12x+8$$
$$f(x) = 3(x^2+4x)+8$$
$$f(x) = 3(x^2+4x+4-4)+8$$
$$f(x) = 3(x^2+4x+4)-12+8$$
$$f(x) = 3(x+2)^2-4$$

$(x, y) \longrightarrow (x-2, 3y-4)$
$(0,0) \longrightarrow (-2,-4)$
$(-1,1) \longrightarrow (-3,-1)$
$(-2,4) \longrightarrow (-4,8)$
$(1,1) \longrightarrow (-1,-1)$
$(2,4) \longrightarrow (0,8)$



13. Solve the following:
    a. f(x) = 3x^2 – 3x -60

$$f(x) = 3x^2-3x-60$$
$$f(x) = 3(x^2-x-20)$$
$$f(x) = 3(x-5)(x+4)$$
$$\boxed{x = 5, -4}$$

b. $f(x) = 5x^2 - 17x - 12$

$$f(x) = 5x^2 - 17x - 12$$
$$f(x) = 5x^2 - 20x + 3x - 12$$
$$f(x) = 5x(x-4) + 3(x-4)$$
$$f(x) = (x-4)(5x+3)$$
$$\boxed{x = 4, -\frac{3}{5}}$$

c. $f(x) = 3x^2 - 27$

$$f(x) = 3x^2 - 27$$
$$f(x) = 3(x^2 - 9)$$
$$f(x) = 3(x-3)(x+3)$$
$$\boxed{x = 3, -3}$$

d. $f(x) = -3x^2 + 5x + 7$

$$f(x) = -3x^2 + 5x + 7$$
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-5 \pm \sqrt{5^2 - 4(-3)(7)}}{2(-3)}$$
$$x = \frac{-5 \pm \sqrt{25 + 84}}{-6}$$
$$\boxed{x = \frac{-5 \pm \sqrt{109}}{-6}}$$

14. If $f(x) = 2x^2 + kx - 2$, determine what value of k will give the function only 1 zero.

$$f(x) = 2x^2 + kx + 2$$
$$b^2 - 4ac = 0$$
$$k^2 - 4(2)(2) = 0$$
$$k^2 + 16 = 0$$
$$k^2 = 16$$
$$k = \pm\sqrt{16}$$
$$\boxed{k = \pm 4}$$

15. Vanessa, after starting a most successful tutoring company, decides to buy the Phoenix Coyotes, who had no fans in their previous city, and move them to Toronto, creating a second NHL team in Toronto to compete with the completely incompetent Leafs. Vanessa has a stadium which seats 25,000 fans, and finds that she sells out when she charges $30/seat. She also finds that if she increases the price by $5, she loses 250 fans.
    a. What price should Vanessa charge for admission to a game in order to maximize her revenue?
    b. How much revenue does Vanessa make at this price?

Let t be tickets sold, P be price, and x be the number of price increases.

$t = 25{,}000 - 250x \qquad p = 30 + 5x$

$$R = (25{,}000 - 250x)(30 + 5x)$$
$$R = -1250x^2 + 117{,}500x + 750{,}000$$
$$R = -1250(x^2 - 94x) + 750{,}000$$
$$R = -1250(x^2 - 94x + 2209 - 2209) + 750{,}000$$
$$R = -1250(x^2 - 94x + 2209) - 2209(-1250) + 750{,}000$$
$$R = -1250(x - 47)^2 + 3{,}511{,}250$$

$47 \cdot 5 + 30 = 265$

$\boxed{\therefore \text{ Vanessa should charge \$265, giving her a revenue of \$3,511,250}}$

16. Dexter claims that you can't figure out two consecutive odd numbered integers that have a product of 323. Prove him wrong and find the answer.

Let $2x+1$ be the first integer, and $2x+3$ be the second.

$$(2x+1)(2x+3) = 323$$
$$4x^2 + 6x + 2x + 3 - 323 = 0$$
$$4x^2 + 8x - 320 = 0$$
$$4(x^2 + 2x - 80) = 0$$
$$4(x-8)(x+10) = 0$$
$$x = 8 \text{ or } x = -10$$

$2(8) + 1 = 17$

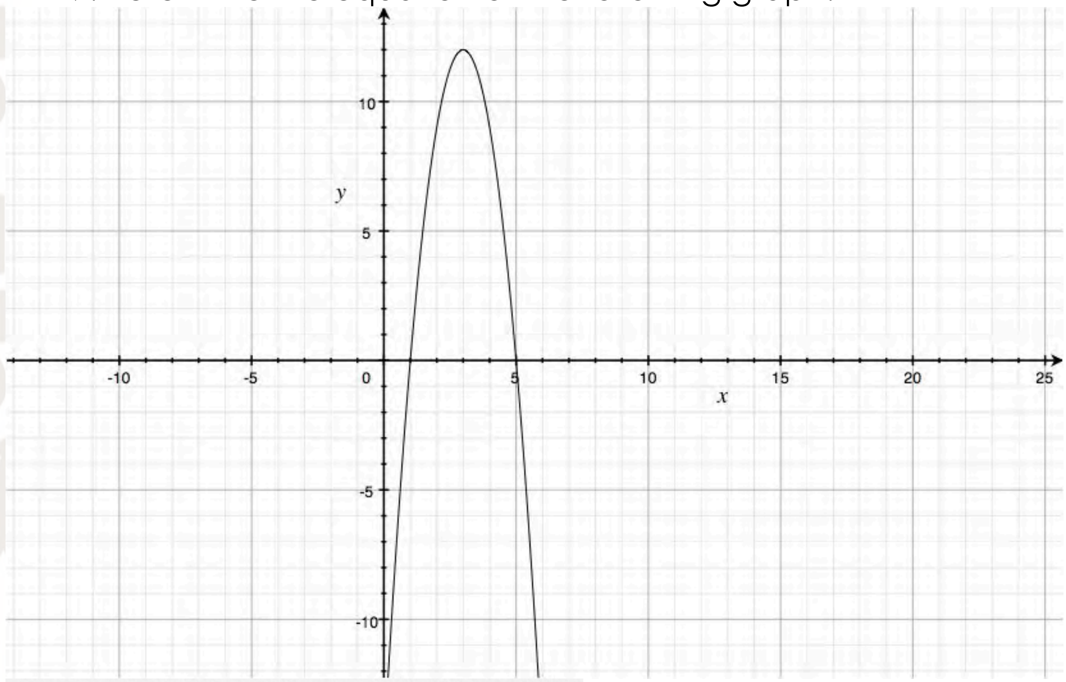
$2(8) + 3 = 19$

$2(-10) + 1 = -19$

$2(-10) + 3 = -17$

$\therefore$ the numbers are 17 and 19 or -17 and -19

17. Determine the equation of the following graph.



Vertex is at $(3,12)$, passes through $(1,0)$

$$f(x) = a(x-3)^2 + 12$$

$$0 = a(1-3)^2 + 12$$

$$-12 = a(-2)^2$$

$$\frac{-12}{4} = \frac{4a}{4}$$

$$a = -3$$

$$\boxed{f(x) = -3(x-3)^2 + 12}$$

Unit IV: Exponential Functions

Important Stuff

Exponent Laws

Product → If it's multiplied, you add; $(x^a)(x^b) = x^{a+b}$

Quotient → If divided, subtract; $\frac{x^a}{x^b} = x^{a-b}$

Multiplication → If brackets, multiply; $(x^a)^b = x^{(a)(b)}$

Reciprocal → If negative, flip it; $\left(\frac{x}{y}\right)^{-a} = \left(\frac{y}{x}\right)^{a}$

Root → If it's a fraction, bottom is a root, top is a power; $x^{\frac{a}{b}} = \sqrt[b]{x^a}$

General Exponential Function

Equation → Looks like $f(x) = a(b)^{k(x-d)} + c$

Asymptote → Where a function can't exist; it has a horizontal one at whatever the vertical shift is

b → It's the base you're working with; if fractional, flips horizontally

Solving

Similar bases → If the numbers can be changed to have similar bases, do that, drop the base, and solve!

Using Exponent Laws → Keep your eyes out for solving using exponent laws. If you see $2^{2x} = 4$, you're going to have to change it to $(2^x)^2 = 4$, and then solve; sometimes, you might have to use quadratic formula

Other Notes

- Mapping equation works the exact same way, just change your points based on the base
- Remember, solving might involve using exponent laws and changing bases! Look for things that they can be broken down into!

Questions

18. Simplify the following to a single positive exponent:

a. $\left(\frac{3x^5 \times 8x^{-4}}{x^3 \times 4x^2}\right)^{-3}$

$$\left(\frac{3x^5 \cdot 8x^{-4}}{x^3 \cdot 4x^2}\right)^{-3}$$

$$\left(\frac{24x^{5-4}}{4x^{3+2}}\right)^{-3}$$

$$\left(\frac{24x^1}{4x^5}\right)^{-3}$$

$$\left(6x^{1-5}\right)^{-3}$$

$$\left(6x^{-4}\right)^{-3}$$

$$6^{-3}x^{12}$$

$$\left(\frac{1}{6}\right)^3 x^{12}$$

$$\boxed{\frac{x^{12}}{216}}$$

b. $\sqrt[3]{\frac{(4^2)(16^{-4})}{(8^5)^{-3}}}$

$$\sqrt[3]{\frac{(4^3)(16^{-4})}{(8^5)^{-3}}}$$

$$\sqrt[3]{\frac{(2^2)^2(2^4)^{-4}}{[(2^3)^5]^{-3}}}$$

$$\sqrt[3]{\frac{2^4 \cdot 2^{-16}}{2^{-45}}}$$

$$\sqrt[3]{\frac{2^{-12}}{2^{-45}}}$$

$$\sqrt[3]{2^{33}}$$

$$(2^{33})^{1/3}$$

$$\boxed{2^{11}}$$

19. Graph $f(x) = -2(\frac{1}{2})^{3(x+2)} + 3$

$f(x) = -2\left(\frac{1}{2}\right)^{3(x+2)} + 3$

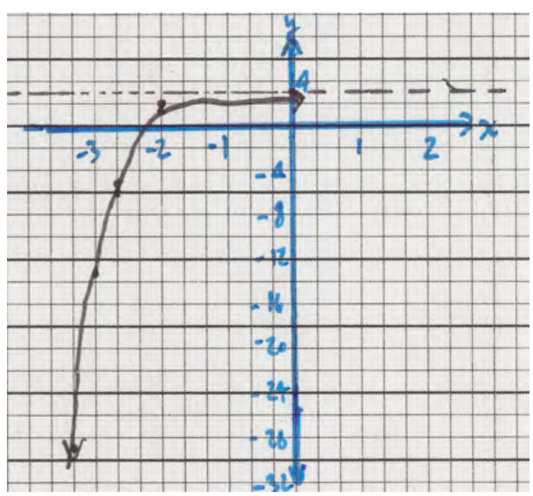
$(x, y) \longrightarrow (\frac{1}{3}x - 2, -2y+3)$

$(-4, 16) \longrightarrow (-3\frac{1}{3}, -29)$

$(-3, 8) \longrightarrow (-3, -13)$

$(-2, 4) \longrightarrow (-2\frac{2}{3}, -5)$

$(0, 1) \longrightarrow (-2, 1)$



20. Solve the following

a. $3^x = 27$

$$3^x = 27$$
$$3^x = 3^3$$
$$\boxed{x = 3}$$

b. $3^{x+3} = 243$

$$3^{x+3} = 243$$
$$(3^x)(3^3) = 243$$
$$\frac{9(3^x)}{9} = \frac{243}{9}$$

c. $2^{2x+3} + 2^x = 9$

$$2^{2x+3} + 2^x = 9$$
$$2^3(2^x)^2 + 2^x - 9 = 0$$
Let $u = 2^x$
$$8u^2 + u - 9 = 0$$
add: 1
mult: -72
$\hookrightarrow 9, -8$
$$8u^2 + 9u - 8u - 9 = 0$$
$$u(8u+9) - (8u+9) = 0$$
$$(8u+9)(u-1) = 0$$
$$u = -\frac{9}{8}, 1$$

$2^x = -\frac{9}{8}$
$\hookrightarrow$ no Solution

$2^x = 1$
$\hookrightarrow \boxed{x = 0}$

21. Geoffrey decides to purchase a limited edition, mint version of the My Little Pony Season 1 Blu-Ray for $40, as it his favourite series. After resisting opening the package to watch his heroes for 10 years, he takes it to a collector to sell it, sure that it would be worth thousands of dollars, but the collector tells him that it's only worth $247.67. Determine the appreciation rate of Geoffrey's My Little Pony Blu-Ray.

$$A = P(r)^t$$
$$\frac{247.67}{40} = \frac{40r^{10}}{40}$$
$$\sqrt[10]{\frac{247.67}{40}} = r$$

$r = 1.2$
$\hookrightarrow 20\%$

$\boxed{\text{∴ the appreciation rate of Geoffrey's Blu-Ray was } \underline{20\%}.}$

22. Dexter is breeding a new type of bacteria, P. *retentiousness*, in an attempt to infect the world and make them all pretentious. If he starts with 20 bacteria, and they double every 15 minutes, how many will he have in 2 days?

For a doubling population

$P(t) = A(2)^t$

$t \to$ # doubling periods

Converting days to minutes

$2 \text{ days} = 48\text{h} \times 60 \frac{\text{min}}{\text{h}} = 2880 \text{ min}$

Doubles every 15 minutes:

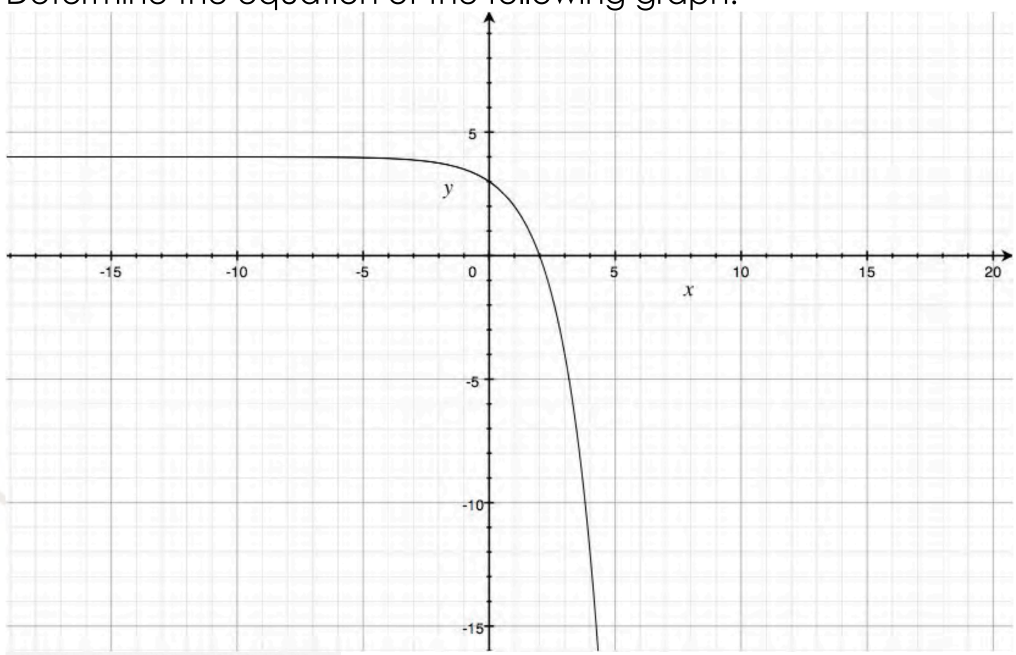
$\frac{2880 \text{ min}}{15 \text{ min}} = 192$ doubling periods

$P(192) = 20(2)^{192}$

$P(192) = 1.26 \times 10^{59}$

$\boxed{\therefore \text{ Dexter will have approximately } 1.26 \times 10^{59} \text{ bacteria.}}$

23. Determine the equation of the following graph.



Asymptote: $y = 4 \to d = 4$

x-intercept: $(2, 0)$

y-intercept: $(0, 3)$

$f(x) = a(b)^{k(x-d)} + c$

increasing by powers of 2 $\to b = 2$

No horizontal stretch or shift

No Vertical stretch, Vert flip

$\boxed{f(x) = -2^x + 3}$

Unit V: Trigonometry I

Important Stuff

Trig Ratios

SOH CAH TOA → Acronym to remember the primary ratios:

Sine → Sin x = opp/hyp
Cosine → Cos x = adj/hyp
Tangent → Tan x = opp/adj

Reciprocal Ratios → The ratios obtained by flipping the primary ratios:

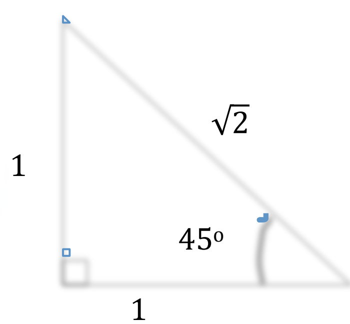
Cosecant → Csc x = 1/sin x = hyp/opp
Secant → Sec x = 1/cos x = hyp/adj
Cotangent → Cot x = 1/tan x = adj/opp

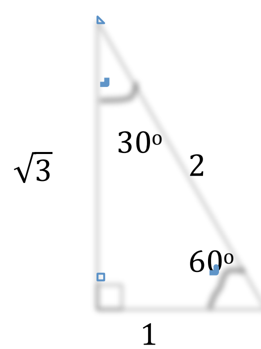
Trig Identities → Changing trig ratios around so that you get the same thing on both sides; use the above as well as: $\sin^2 x + \cos^2 x = 1$

CAST Rule

CAST Rule → The rule which tells when the values of ratios will be positive; they are all positive in quadrant I, Sine is positive in quadrant 2, Tangent in 3, and Cosine in 4

Special Triangles → Triangles which give the exact values for 30°, 45°, and 60°; they are:





Solving Exact Values → Identify the quadrant, find the related angle, and change the sine based one CAST

Solving Angles → Find the related angle, determine what quadrant signs match in, state possible angles

Sine and Cosine Law

Sine Law → Formula $\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$; Used to solve when two side and two opposite angles are present in non-right triangle

Cosine Law → Formula $a^2 = b^2 + c^2 - 2bc\ Cos\ A$; used to solve when three sides and one opposite angle are present in non-right triangle

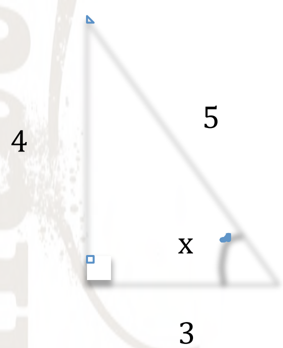
Ambiguous Case of Sine Law → Happens when the opposite side of known acute angle is shorter than the other side that is unknown; yields two results; do 180-1st answer to get second possible angle.

Other Notes

- Remember, angles are always measured to the nearest horizontal on the graph
- Uppercase letters represent angles; lowercase represent opposite sides
- If it asks for an exact answer, give a fraction!

Questions

24. State the primary and reciprocal trigonometric ratios for the following triangle.



$\sin x = \frac{4}{5}$

$\cos x = \frac{3}{5}$

$\tan x = \frac{4}{3}$

$\csc x = \frac{5}{4}$

$\sec x = \frac{5}{3}$

$\cot x = \frac{3}{4}$

25. Evaluate using a calculator. Round to 3 decimal places.
    a. Cos 50°

    $\cos 50° = 0.643$

    b. Sin 42°

    $\sin 42° = 0.669$

    c. Cot 37°

    $\cot 37° = 1.327$

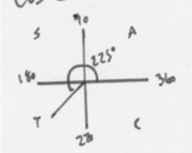
    d. Csc 72°

    $\csc 72° = 1.051$

26. Determine the exact value of the following:

a. Cos 225°

$\cos 225^\circ = ?$



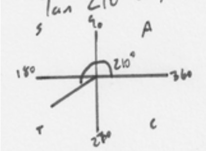
$x_{ref} = 225 - 180 = 45^\circ$

$\cos 45^\circ = \frac{1}{\sqrt{2}}$

$\boxed{\cos 225^\circ = -\frac{1}{\sqrt{2}}}$

b. Tan 210°

$\tan 210^\circ = ?$



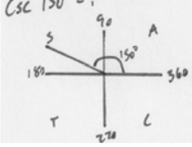
$x_{ref} = 210 - 180 = 30^\circ$

$\tan 30^\circ = \frac{1}{\sqrt{3}}$

$\boxed{\tan 210^\circ = +\frac{1}{\sqrt{3}}}$

c. Csc 150°

$\csc 150^\circ = ?$



$x_{ref} = 180 - 150$

$= 30^\circ$

$\csc 30^\circ = \frac{2}{1}$

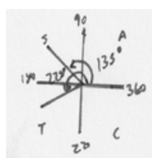
$\boxed{\csc 150^\circ = +2}$

27. Determine all possible angles for {0°<x<360°}

a. Cos x = $-\frac{1}{\sqrt{2}}$

$\cos x = -\frac{1}{\sqrt{2}}$

$x_{ref} = 45^\circ$

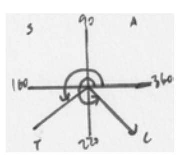


$\therefore x = 135^\circ$ or 225°

b. Sin x = -0.6691

$\sin x = -0.6691$

$x_{ref} = 42^\circ$



$180 + 42 = 222^\circ$

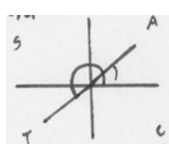
$360 - 42 = 318^\circ$

$\boxed{\therefore x = 222^\circ \text{ or } 318^\circ}$

c. Cot x = $\sqrt{3}$

$\cot x = \sqrt{3}$

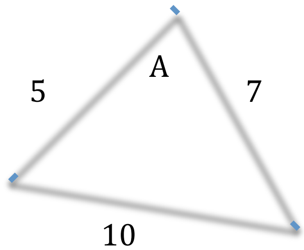
$x_{ref} = 30^\circ$



$180 + 30^\circ = 210^\circ$

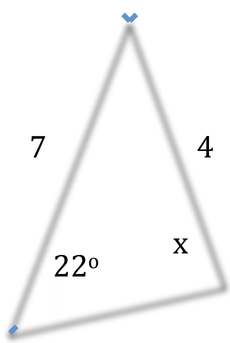
$\therefore x = 30^\circ$ or 210°

28. Determine the value of A



$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$10^2 = 5^2 + 7^2 - 2(5)(7)\cos A$$
$$\frac{10^2 - 5^2 - 7^2}{-2(5)(7)} = \cos A$$
$$A = \cos^{-1}\left(\frac{10^2 - 5^2 - 7^2}{-2(5)(7)}\right)$$
$$\boxed{A = 111.8^\circ}$$

29. State all possible values for x.



$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
$$\frac{4}{\sin 22} = \frac{7}{\sin x}$$
$$4\sin x = 7\cdot \sin 22$$
$$x = \sin^{-1}\left(\frac{7\cdot \sin 22}{4}\right)$$
$$x = 41^\circ$$

Since $a < b$, ambiguous.

$$180 - 41^\circ = 139^\circ$$
$$\boxed{\therefore\ x = 41^\circ \text{ or } 139^\circ}$$

30. Prove the following

a.

$$Sin\,x\,Tan\,x + Sec\,x = \frac{Sin^2x + 1}{Cos\,X}$$

$$\text{Sin}\,x\,\text{Tan}\,x + \text{Sec}\,x = \frac{\text{Sin}^2 x + 1}{\text{Cos}\,x}$$

L.S.

$$\text{Sin}\,x \cdot \frac{\text{Sin}\,x}{\text{Cos}\,x} + \frac{1}{\text{Cos}\,x} =$$

$$\frac{\text{Sin}^2 x + 1}{\text{Cos}\,x} = \frac{\text{Sin}^2 x + 1}{\text{Cos}\,x}$$

L.S. = R.S.

Q.E.D.

b.

$$\frac{Cos\,x}{1 + Sin\,x} + \frac{Cos\,x}{1 - Sin\,x} = 2\,Sec\,x$$

$$\frac{\text{Cos}\,x}{1+\text{Sin}\,x} + \frac{\text{Cos}\,x}{1-\text{Sin}\,x} = 2\,\text{Sec}\,x$$

L.S.

$$\frac{(1-\text{Sin}\,x)}{(1-\text{Sin}\,x)} \cdot \frac{\text{Cos}\,x}{1+\text{Sin}\,x} + \frac{\text{Cos}\,x}{1-\text{Sin}\,x} \cdot \frac{1+\text{Sin}\,x}{1+\text{Sin}\,x} =$$

$$\frac{\text{Cos}\,x - \text{Sin}\,x\,\text{Cos}\,x + \text{Cos}\,x + \text{Sin}\,x\,\text{Cos}\,x}{(1-\text{Sin}\,x)(1+\text{Sin}\,x)}$$

$$\frac{2\,\text{Cos}\,x}{1-\text{Sin}^2 x} =$$

$$\frac{2\,\cancel{\text{Cos}\,x}}{\text{Cos}^{\cancel{2}} x} =$$

$$2 \cdot \frac{1}{\text{Cos}\,x} =$$

$$2\,\text{Sec}\,x = 2\,\text{Sec}\,x$$

L.S. = R.S.

Q.E.D.

Unit VI: Trigonometry II

Important Stuff

Periodic Function

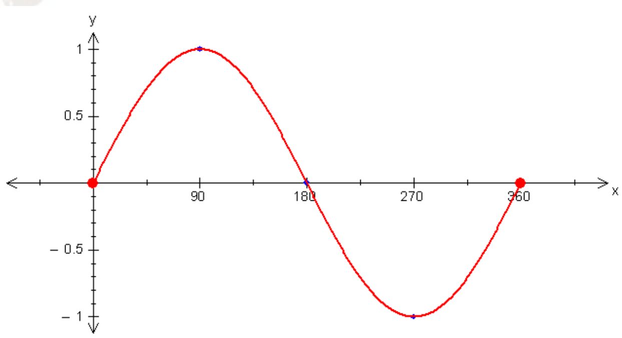
Periodic Function → Function that repeats itself over time

Period → How long it takes a periodic function to repeat

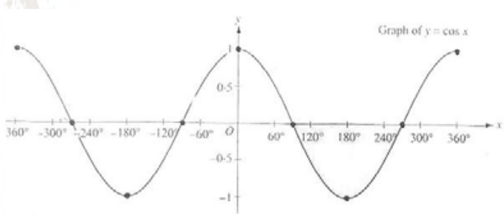
Amplitude → The half of the maximum or minimum height the function reaches; $(y_{max} – y_{min})/2$

Sine and Cosine

Sine function → Has general formula f(x) = a sin[k(x-d)] + c; works same way as regular mapping formula; period is 360°; looks like:



Cosine function → Has general formula f(x) = a cos[k(x-d)] + c; works same as regular mapping formula; period is 360°; looks like:



k → Gives value of period by doing 360/k

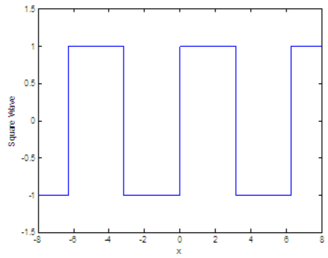
Application Problems → Questions which are modeled by Sine or cosine functions; the change in height/width represents amplitude, time is the period; starting point and amount up/down/left/right will change vertical and horizontal shift

Other Notes

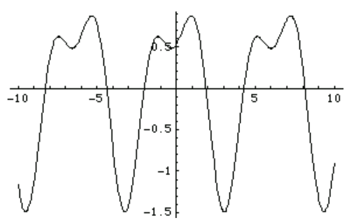
- Sine starts and 0 and goes up, Cos starts at 1 and goes down; they look the same other than where they start!

Questions

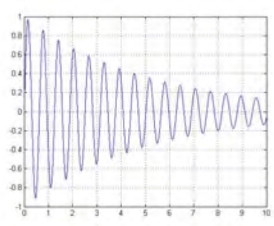
31. State whether the following are periodic

a. 

Periodic

b. 

Periodic

c. 

Not Periodic

32. Graph f(x) = 4 Sin[90(x+2)] + 3

$(x, y) \rightarrow (\frac{1}{90}x - 2, 4y + 3)$

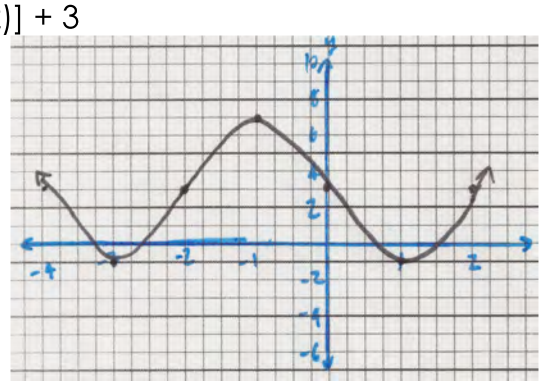
$(0, 0) \rightarrow (-2, 3)$

$(90, 1) \rightarrow (-1, 7)$

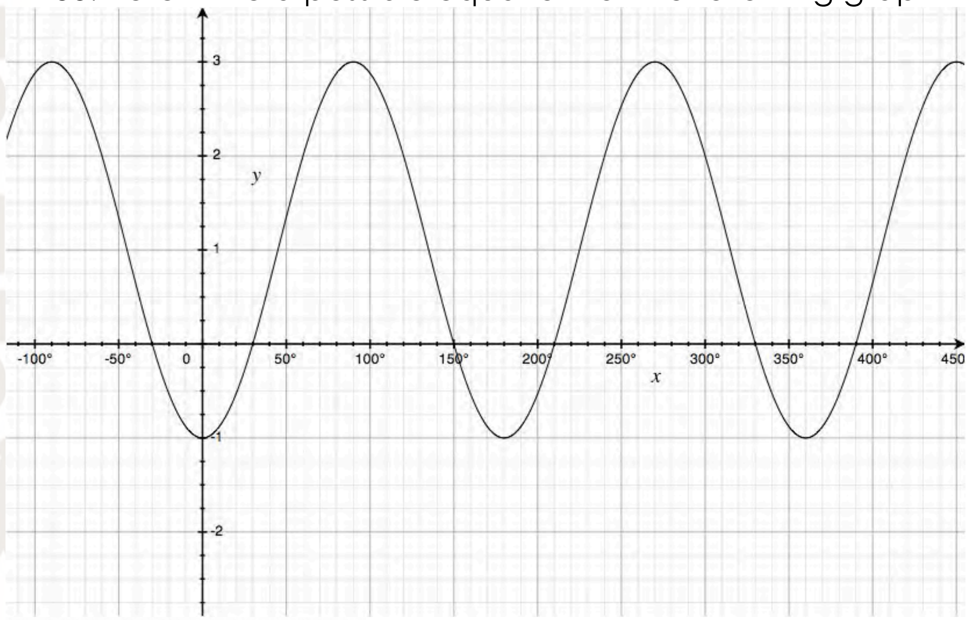
$(180, 0) \rightarrow (0, 3)$

$(270, -1) \rightarrow (1, -1)$

$(360, 0) \rightarrow (2, 3)$



33. Determine a possible equation for the following graph



Max = 3 Min = −1

$\frac{3-(-1)}{2} = 2 \rightarrow$ amplitude

$\frac{3+(-1)}{2} = 1 \rightarrow$ axis

Period : 180 → k = 2

− Cosine

$\boxed{f(x) = -2\cos(2x) + 1}$

34. At Nicole's Wonky Wave-pool, a water attraction at the newest and most awesome theme park, Math Guru Land, the waves are said to be over 5m tall! They occur at 15 second intervals, and the water has a standing height of 1.2m. Determine the equation of the line, and draw the graph which represents the motion of the wave-pool assuming that you start in between two waves (i.e. when no wave is present).

Period = 15 Seconds

$k = \frac{360}{15} = 24$

amplitude = $\frac{5}{2} = 2.5$

$c = 1.2 + 2.5 = 3.7$

− Cosine

$\boxed{h(t) = -2.5\cos(24t) + 3.7}$

Unit VII: Sequences and Series

Important Stuff

Sequences

Sequences → Set of numbers that follow an equation based on their order in number set

Arithmetic Sequence → Numbers that change by the same amount; general formula is $t_n = a + (n - 1)d$; is your start, d is your difference, n is the position in sequence, and t_n is the value!

Geometric Sequence → Set of numbers changing by the same ratio; general formula $t_n = a\, r^{(n-1)}$; t_n, n and a are the same, r represents the ratio

Series

Series → The sum of a sequence

Arithmetic Series → Has the formula $s_n = \frac{n}{2}[2a + (n-1)d]$; determine number of terms being summed, and provides answer

Geometric Series → Has formula $S_n = \frac{a(r^n - 1)}{r-1}$; determine number of terms being summed, and provides answer

Pascal's Triangle

Pascal's Triangle → Mathematical tool created by Pascal in which the numbers of rows are the sum of the numbers above; looks like:

$$
\begin{array}{ccccccccccc}
 & & & & & 1 & & & & & \\
 & & & & 1 & & 1 & & & & \\
 & & & 1 & & 2 & & 1 & & & \\
 & & 1 & & 3 & & 3 & & 1 & & \\
 & 1 & & 4 & & 6 & & 4 & & 1 & \\
1 & & 5 & & 10 & & 10 & & 5 & & 1
\end{array}
$$

Binomial Expansion → How expand a binomial to a specific power; the power corresponds to the row on the triangle, which provides coefficients; raise the first number to the highest power and go down each term, where as the second number is raised to zero and goes up; i.e.

$$(a + b)^3 = 1a^3b^0 + 3a^2b^1 + 3a^1b^2 + 1a^0b^3$$

Other Notes

- t_n IS THE ACTUAL VALUE OF THE SEQUENCE AT POSITION n; n represents what position we're at!

Questions

35. Determine the general term of the arithmetic sequence with $t_5 = 74$, and $t_9 = 46$. If the final term is -45, determine the number of terms in the sequence.

$t_9 = 46 \quad t_5 = 74$

$t_n = a + (n-1)d$

$46 = a + (9-1)d$

$46 = a + 8d$

$46 - 8d = a$

$74 = a + (5-1)d$

$74 = 46 - 8d + 4d$

$74 - 46 = -4d$

$\frac{28}{-4} = d$

$d = -7$

$a = 46 - 8(-7)$

$a = 102$

$t_n = 102 + (n-1)(-7)$

$t_n = 102 - 7n + 7$

$t_n = 109 - 7n$

$-45 = 109 - 7n$

$\frac{-45 - 109}{-7} = n$

$n = 22$

$\boxed{\therefore\ t_n = 109 - 7n \text{ and there are 22 terms}}$

36. Find the general term of the following sequence: 3, 4.5, 6.75, 10.125,…

Geometric Sequence

$r = \frac{4.5}{3}$

$r = 1.5$

$\boxed{t_n = 3 \cdot 1.5^{n-1}}$

37. Find the sum of the series 7+14 + 28 + 56 + … + 1792

$r = \frac{14}{7} = 2$

$t_n = 7 \cdot 2^{n-1}$

$\frac{1792}{7} = 2^{n-1}$

$256 = 2^{n-1}$

$2^8 = 2^{n-1}$

$8 = n - 1$

$9 = n$

$S_9 = \frac{a(r^n - 1)}{r - 1}$

$S_9 = \frac{7(2^9 - 1)}{2 - 1}$

$\boxed{S_9 = 3577}$

38. Expand the following using Pascal's Triangle: $(2x-4)^6$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1

$$1(2x)^6(-4)^0 + 6(2x)^5(-4)^1 + 15(2x)^4(-4)^2 + 20(2x)^3(-4)^3 + 15(2x)^2(-4)^4 + 6(2x)^1(-4)^5 + 1(2x)^0(-4)^6$$

$$\boxed{64x^6 - 768x^5 + 3840x^4 - 10{,}240x^3 + 15{,}360x^2 - 12{,}288x + 4096}$$

39. Lindsay has gone crazy after making a pizza, and starts cutting the pizza repeatedly. She cuts the pizza in half once, and then cuts those halves in halves, and continues this 10 times. How many slices has Lindsay cut her pizza into?

$a = 1$

$r = 2$

$n = 10$

$t_{10} = 1 \cdot (2)^{10}$

$t_{10} = 1024$

$\boxed{\therefore \text{ She has cut } \underline{1024} \text{ slices}}$

Unit VIII: Financial Applications

Important Stuff

Simple and Compound Interest

Interest → A percentage of money from a total amount that is given on top of that amount

Simple Interest → When interest is accumulated only on the initial amount (called the principal), it is simple interest; the equation is A=P+Prt, where A is the total amount, P is the principle, r is the rate, and t is the time.

Compound Interest → This is when interest is taken based on the principle amount along with any interest which has been earned; the equation is $A=P(1+i)^n$; if there are multiple compounding periods, divide the interest rate by the number of compounding periods, and multiply the time by the number of compounding periods (it would look like $A = P\left(1+\frac{i}{m}\right)^{nm}$, where m is the number of compounding periods)

Present Value → How much you need to invest now to be able to have some amount in the future; equation is PV = $\frac{FV}{\left(1+\frac{i}{m}\right)^{nm}}$, where PV is present value and FV is Future value

Annuities

Annuity → Where there is a certain amount being paid in or withdrawn from an investment for an amount of time at a particular interest rate; the equation for it is $A = \frac{R[\left(1+\frac{i}{m}\right)^{n\,m}-1]}{\frac{i}{m}}$; R is the amount which is being deposited.

Present Value of Annuity → This determines the amount of money that needs to be invested in order to be able to take out a certain amount for a time; the equation is $PV = \frac{R[1-\left(1+\frac{i}{m}\right)^{-nm}]}{\frac{i}{m}}$; works same as other equations

Other Notes

- Don't forget to change your rates based on the compounding periods!

Questions

40. Determine the amount of money made on a $1000 investment for 6 years at a simple interest rate of 3.4% annually.

$P = 1000$

$r = 0.034$

$t = 6$

$A = 1000 + 1000(0.034)(6)$

$\boxed{A = \$1204}$

41. Dexter sold his soul for $2.71 and a sandwich. He invests the money for 50 years at 5.8% compounded daily. How much money does his soul make?

$P = \$2.71$

$i = 0.058$

$n = 50$

$m = 365$

$A = 2.71\left(1 + \frac{0.058}{365}\right)^{50 \cdot 365}$

$\boxed{A = \$49.24}$

42. Joel, being frugal and paranoid, deposits $750 a month into an account guarded by a robot and an Italian plumber wearing a red shirt and blue overalls. If the amount in his account has an interest rate of 3.5% annually, compounded monthly, how much will Joel make in 7 years?

$R = 750$

$i = 0.035$

$n = 7$

$m = 12$

$A = \dfrac{750\left[\left(1 + \frac{0.035}{12}\right)^{7 \cdot 12} - 1\right]}{0.035/12}$

$\boxed{A = \$71,271.20}$

43. If Vanessa wants to withdraw $200 from an account every month for the next 5 years so that she can buy tea, and she has an annual interest rate of 5.7% compounded monthly, how much should she put into the account now?

$R = 200$

$n = 5$

$i = 0.057$

$m = 12$

$$PV = \frac{200\left[1 - \left(1 + \frac{0.057}{12}\right)^{-5 \cdot 12}\right]}{0.057/12}$$

$$\boxed{PV = \$10,420.13}$$