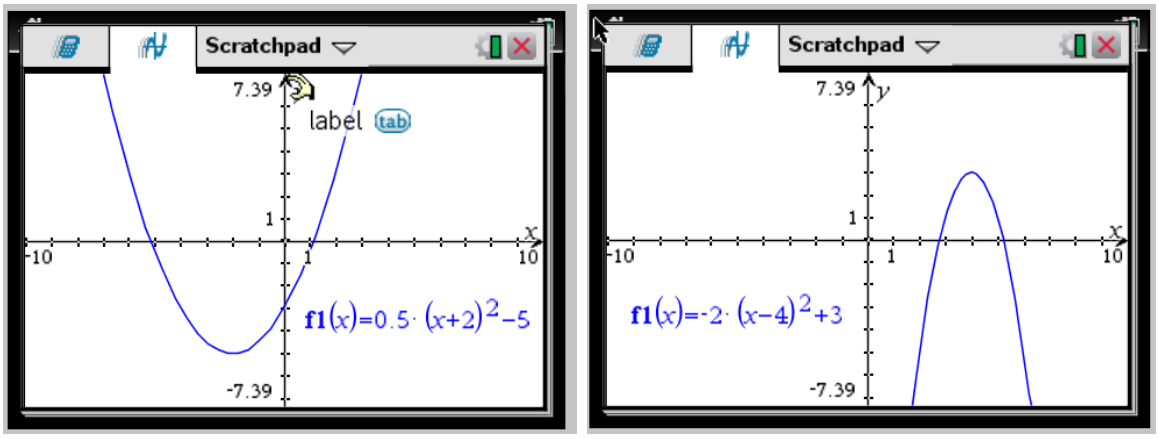
Determining Maximum and Minimum Values of a Quadratic Function

Learning Goals

- recall how to convert a quadratic function to vertex form by completing the square
- determine four ways to find the max /min of a quadratic function
- investigate "Max / Min " word problems

Maximum / Minimum Value (Optimum Value)



Finding vertex from standard form: $f(x) = ax^2 + bx + c$

MCR3U *3.2 Maximum and Minimum of Quadratic Functions*

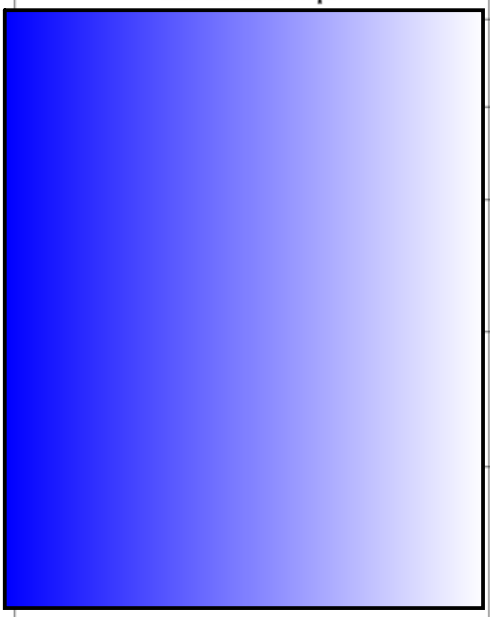
The vertex is the optimal value of a quadratic function. The x-coordinate is the input value required to achieve the optimal value, and the y-coordinate is the optimal value of the function.

If the parabola opens up the y-coordinate is a _____ value

If the parabola opens down, the y-coordinate is a _____ value

Completing the Square

Completing the square allows us to convert from Standard Form to Vertex Form

The essential steps:	Example: $f(x) = 2x^2 + 12x - 7$
	

Finish handout for homework

1. $f(x) = -3x^2 + 6x + 1$

3. $f(x) = 0.5x^2 + 5x + 7$

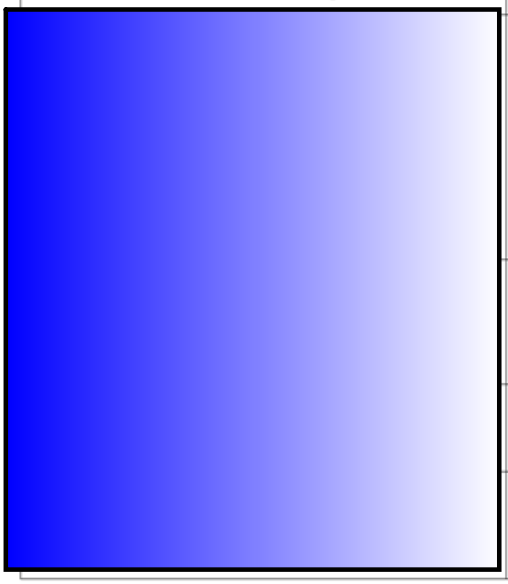
Answers:

$f(x)=-3(x-1)^2+4$

$f(x)=0.5(x+5)^2-5.5$

Factoring and Using the Roots

This allows us to convert from Standard Form to Factored Form and then to Vertex Form

The essential steps:	Example: $f(x) = 4x^2 - 12x - 40$
	

Practise: Finish handout for homework

1. $f(x) = x^2 - 8x + 12$

2. $f(x) = -3x^2 - 12x + 15$

Answers:

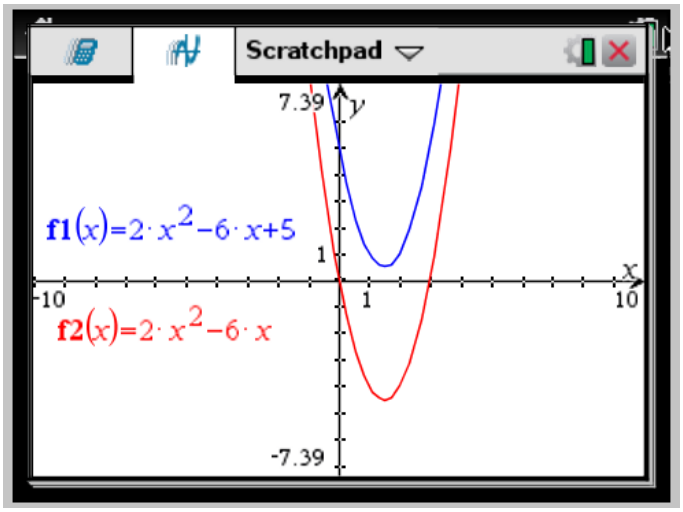
$f(x)=(x-4)^2-4$

$f(x)=-3(x+2)^2+27$

Partial Factoring

Graph $y=2x^2-6x+5$

$y=2x^2-6x$



What characteristic do these quadratic functions share?

Partial Factoring
(When function cannot be factored)

$y=2x^2-6x+5$

This allows us to find the x-value of the vertex by determining the Axis of Symmetry of the parabola.

The essential steps:	Example: $f(x) = -2x^2 + 8x + 15$
1. Create ***a new function*** that shares the Axis of Symmetry by dropping the "c"	
2. Factor the new function, to determine the roots.	
2. Determine the Axis of Symmetry by averaging the zeroes. (this is the x co-ordinate of the Vertex)	
3. Determine the y co-ordinate of the Vertex	
4. State the function in Vertex Form	

Practise:

1. $f(x) = 7x^2 + 14x + 11$

2. $f(x) = -3x^2 - 12x + 15$

Using the formula

$$\left(-\frac{b}{2a}\ ,\ f\left(-\frac{b}{2a}\right)\right)$$

Find the vertex using the formula

$g(x)=x^2+6x+1$

Using the TI-Nspire

1. Graph

2.  (menu)

Analyze Graph

- Max/Min

3. Trace

On the Boards...

1. Find the vertex by Completing the Square $f(x) = x^2 - 10x + 1$

2. Find the vertex by Using the Roots $g(x)=6x^2-x-2$

3. Find the vertex using partial factoring. $f(x)=x^2-10x+1$

4. Find the vertex using the formula $g(x)=3x^2+8x+1$

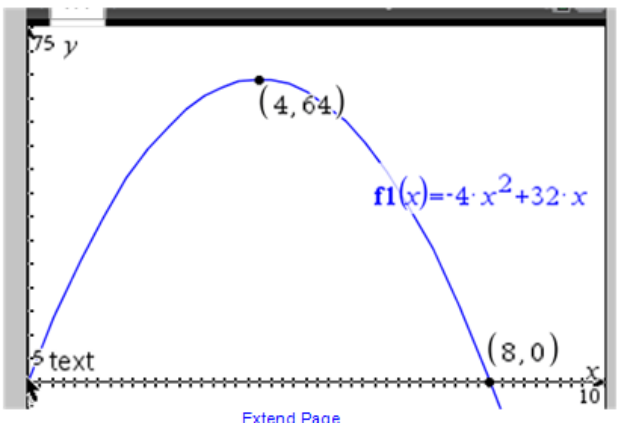
On the Boards...

11. The height of a rocket above the ground is modelled by the quadratic function $h(t) = -4t^2 + 32t$, where $h(t)$ is the height in metres t seconds after the rocket was launched.

What is the maximum height that the rocket will reach?

What part of the parabola are we looking for?

How are we going to find it?



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A

What is the maximum height that the rocket will reach?

What part of the parabola are we looking for?

y-coordinate of vertex

How are we going to find it?

complete the square

$h = -4t^2 + 32t$

$= -4(t^2 - 8t)$

$= -4(t^2 - 8t + 16 - 16)$

$= -4(t^2 - 8t + 16) + 64$

$= -4(t - 4)^2 + 64$

$\therefore$ height is 64

factor

$h = -4t^2 + 32t$

$= -4t(t - 8)$

zeros 0 8

AOS $\frac{0+8}{2} = 4$

$h(4) = -4(4)^2 + 32(4)$

$= 64$

Seatwork

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