Pause the video and try this Warm-up

Simplify

$$\frac{2t}{t^2-1}-\frac{t+2}{t^2+3t-4}$$

Warm-up

Simplify

$$\frac{2t}{t^2-1}-\frac{t+2}{t^2+3t-4}$$

$$=\frac{2t}{(t-1)(t+1)}-\frac{t+2}{(t+4)(t-1)}$$

restrictions

$t \neq 1, -1, -4$

$$=\frac{(t+4)2t}{(t-1)(t+1)(t+4)}-\frac{(t+1)(t+2)}{(t+1)(t-1)(t+4)}$$

$$=\frac{2t^2+8t-t^2-2t-t-2}{(t-1)(t+1)(t+4)}$$

$$=\frac{t^2+5t-2}{(t-1)(t+1)(t+4)}$$

Graphs of Rational Functions

Learning Goals:
- **simplify rational functions**
- **determine position of asymptotes and holes**

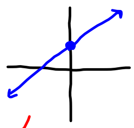
Please print the handout if possible

You will need your TI-Nspire for this activity
(Desmos will also work)

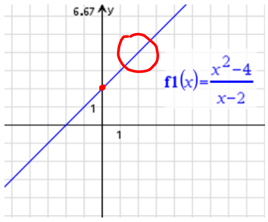
Turn on the Video

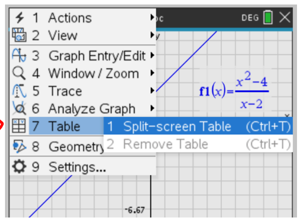
I am going to show you the first one.

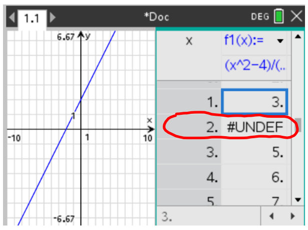
For each of the following:
1. Simplify the function.
2. Graph the **Original Function** using Nspire.
3. Sketch the graph.
4. Look at the Table of Values and complete the fourth column
5. **Adjust your graph as necessary and answer the question in the fifth column.**

Original Function	Simplified Function	Sketch the Graph	Table of Values	How does this point show up on the graph?
$f(x) = \frac{x^2-4}{x-2}$	$= \frac{(x+2)\cancel{(x-2)}}{\cancel{x-2}}$ $= x+2$		(2, und)	nothing there

$x \neq 2$







MCR 3U *Graphing Rational Functions*

For each of the following:
1. Simplify the function.
2. Graph the **Original Function** using Nspire.
3. Sketch the graph.
4. Look at the Table of Values and complete the fourth column
5. **Adjust your graph as necessary and answer the question in the fifth column.**

Original Function	Simplified Function	Sketch the Graph	Table of Values	How does this point show up on the graph?
$f(x) = \frac{x^2-4}{x-2}$			(2, ______)	
$g(x) = \frac{2x^2-x-1}{x-1}$			(1, ______)	
$h(x) = \frac{1}{x-3}$			(3, ______)	
$i(x) = \frac{2x+1}{x-2}$			(2, ______)	

MCR 3U *Graphing Rational Functions*

Original Function	Simplified Function	Sketch the Graph	Table of Values	How does this point show up on the graph?
$j(x)=\frac{2x+1}{(x+1)(x-2)}$			$(-1,\_\_\_\_)$ $(2,\_\_\_\_)$	
$k(x)=\frac{2x+4}{(x+2)(x+5)}$			$(2,\_\_\_\_)$ $(-5,\_\_\_\_)$	
$l(x)=\frac{1}{x}+2$			$(0,\_\_\_\_)$	
$m(x)=\frac{x^3+3}{x^2+2}$			$(-2,\_\_\_\_)$ $(0,\_\_\_\_)$ $(3,\_\_\_\_)$	

Summary:

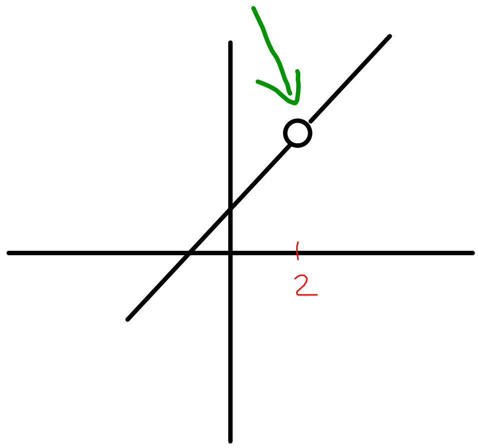
There is a __________ from a factor that is both in the __________ and __________

There is a __________ from a factor that is still in the __________ after __________

Using your TI-Nspire graph

$$f(x)=\frac{x^2-4}{x-2}$$

$$=\frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}}$$



What do you observe?

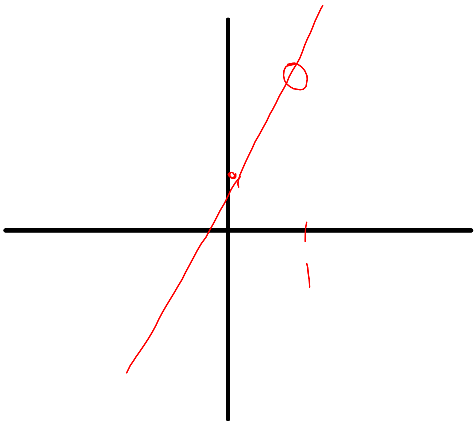
straight line

restrictions $x \neq 2 \Rightarrow$ Hole

Using your TI-Nspire graph

$g(x)=\dfrac{2x^2-x-1}{x-1}$ $\leftarrow x \neq 1$

$=\dfrac{(2x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$



What do you observe?

straight line

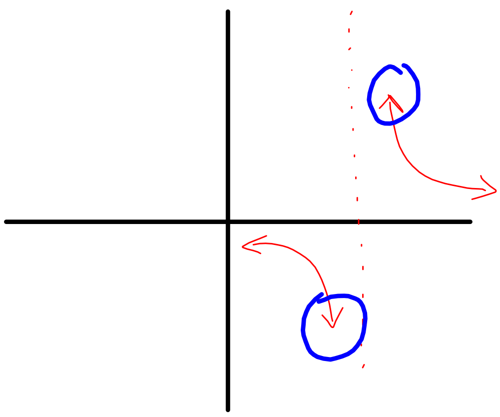
but there is a HOLE

at $x=1$

Using your TI-Nspire graph

$h(x)=\dfrac{1}{x-3}$

$x \neq 3$



What do you observe?

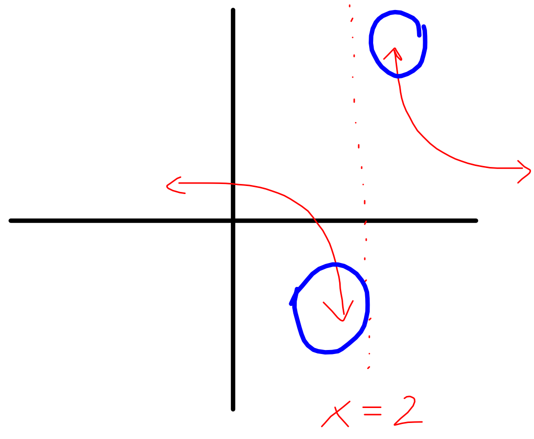
No hole

There is an asymptote

Using your TI-Nspire graph

$$i(x) = \frac{2x+1}{x-2}$$

$x \neq 2$

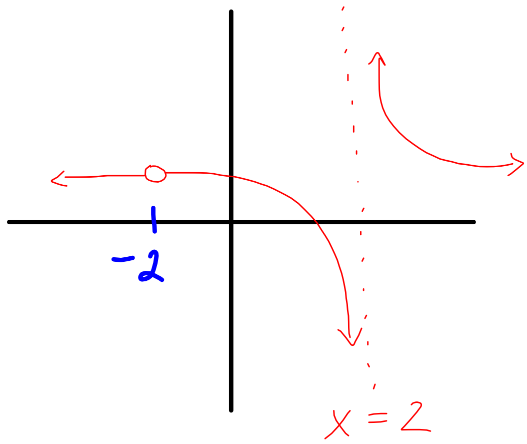


What do you observe?

Using your TI-Nspire graph

$$i(x) = \frac{(2x+1)(x+2)}{x^2 - 4}$$

$$= \frac{(2x+1)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}$$



$x \neq 2, -2$

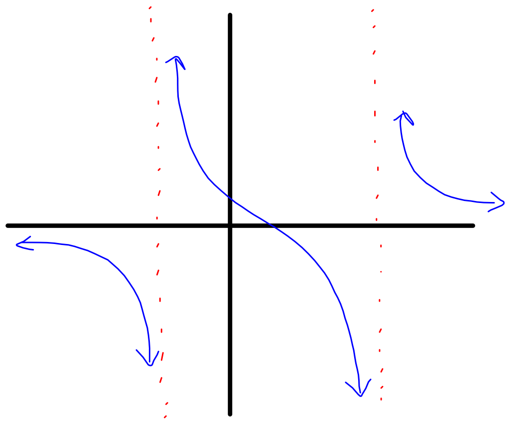
What do you observe?

$x = 2$ asymptote

$x = -2$ hole

Using your TI-Nspire graph

$$j(x)=\frac{2x+1}{(x+1)(x-2)}$$



$x \neq -1, 2$

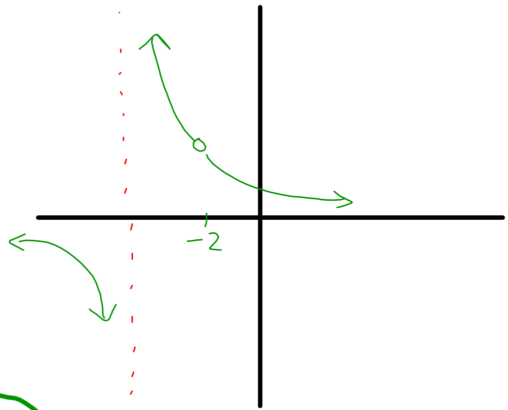
What do you observe?

2 asymptotes

Using your TI-Nspire graph

$$k(x)=\frac{2x+4}{(x+2)(x+5)}$$

$$=\frac{2\cancel{(x+2)}}{\cancel{(x+2)}(x+5)}$$



$x \neq -2, -5$

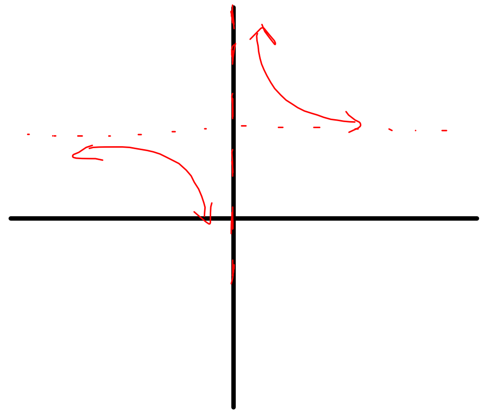
What do you observe?

Hole at $x=-2$

Asymptote $x=-5$

Using your TI-Nspire graph

$$l(x)=\frac{1}{x}+2$$



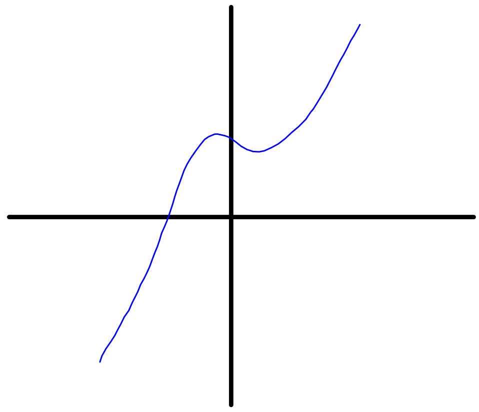
$x \neq 0$

What do you observe?

V. Translation up 2

Using your TI-Nspire graph

$$m(x)=\frac{x^3+3}{x^2+2}$$



What do you observe?

$x^2+2 \neq 0$

$x^2 \neq -2$

$\therefore$ no restriction

Rules:

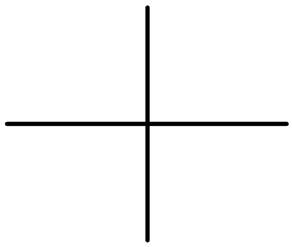
There is a Hole from a factor that is both in the numerator and denominator.

There is a Asymptote from a factor that is still in the denominator after simplification

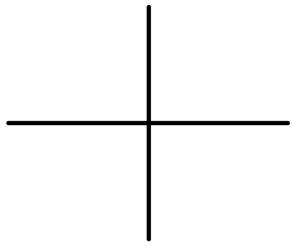
Try on your own...WITHOUT technology

Identify the holes and asymptotes

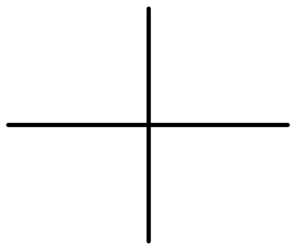
$$f(x) = \frac{3}{x-5}$$



$$g(x) = \frac{3}{x^2 - 3x - 10}$$



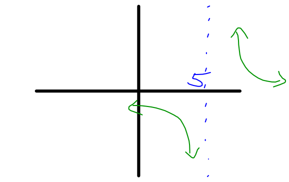
$$h(x) = \frac{x+4}{2x^2 - 32}$$



Identify the holes and asymptotes

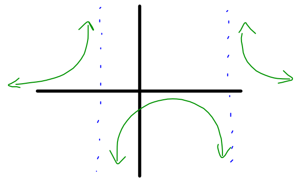
$$f(x) = \frac{3}{x-5}$$

A. at $x=5$

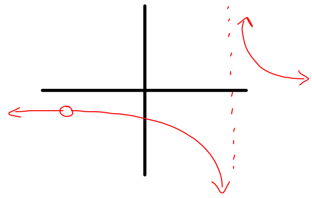


$$g(x) = \frac{3}{x^2 - 3x - 10}$$

$$= \frac{3}{(x-5)(x+2)}$$



$$h(x) = \frac{x+4}{2x^2 - 32}$$



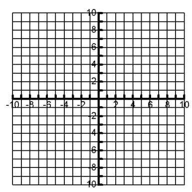
$$= \frac{x+4}{2(x^2-16)}$$

$$= \frac{\cancel{x+4}}{2\cancel{(x+4)}(x-4)}$$

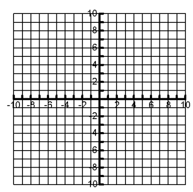
Try On Your Own

MCR 3U *Graphing Rational Functions*

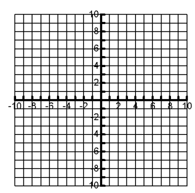
1. $f(x) = \dfrac{2x^2 - 15x + 25}{x-5}$



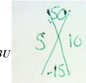
2. $f(x) = \dfrac{5x^3 - 10x^2 - 15x}{5x}$



3. $f(x) = \dfrac{3-3x}{3x^2 - 18x + 15}$



MCR 3U



Simplify and State Restrictions

Graphing Rational Functions

1. $f(x)=\dfrac{2x^2-15x+25}{x-5}$

$=\dfrac{2x^2-10x-5x+25}{(x-5)}$

$=\dfrac{2x(x-5)-5(x-5)}{(x-5)}$

$=\dfrac{(2x-5)(x-5)}{(x-5)}$

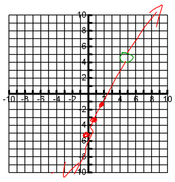
$\boxed{f(x)=2x-5,\ x\neq 5}$

To graph a Line ...

$y=mx+b$

↑ m ↑ y intercept

$\frac{m}{1}=$ slope $=\frac{\text{Rise}}{\text{Run}}$



To graph the Hole

Use Simplified Function

$f(5)=2(5)-5$

$=10-5$

$=5$

Hole $(5,5)$

2. $f(x)=\dfrac{5x^3-10x^2-15x}{5x}$

$=\dfrac{5x(x^2-2x-3)}{5x},\ x\neq 0$

$=x^2-2x-3$

$f(x)=(x-3)(x+1),\ x\neq 0$

To graph a Parabola ...

Three Points

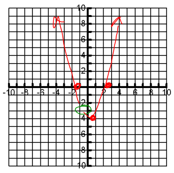
Zeros $(3,0)$ $(-1,0)$

$AOS=\dfrac{3+(-1)}{2}$

$=1$

$f(1)=(1-3)(1+1)$

$=-4$ $\therefore v(1,-4)$



To graph the Hole

Use Simplified Function

$f(0)=(0-3)(0+1)$

$=-3$

Hole $(0;3)$

3. $f(x)=\dfrac{3-3x}{3x^2-18x+15}$

$=\dfrac{-3x+3}{3(x^2-6x+5)}$

$=\dfrac{-3(x-1)}{3(x-1)(x-5)}$

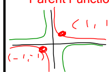
$\boxed{f(x)=\dfrac{-1}{x-5},\ x\neq 1}$

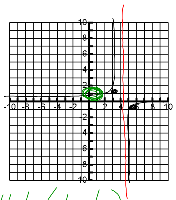
$x\neq 5$

Vertical Asymptote

To graph a Reciprocal Function ...

Parent Function & Transformations





To graph the Hole

Use Simplified Function

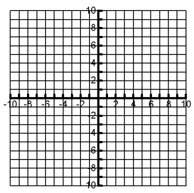
$f(1)=\dfrac{-1}{1-5}$

$=\dfrac{-1}{-4}=\dfrac{1}{4}$

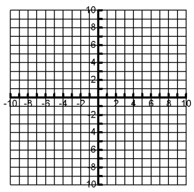
Hole $\left(1,\frac{1}{4}\right)$

MCR 3U *Graphing Rational Functions*

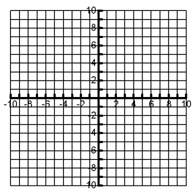
4. $f(x)=\dfrac{x^2+4x-12}{2x+12}$



5. $f(x)=\dfrac{x^2+4x+4}{x-2}\times\dfrac{x^2-6x+8}{3x+6}$



6. $f(x)=\dfrac{5x-x^2}{-x^3+2x^2+15x}$



MCR 3U *Graphing Rational Functions*

4. $f(x)=\dfrac{x^2+4x-12}{2x+12}$

$=\dfrac{(x+\cancel{6})(x-2)}{2(x+\cancel{6})}$

$=\dfrac{x-2}{2}$

$=\frac{1}{2}(x-2)$

$\boxed{f(x)=\frac{1}{2}x-1,\ x\neq -6}$

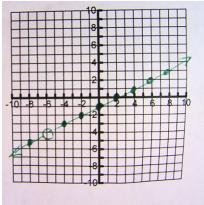
Hole

$f(-6)=\frac{1}{2}(-6-2)$

$=\frac{1}{2}(-8)$

$=-4$

$(-6,-4)$



5. $f(x)=\dfrac{x^2+4x+4}{x-2}\times\dfrac{x^2-6x+8}{3x+6}$

$=\dfrac{(x+2)(x+\cancel{2})}{(x-\cancel{2})}\times\dfrac{(x-4)(\cancel{x-2})}{3(\cancel{x+2})}$

$=\dfrac{(x+2)(x-4)}{3}$

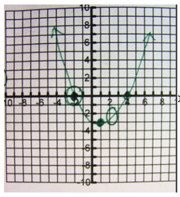
$\boxed{f(x)=\frac{1}{3}(x+2)(x-4),\ x\neq 2,-2}$

Hole

$f(2)=\frac{1}{3}(2+2)(2-4)$

$=\frac{1}{3}(4)(-2)$

$=\frac{-8}{3}$



Parabola

Zeroes $(-2,0)$ $(4,0)$

AoS $=\dfrac{-2+4}{2}=1$

$f(1)=\frac{1}{3}(1+2)(1-4)=\frac{1}{3}(3)(-3)=-3$ $V(1,-3)$

6. $f(x)=\dfrac{5x-x^2}{-x^3+2x^2+15x}$

$=\dfrac{-x^2+5x}{-x(x^2-2x-15)}$

$=\dfrac{\cancel{-x}(\cancel{x-5})}{\cancel{-x}(\cancel{x-5})(x+3)}$, $x\neq 0$, $x\neq 5$

$=\dfrac{1}{x+3}$, $x\neq -3$

Vertical Asymptote

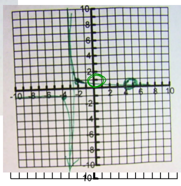
Hole

$f(5)=\dfrac{1}{5+3}$

$=\dfrac{1}{8}$

$f(0)=\dfrac{1}{0+3}$

$=\dfrac{1}{3}$



Seatwork

pg 116 #1