Polynomials

A polynomial is an algebraic expression with real coefficients and non-negative integer exponents.

A polynomial with 1 term is called a monomial, $7x$.
A polynomial with 2 terms is called a binomial, $3x^2 - 9$.
A polynomial with 3 terms is called a trinomial, $3x^2 + 7x - 9$.

The degree of the polynomial is determined by the value of the highest exponent of the variable in the polynomial.

e.g. $3x^2 + 7x - 9$, degree is 2.

For polynomials with one variable, if the degree is 0, then it is called a constant.
If the degree is 1, then it is called linear.
If the degree is 2, then it is called quadratic.
If the degree is 3, then it is called cubic.

We can add and subtract polynomials by collecting like terms.
e.g. Simplify.

$$
\begin{aligned}
&(5x^4 - x^2 - 2) - (x^4 - 2x^3 + 3x^2 - 5) \\
&= 5x^4 - x^2 - 2 - x^4 + 2x^3 - 3x^2 + 5 \\
&= 5x^4 - x^4 + 2x^3 - x^2 - 3x^2 - 2 + 5 \\
&= 4x^4 + 2x^3 - 4x^2 + 3
\end{aligned}
$$

The negative in front of the brackets applies to every term inside the brackets. That is, you multiply each term by –1.

To multiply polynomials, multiply each term in the first polynomial by each term in the second.

e.g. Expand and simplify.

$$
\begin{aligned}
&(x^2 + 4)(x^2 - 2x + 3) \\
&= x^4 - 2x^3 + 3x^2 + 4x^2 - 8x + 12 \\
&= x^4 - 2x^3 + 7x^2 - 8x + 12
\end{aligned}
$$

Factoring Polynomials

To **expand** means to write a product of polynomials as a sum or a difference of terms.
To **factor** means to write a sum or a difference of terms as a product of polynomials.

Factoring is the inverse operation of expanding.

Expanding →

$$(2x + 3)(3x - 7) = 6x^2 - 5x - 21$$

← Factoring

Product of polynomials

Sum or difference of terms

Types of factoring:

Common Factors: factors that are common among each term.
e.g. Factor,

$$35m^3n^3-21m^2n^2+56m^2n$$
$$=7m^2n\left(5mn^2-3n+8\right)$$

Each term is divisible by $7m^2n$.

Factor by grouping: group terms to help in the factoring process.
e.g. Factor,

$$A: 4mx+ny-4nx-my$$
$$=4mx-4nx+ny-my$$
$$=4x(m-n)+y(n-m)$$
$$=4x(m-n)-y(m-n)$$
$$=(4x-y)(m-n)$$

Group $4mx - 4nx$ and $ny - my$, factor each group

Recall $n - m = -(m - n)$

Common factor

$$B: 1+6x+9x^2-4y^2$$
$$=(1+3x)^2-4y^2$$
$$=[(1+3x)+2y][(1+3x)-2y]$$
$$=(1+3x+2y)(1+3x-2y)$$

$1+6x+9x^2$ is a perfect square trinomial

Difference of squares

Factoring ax^2+bx+c
Find the product of *ac*. Find two numbers that multiply to *ac* and add to *b*.
e.g. Factor,

$$A: y^2+9y+14$$
$$=y^2+7y+2y+14$$
$$=y(y+7)+2(y+7)$$
$$=(y+2)(y+7)$$

Product = 14 = 2(7)
Sum = 9 = 2 + 7

$$B: 3x^2-7xy-6y^2$$
$$=3x^2-9xy+2xy-6y^2$$
$$=3x(x-3y)+2y(x-3y)$$
$$=(3x+2y)(x-3y)$$

Product = 3(−6) = −18 = −9(2)
Sum = −7 = −9 + 2
Decompose middle term $-7xy$ into $-9xy + 2xy$.
Factor by grouping.

Sometimes polynomials can be factored using **special patterns**.

Perfect square trinomial $a^2+2ab+b^2=(a+b)(a+b)$ or $a^2-2ab+b^2=(a-b)(a-b)$
e.g. Factor,

$$A: 4p^2+12p+9$$
$$=(2p+3)^2$$

$$B: 100x^2-80xy+16y^2$$
$$=4(25x^2-20xy+4y^2)$$
$$=4(5x-2y)(5x-2y)$$

Difference of squares $a^2-b^2=(a+b)(a-b)$
e.g. Factor, $9x^2-4y^2=(3x+2y)(3x-2y)$

Things to think about when factoring:

- Is there a common factor?
- Can I factor by grouping?
- Are there any special patterns?
- Check, can I factor x^2+bx+c?
- Check, can I factor ax^2+bx+c?

Rational Expressions

For polynomials F and G, a rational expression is formed when $\frac{F}{G}$, $G \neq 0$.

e.g. $\frac{3x+7}{21x^2+14x+9}$

Simplifying Rational Expressions

e.g. Simplify and state the restrictions.

$$\frac{m^2-9}{m^2+6m+9}=\frac{(m+3)(m-3)}{(m+3)(m+3)}$$

Factor the numerator and denominator.
Note the restrictions. $m \neq -3$

$$=\frac{\cancel{(m+3)}(m-3)}{\cancel{(m+3)}(m+3)}$$

Simplify.

$$=\frac{m-3}{m+3},\ m \neq -3$$

State the restrictions.

Multiplying and Dividing Rational Expressions

e.g. Simplify and state the restrictions.

$$A: \frac{x^2+7x}{x^2-1}\times\frac{x^2+3x+2}{x^2+14x+49}$$

$$=\frac{x(x+7)}{(x+1)(x-1)}\times\frac{(x+1)(x+2)}{(x+7)(x+7)}$$

Factor.
Note restrictions.

$$=\frac{x\cancel{(x+7)}}{\cancel{(x+1)}(x-1)}\times\frac{\cancel{(x+1)}(x+2)}{\cancel{(x+7)}(x+7)}$$

Simplify.

$$=\frac{x(x+2)}{(x-1)(x+7)},\ x \neq \pm 1, -7$$

State restrictions.

$$B: \frac{x^2-9}{x^2+5x+4}\div\frac{x^2-4x+3}{x^2+5x+4}$$

$$=\frac{(x+3)(x-3)}{(x+4)(x+1)}\div\frac{(x-1)(x-3)}{(x+4)(x+1)}$$

Factor.
Note restrictions.

$$=\frac{(x+3)(x-3)}{(x+4)(x+1)}\times\frac{(x+4)(x+1)}{(x-1)(x-3)}$$

Invert and multiply.
Note any new restrictions.

$$=\frac{(x+3)\cancel{(x-3)}}{\cancel{(x+4)(x+1)}}\times\frac{\cancel{(x+4)(x+1)}}{(x-1)\cancel{(x-3)}}$$

Simplify.

$$=\frac{(x+3)}{(x-1)},\ x \neq -4, \pm 1, 3$$

State restrictions.

Adding and Subtracting Rational Expressions

e.g. Simplify and state the restrictions.

$$A: \frac{3}{x^2-4}+\frac{5}{x+2}$$

Factor.
Note restrictions.
Simplify if possible.

$$=\frac{3}{(x-2)(x+2)}+\frac{5}{x+2}$$

$$=\frac{3}{(x-2)(x+2)}+\frac{5(x-2)}{(x+2)(x-2)}$$

Find LCD.
Write all terms using LCD.

$$=\frac{3+5x-10}{(x+2)(x-2)}$$

Add.

$$=\frac{5x-7}{(x+2)(x-2)},\ x \neq \pm 2$$

State restrictions.

$$B: \frac{2}{x^2-xy}-\frac{3}{xy-y^2}$$

Factor.
Note restrictions.
Simplify if possible.

$$=\frac{2}{x(x-y)}-\frac{3}{y(x-y)}$$

Find LCD.
Write all terms using LCD.

$$=\frac{2y}{xy(x-y)}-\frac{3x}{xy(x-y)}$$

$$=\frac{2y-3x}{xy(x-y)},\ x \neq 0, y,\ y \neq 0$$

Subtract.
State restrictions.

Note that after addition or subtraction it may be possible to factor the numerator and simplify the expression further. Always reduce the answer to lowest terms.

Radicals

e.g. $\sqrt[n]{a}$, $\sqrt{}$ is called the radical sign, n is the index of the radical, and a is called the radicand.
$\sqrt{3}$ is said to be a radical of order 2. $\sqrt[3]{8}$ is a radical of order 3.

Like radicals: $\sqrt{5}, 2\sqrt{5}, -3\sqrt{5}$

Same order, like radicands

Unlike radicals: $\sqrt{5}, \sqrt[3]{5}, \sqrt{3}$

Different order

Different radicands

Entire radicals: $\sqrt{8}, \sqrt{16}, \sqrt{29}$

Mixed radicals: $4\sqrt{2}, 2\sqrt{3}, 5\sqrt{7}$

A radical in **simplest form** meets the following conditions:

For a radical of order n, the radicand has no factor that is the nth power of an integer.

Not simplest form

$$\sqrt{8} = \sqrt{4\times 2} = \sqrt{2^2\times 2} = 2\sqrt{2}$$

Simplest form

The radicand contains no fractions.

$$\sqrt{\frac{3}{2}} = \sqrt{\frac{3}{2}\times\frac{2}{2}} = \sqrt{\frac{6}{2^2}} = \frac{\sqrt{6}}{\sqrt{2^2}} = \frac{\sqrt{6}}{2}$$

Simplest form

The radicand contains no factors with negative exponents.

$$\sqrt{a^{-1}} = \sqrt{\frac{1}{a}} = \sqrt{\frac{1}{a}\times\frac{a}{a}} = \sqrt{\frac{a}{a^2}} = \frac{\sqrt{a}}{a}$$

Simplest form

The index of a radical must be as small as possible.

$$\sqrt[4]{3^2} = \sqrt{\sqrt{3^2}} = \sqrt{3}$$

Simplest form

Addition and Subtraction of Radicals

To add or subtract radicals, you add or subtract the coefficients of each radical.

e.g. Simplify.

$$\begin{aligned}2\sqrt{12}-5\sqrt{27}+3\sqrt{40} &= 2\sqrt{4\times 3}-5\sqrt{9\times 3}+3\sqrt{4\times 10}\\ &= 2\left(2\sqrt{3}\right)-5\left(3\sqrt{3}\right)+3\left(2\sqrt{10}\right)\\ &= 4\sqrt{3}-15\sqrt{3}+6\sqrt{10}\\ &= -11\sqrt{3}+6\sqrt{10}\end{aligned}$$

Express each radical in simplest form.

Collect like radicals. Add and subtract.

Multiplying Radicals

$$\boxed{\sqrt{a}\times\sqrt{b}=\sqrt{ab},\ a\geq 0,\ b\geq 0}$$

e.g. Simplify.

$$\begin{aligned}\left(\sqrt{2}+2\sqrt{3}\right)\left(\sqrt{2}-3\sqrt{3}\right) &= \left(\sqrt{2}\right)\left(\sqrt{2}\right)-\left(\sqrt{2}\right)\left(3\sqrt{3}\right)+\left(2\sqrt{3}\right)\left(\sqrt{2}\right)-\left(2\sqrt{3}\right)\left(3\sqrt{3}\right)\\ &= 2-3\sqrt{6}+2\sqrt{6}-6(3)\\ &= 2-18-3\sqrt{6}+2\sqrt{6}\\ &= -16-\sqrt{6}\end{aligned}$$

Use the distributive property to expand

Multiply coefficients together. Multiply radicands together.

Collect like terms. Express in simplest form.

Conjugates

Opposite signs

$(a\sqrt{b} + c\sqrt{d})$ and $(a\sqrt{b} - c\sqrt{d})$ are called conjugates.

Same terms Same terms

When conjugates are multiplied the result is a rational expression (no radicals).
e.g. Find the product.

$$\begin{aligned}(\sqrt{5}+3\sqrt{2})(\sqrt{5}-3\sqrt{2}) &= (\sqrt{5})^2-(3\sqrt{2})^2\\ &= 5-9(2)\\ &= 5-18\\ &= -13\end{aligned}$$

Dividing Radicals

$$\boxed{\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}},\ a,b \in \mathbf{R},\ a \geq 0, b \geq 0}$$

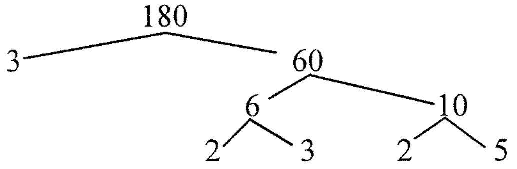
e.g. Simplify.

$$\begin{aligned}\frac{2\sqrt{10}+3\sqrt{30}}{\sqrt{5}} &= \frac{2\sqrt{10}}{\sqrt{5}}+\frac{3\sqrt{30}}{\sqrt{5}}\\ &= 2\sqrt{\frac{10}{5}}+3\sqrt{\frac{30}{5}}\\ &= 2\sqrt{2}+3\sqrt{6}\end{aligned}$$

Prime Factorization

Factor a number into its prime factors using the tree diagram method.

e.g.



$180 = (2^2)(3^2)(5)$

Exponent Rules

Rule	Description	Example
Product	$a^m \times a^n = a^{m+n}$	$4^2 \times 4^5 = 4^7$
Quotient	$a^m \div a^n = a^{m-n}$	$5^4 \div 5^2 = 5^2$
Power of a power	$(a^m)^n = a^{m \times n}$	$(3^2)^4 = 3^8$
Power of a quotient	$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}, b \neq 0$	$\left(\frac{3}{4}\right)^5 = \frac{3^5}{4^5}$
Zero as an exponent	$a^0 = 1$	$7^0 = 1$
Negative exponents	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$9^{-2} = \frac{1}{9^2}$
Rational Exponents	$a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$	$27^{\frac{4}{3}} = \sqrt[3]{27^4} = (\sqrt[3]{27})^4$

e.g. Evaluate.

$$\begin{aligned}(3^0+3^2)^{-2} &= (1+9)^{-2}\\ &= 10^{-2}\\ &= \frac{1}{10^2}\\ &= \frac{1}{100}\end{aligned}$$

Follow the order of operations. Evaluate brackets first.

e.g. Simplify.

$$\begin{aligned}\left(\frac{b^3}{2a^{-3}}\right)^{-2} &= \frac{b^{3(-2)}}{(2a^{-3})^{-2}}\\ &= \frac{b^{-6}}{2^{-2}a^{-3(-2)}}\\ &= \frac{2^2b^{-6}}{a^6}\\ &= \frac{4}{a^6b^6}\end{aligned}$$

Power of a quotient.

Power of a product.

Solving Exponential Equations

e.g. Solve for x.

$$\begin{aligned}9^{x-2}-8&=73\\ 9^{x-2}&=73+8\\ 9^{x-2}&=81\\ 9^{x-2}&=9^2\end{aligned}$$

Add 8 to both sides. Simplify.

Note LS and RS are powers of 9, so rewrite them as powers using the same base.

$$\begin{aligned}x-2&=2\\ x&=2+2\\ x&=4\end{aligned}$$

When the bases are the same, equate the exponents. Solve for x.

Don't forget to check your solution!

$$\begin{aligned}LS &= 9^{x-2}-8 \qquad RS = 73\\ &= 9^{4-2}-8\\ &= 81-8\\ &= 73 = RS \qquad x = 4 \text{ checks}\end{aligned}$$

Functions

A **relation** is a relationship between two sets. Relations can be described using:

an equation

$y=3x^2-7$

in words

"output is three more than input"

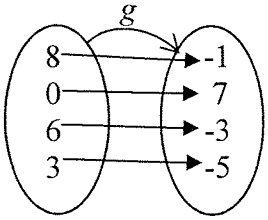
a set of ordered pairs

$\{(1,2),(0,3),(4,8)\}$

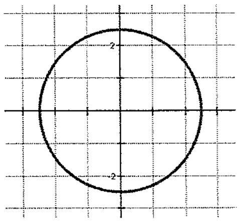
function notation

$f(x)=x^2-3x$

an arrow diagram



a graph



a table

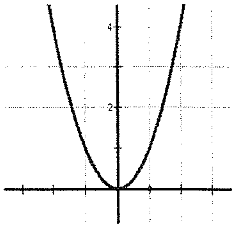
x	y
1	2
2	3
3	4
4	3

The **domain** of a relation is the set of possible input values (x values).
The **range** is the set of possible output values (y values).

e.g. State the domain and range.

A: $\{(1,2),(0,3),(4,8)\}$
Domain = $\{0, 1, 4\}$
Range = $\{2, 3, 8\}$

B:

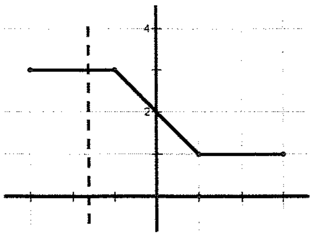


C: $y=\sqrt{x-5}$
What value of x will make $x-5=0$? $x=5$
The radicand cannot be less than zero, so
Domain = $\{x \in \Re \mid x \geq 5\}$
Range = $\{y \in \Re \mid y \geq 0\}$

A **function** is a special type of relation in which every element of the domain corresponds to exactly one element of the range.
$y = x - 7$ and $y = x^2 + 15$ are examples of functions. $y = \pm\sqrt{x}$ is not a function because for every value of x there are two values of y.

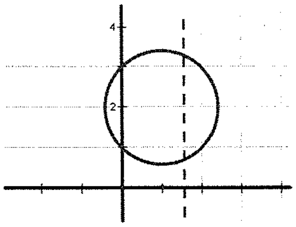
The **vertical line test** is used to determine if a graph of a relation is a function. If a vertical line can be passed along the entire length of the graph and it never touches more than one point at a time, then the relation is a function.

e.g. A:



This passes the vertical line test, so it is a function.

B:



The line passes through more than one point, so this relation fails the vertical line test. It is not a function.

Inverse Functions

The inverse, f^{-1}, of a relation, f, maps each output of the original relation back onto the corresponding input value. The domain of the inverse is the range of the function, and the range of the inverse is the domain of the function. That is, if $(a,b) \in f$, then $(b,a) \in f^{-1}$. The graph of $y = f^{-1}(x)$ is the reflection of the graph $y = f(x)$ in the line $y = x$.

e.g. Given $f(x) = \dfrac{3x-1}{5}$.

Evaluate $f(-3)$.

$$f(-3) = \frac{3(-3)-1}{5}$$
$$f(-3) = \frac{-9-1}{5}$$
$$f(-3) = \frac{-10}{5}$$
$$f(-3) = -2$$

Replace all x's with −3.
Evaluate.

Evaluate $3f(2)+1$

$$3f(2)+1 = 3\left[\frac{3(2)-1}{5}\right]+1$$
$$= 3\left[\frac{6-1}{5}\right]+1$$
$$= 3\left(\frac{5}{5}\right)+1$$
$$= 3(1)+1$$
$$3f(2)+1 = 4$$

You want to find the value of the expression $3f(2)+1$.
You are <u>not</u> solving for $f(2)$.

Determine $f^{-1}(x)$.

$$y = \frac{3x-1}{5}$$
$$x = \frac{3y-1}{5}$$
$$5x = 3y - 1$$
$$3y = 5x + 1$$
$$y = \frac{5x+1}{3}$$
$$\therefore f^{-1}(x) = \frac{5x+1}{3}$$

Rewrite $f(x)$ as $y = \dfrac{3x-1}{5}$

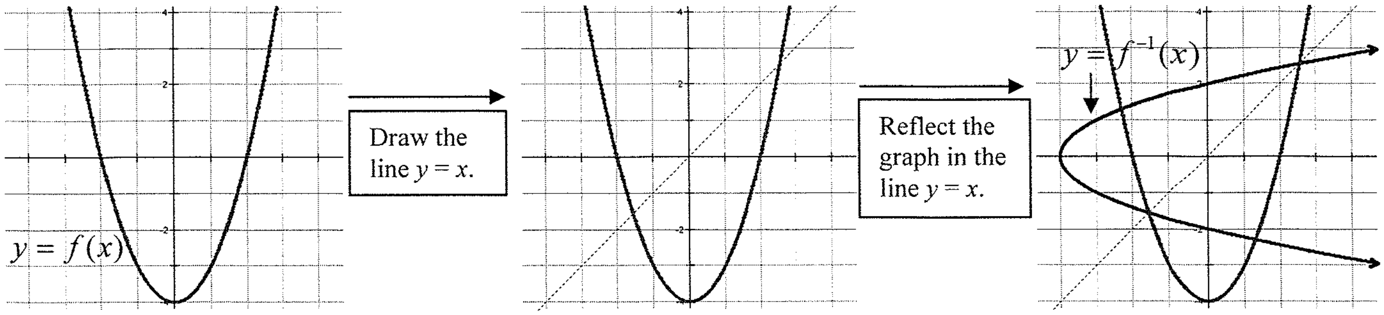
Interchange x and y.
Solve for y.

Evaluate $f^{-1}(2)$

$$f^{-1}(x) = \frac{5x+1}{3}$$
$$f^{-1}(2) = \frac{5(2)+1}{3}$$
$$= \frac{10+1}{3}$$
$$f^{-1}(2) = \frac{11}{3}$$

If you have not already determined $f^{-1}(x)$ do so.
Using $f^{-1}(x)$, replace all x's with 2.
Evaluate.

e.g. Sketch the graph of the inverse of the given function $y = f(x)$.



The inverse of a function is not necessarily going to be a function. If you would like the inverse to also be a function, you may have to restrict the domain or range of the original function. For the example above, the inverse will only be a function if we restrict the domain to $\{x \mid x \geq 0, x \in \mathbf{R}\}$ or $\{x \mid x \leq 0, x \in \mathbf{R}\}$.

Transformations of Functions

To graph $y = af[k(x-d)] + c$ from the graph $y = f(x)$ consider:

a – determines the vertical stretch. The graph $y = f(x)$ is stretched vertically by a factor of a. If $a < 0$ then the graph is reflected in the x-axis, as well.

k – determines the horizontal stretch. The graph $y = f(x)$ is stretched horizontally by a factor of $\frac{1}{k}$. If $k < 0$ then the graph is also reflected in the y-axis.

d – determines the horizontal translation. If $d > 0$ the graph shifts to the right by d units. If $d < 0$ then the graph shifts left by d units.

c – determines the vertical translation. If $c > 0$ the graph shifts up by c units. If c < 0 then the graph shifts down by c units.

When applying transformations to a graph the stretches and reflections should be applied before any translations.

e.g. The graph of $y = f(x)$ is transformed into $y = 3f(2x-4)$. Describe the transformations.

First, factor inside the brackets to determine the values of k and p.

$y = 3f(2(x-2))$

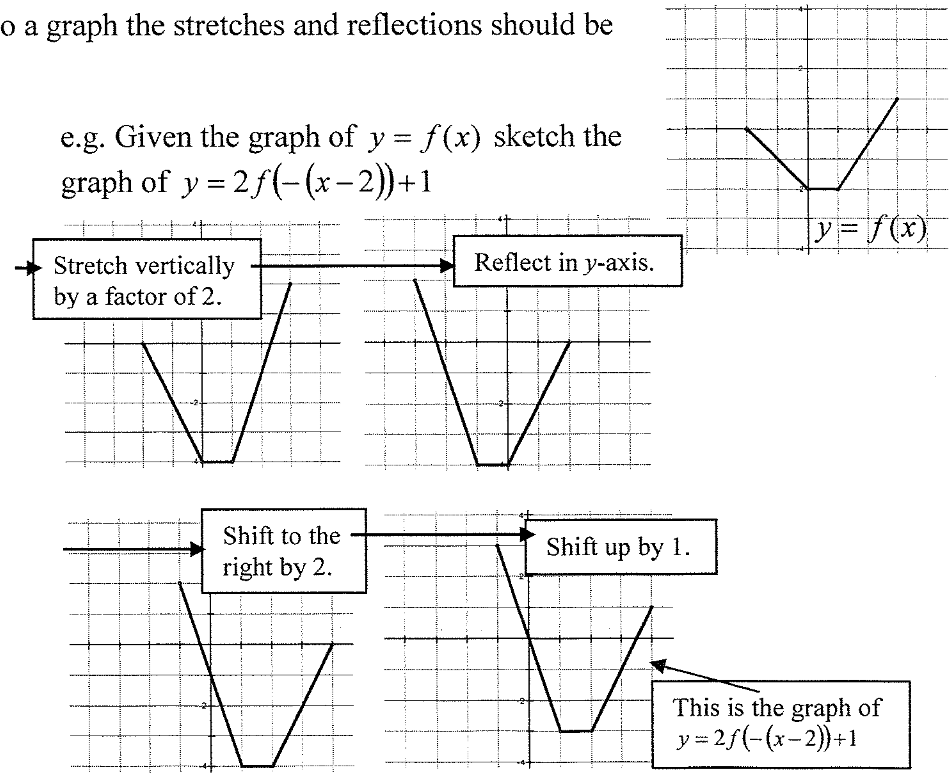
$a = 3, k = 2, p = 2$

There is a vertical stretch of 3.

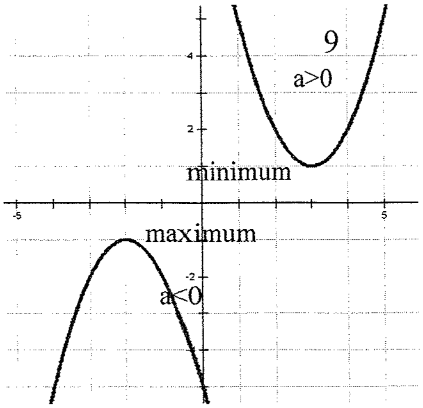
A horizontal stretch of $\frac{1}{2}$.

The graph will be shifted 2 units to the right.

e.g. Given the graph of $y = f(x)$ sketch the graph of $y = 2f(-(x-2)) + 1$



MCR3U Exam Review



Quadratic Functions

The graph of the quadratic function, $f(x) = ax^2 + bx + c$, is a parabola. When $a > 0$ the parabola opens up. When $a < 0$ the parabola opens down.

Vertex Form: $f(x) = a(x-h)^2 + k$

The vertex is (h,k). The maximum or minimum value is *k*.

The axis of symmetry is $y = h$.

Factored Form: $f(x) = a(x-p)(x-q)$

The zeroes are $x = p$ and $x = q$.

Standard Form: $f(x) = ax^2 + bx + c$

The *y*-intercept is *c*.

Complete the square to change the standard form to vertex form.

e.g.

$f(x) = -2x^2 - 12x + 7$

Factor the coefficient of x^2 (a-value) from the terms with x^2 and x.

$f(x) = -2(x^2 + 6x) + 7$

Divide the coefficient of x (b-value) by 2. Square this number. Add and subtract it.

$f(x) = -2(x^2 + 6x + 9 - 9) + 7$

Bring the last term inside the bracket outside the brackets.

$f(x) = -2(x^2 + 6x + 9) - 2(-9) + 7$

Factor the perfect square trinomial inside the brackets.

$f(x) = -2(x+3)^2 - 2(-9) + 7$

Simplify.

$f(x) = -2(x+3)^2 + 25$

Maximum and Minimum Values

Vertex form, maximum/minimum value is *k*.

Factored form:

e.g. Determine the maximum or minimum value of $f(x) = (x-1)(x-7)$.

The zeroes of $f(x)$ are equidistant from the axis of symmetry. The zeroes are $x = 1$ and $x = 7$.

$$x = \frac{1+7}{2}$$

$$x = 4$$

The axis of symmetry is $x = 4$. The axis of symmetry passes through the vertex. The *x*-coordinate of the vertex is 4. To find the *y*-coordinate of the vertex, evaluate $f(4)$.

$$f(4) = (4-1)(4-7)$$

$$f(4) = 3(-3)$$

$$f(4) = -9$$

The vertex is $(4,-9)$. Because *a* is positive ($a = 1$), the graph opens up. The minimum value is –9.

Zeroes

To determine the number of zeroes of a quadratic function consider the form of the function.

Vertex form: If *a* and *k* have opposite signs there are 2 zeroes (2 roots).
If *a* and *k* have the same sign there are no zeroes (0 roots).
If $k = 0$ there is one zero (1 root).

Factored form: $f(x) = a(x-p)(x-q) \rightarrow$ 2 zeroes. The zeroes are $x = p$ and $x = q$.
$f(x) = a(x-p)^2 \rightarrow$ 1 zero. The zero is $x = p$.

Standard form: Check discriminant. $D = b^2 - 4ac$
If $D < 0$ there are no zeroes.
If $D = 0$ there is 1 zero.
If $D > 0$ there are 2 zeroes.

To determine the zeroes of from the standard form use the **quadratic formula**.

For , $ax^2+bx+c=0$ use $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$ to solve for x.

Reciprocal functions

The reciprocal function of a function, f, is defined as $\dfrac{1}{f}$. To help you graph $y=\dfrac{1}{f(x)}$, you should use the following:

The vertical asymptotes of $y=\dfrac{1}{f(x)}$ will occur where $f(x)=0$

As $f(x)$ increases, $\dfrac{1}{f(x)}$ decreases. As $f(x)$ decreases, $\dfrac{1}{f(x)}$ increases.

For $f(x)>0$, $\dfrac{1}{f(x)}>0$. For $f(x)<0$, $\dfrac{1}{f(x)}<0$.

The graph of $y=\dfrac{1}{f(x)}$ always passes through the points where $f(x)=1$ or $f(x)=-1$.

You may find it helpful to sketch the graph of $y=f(x)$ first, before you graph the reciprocal.

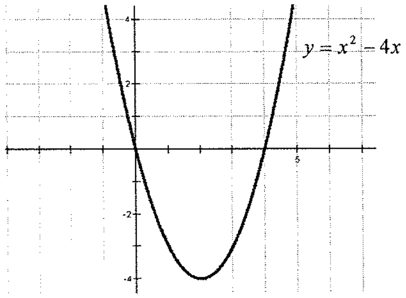
e.g. Sketch the graph of $y=\dfrac{1}{x^2-4x}$.

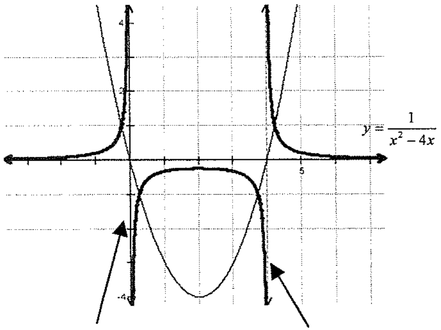
Look at the function $f(x)=x^2-4x$.

Factor it. $f(x)=x(x-4)$.

The zeroes are $x = 0$, and $x = 4$. The vertical asymptotes will be at $x = 0$, and $x = 4$.

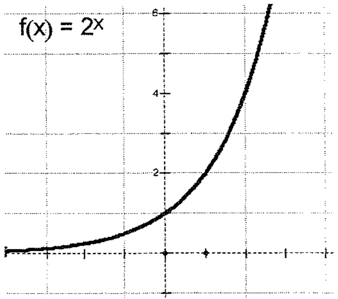
You could sketch the graph of $f(x)=x^2-4x$ to see where the function increases and decreases, where $f(x)=1$ or –1. Use the information above to help you sketch the reciprocal.



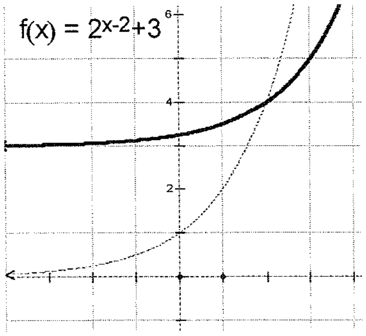


Vertical asymptotes

Exponential Functions



In general, the exponential function is defined by the equation, $y=a^x$ or $f(x)=a^x$, $a>0, x\in R$. Transformations apply to exponential functions the same way they do to all other functions.



Exponential Growth and Decay

Population growth and radioactive decay can be modelled using exponential functions.

$y = ab^x$

a – is the initial amount/value
b- is the ratio (b>1 growth rate, 0<b<1 decay rate)
x – the number of growth/decay period

Compound Interest

Calculating the future amount: $A = P(1+i)^n$

Calculating the present amount: $P = A(1+i)^{-n}$

A – future amount
P – present (initial) amount
i – interest rate per conversion period
n – number of conversion periods

Annuities

Regular payments at regular intervals

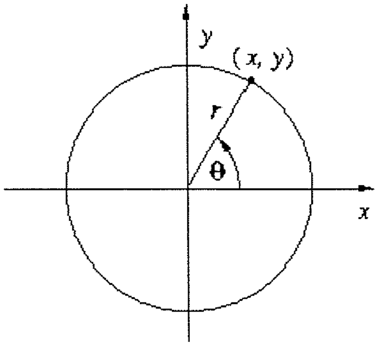
Calculating the future amount: $A = R\left(\frac{(1+i)^n - 1}{i}\right)$

R – the amount of the regular payments

Calculating the present amount: $P = R\left(\frac{1-(1+i)^{-n}}{i}\right)$

Trigonometry

For any point P(x, y) in the Cartesian plane, the trigonometric ratios For angles in standard position can be expressed in terms of x, y and r.



$r^2 = x^2 + y^2$ (from the Pythagorean theorem)

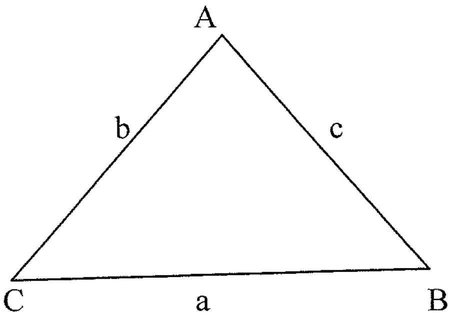
Primary Trigonometric Ratios

$\sin\theta = \frac{y}{r}$ $\cos\theta = \frac{x}{r}$ $\tan\theta = \frac{y}{x}$

Reciprocal Trigonometric Ratios

$\csc\theta = \frac{r}{y} = \frac{1}{\sin\theta}$ $\sec\theta = \frac{r}{x} = \frac{1}{\cos\theta}$ $\cot\theta = \frac{x}{y} = \frac{1}{\tan\theta}$

Trigonometry of Oblique Triangles



Sine Law

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Can be used when you know ASA, AAS, SSA

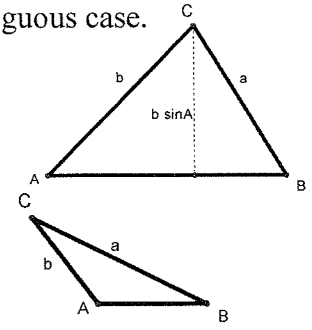
Cosine Law

$a^2 = b^2 + c^2 - 2bc\cos A$

Can be used when you know SSS, SAS

When you know SSA it is considered the ambiguous case.

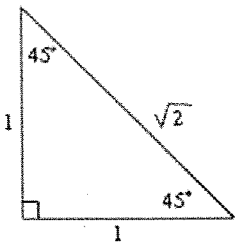
Angle	Conditions	# of Triangles
$\angle A < 90^\circ$	$a < b\sin A$	0
	$a = b\sin A$	1
	$a > b\sin A$	2
$\angle A > 90^\circ$	$a \le b$	0
	$a > b$	1

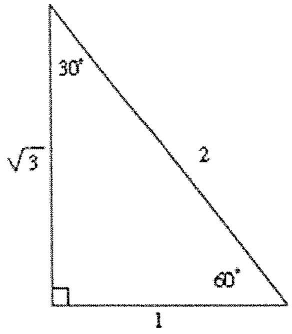


Trigonometric Ratios for Special Angles

The exact values of the trigonometric ratios for 30°, 45°, and 60° can be found using the appropriate sides of the special triangles.

θ	sin θ	cos θ	tan θ
30°	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$
45°	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1
60°	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$





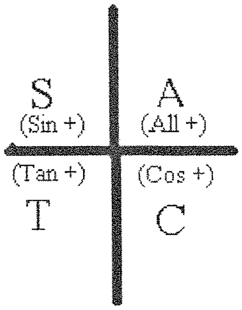
The special triangle can also be used to find the exact values of angles related to 30°, 45°, and 60° using the CAST rule.

e.g. Find the exact value of sin225°

$$\sin 225^\circ = -\sin 45^\circ$$
$$= -\frac{1}{\sqrt{2}}$$

The angle 225° is located in the third quadrant where only tan is positive, so sin225° will be negative.

The related angle is 45° because 180 + 45 = 225°.



Trigonometric Identities

Pythagorean Identity: $\sin^2\theta + \cos^2\theta = 1$ Quotient Identity: $\tan\theta = \frac{\sin\theta}{\cos\theta}$

e.g. Prove the identity. $\sin^2\theta + 2\cos^2\theta - 1 = \cos^2\theta$

$$LS = \sin^2\theta + 2\cos^2\theta - 1$$
$$= \sin^2\theta + \cos^2\theta + \cos^2\theta - 1$$
$$= 1 + \cos^2\theta - 1$$
$$= \cos^2\theta = RS$$

Work with each side separately.
Look for the quotient or Pythagorean identities.
You may need to factor, simplify or split terms up.
When you are done, write a concluding statement.

Since LS=RS then $\sin^2\theta + 2\cos^2\theta - 1 = \cos^2\theta$ is true for all values of θ.

Periodic Functions

A periodic function has a repeating pattern.
The **cycle** is the smallest complete repeating pattern.
The **axis of the curve** is a horizontal line that is midway between the maximum and minimum values of the graph. The equation is

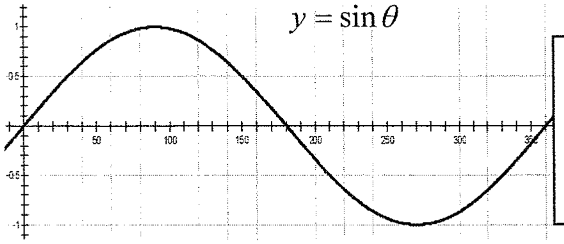
$$y = \frac{\text{max value} + \text{min value}}{2}.$$

The **period** is the length of the cycle.
The **amplitude** is the magnitude of the vertical distance from the axis of the curve to the maximum or minimum value. The equation is

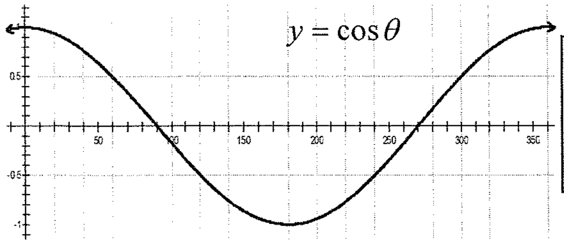
$$a = \frac{\text{max value} - \text{min value}}{2}$$

Trigonometric Functions

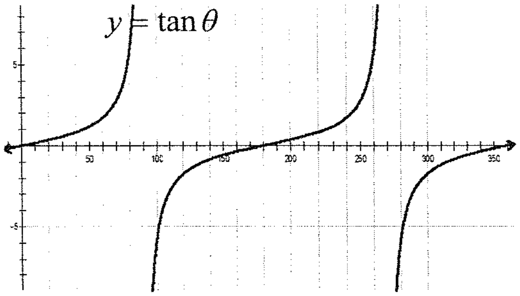
The graphs of $y = \sin\theta$, $y = \cos\theta$, and $y = \tan\theta$ are shown below.



$y = \sin\theta$
Period = 360°
Amplitude = 1
Zeroes = 0°, 180°, 360°…



$y = \cos\theta$
Period = 360°
Amplitude = 1
Zeroes = 90°, 270°…



$y = \tan\theta$
Period = 180°
Zeroes = 0°, 180°, 360°…
Vertical asymptotes = 90°, 270°…

Transformations of Trigonometric Functions

Transformations apply to trig functions as they do to any other function.

The graphs of $y = a\sin k(\theta + d) + c$ and $y = a\cos k(\theta + b) + d$ are transformations of the graphs $y = \sin\theta$ and $y = \cos\theta$ respectively.

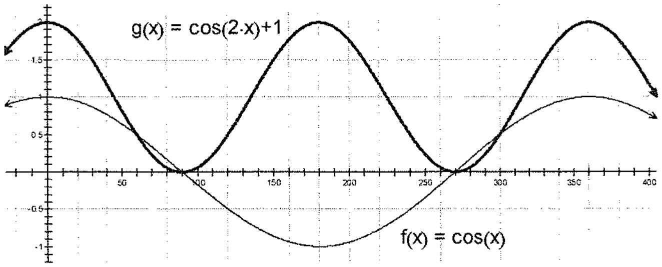
The value of a determines the vertical stretch, called the **amplitude**.
It also tells whether the curve is reflected in the θ-axis.

The value of k determines the horizontal stretch. The graph is stretched by a factor of $\frac{1}{k}$. We can use this value to determine the **period** of the transformation of $y = \sin\theta$ or $y = \cos\theta$.

The period of $y = \sin k\theta$ or $y = \cos k\theta$ is $\frac{360^\circ}{k}$, $k > 0$. The period of $y = \tan k\theta$ is $\frac{180^\circ}{k}$, $k > 0$.
The value of d determines the horizontal translation, known as the **phase shift**.
The value of c determines the vertical translation. $y = d$ is the equation of the **axis of the curve**.

e.g.

$y = \cos 2\theta + 1$



e.g.

$y = \frac{1}{2}\sin\left(\theta + 45^\circ\right)$