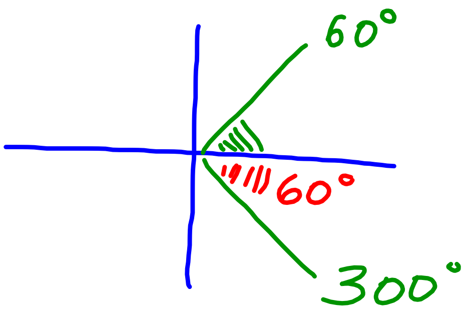
Warm - up $0^\circ \leq \theta \leq 360^\circ$

Find all possible angles for cos θ = 0.5

Find all possible angles for tan θ = - 1.7

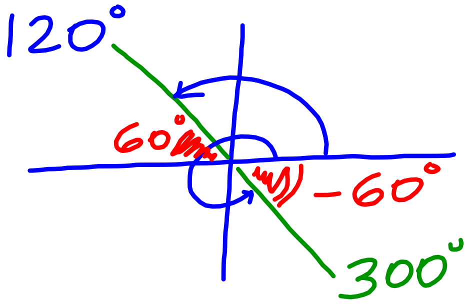
Warm - up $0^\circ \leq \theta \leq 360^\circ$

Find all possible angles for cos θ = 0.5



$\theta = 60^\circ$

Find all possible angles for tan θ = - 1.7

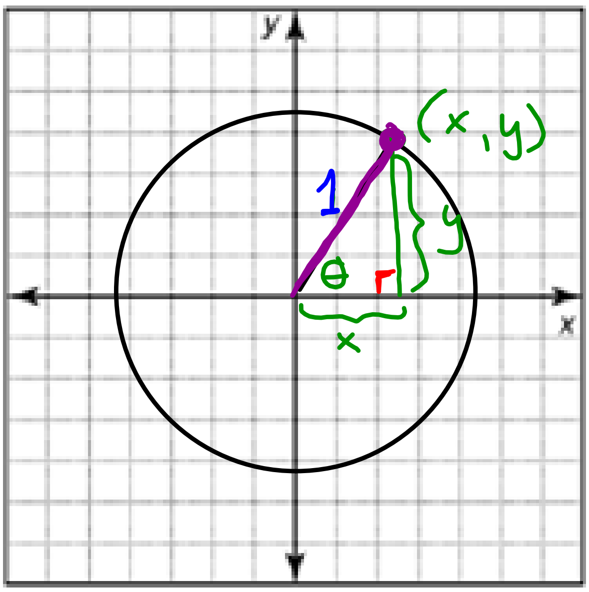


$\theta = -60^\circ$

Evaluating Trigonometric Ratios using Coordinates

Learning Goals

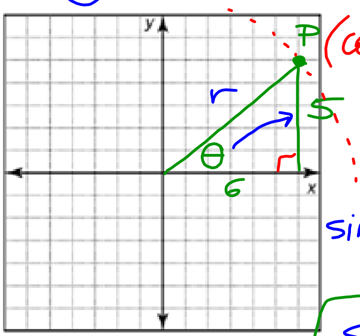
- determine the exact trigonometric ratio of any angle using the coordinate plane



What happens when you have an angle defined by a point ???

The coordinates of a point P on a terminal arm of an $\angle\theta$ in standard position are given, where $0 < \theta < 360°$. Determine the exact values of $\sin\theta$, $\cos\theta$, and $\tan\theta$.

P(6, 5)



$P(\cos\theta, \sin\theta)$

$\sin\theta = \frac{opp}{hyp} = \frac{y}{r}$

$\boxed{\sin\theta\,(r) = y}$

$\cos\theta = \frac{adj}{hyp}$

$\cos\theta = \frac{x}{r}$

$\boxed{r\cdot\cos\theta = x}$

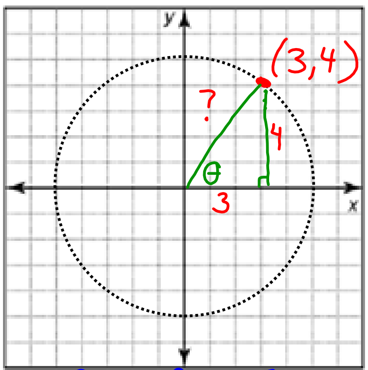
$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{\cancel{r}\cdot\sin\theta}{\cancel{r}\cdot\cos\theta}$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Find trigonometric ratios using the point P(3,4)



$\sin\theta = \frac{y}{r} = \frac{4}{5}$

$\cos\theta = \frac{x}{r} = \frac{3}{5}$

$\tan\theta = \frac{y}{x} = \frac{4}{3}$

$a^2 + b^2 = c^2$

$y^2 + x^2 = r^2$

$\sqrt{x^2 + y^2} = r$

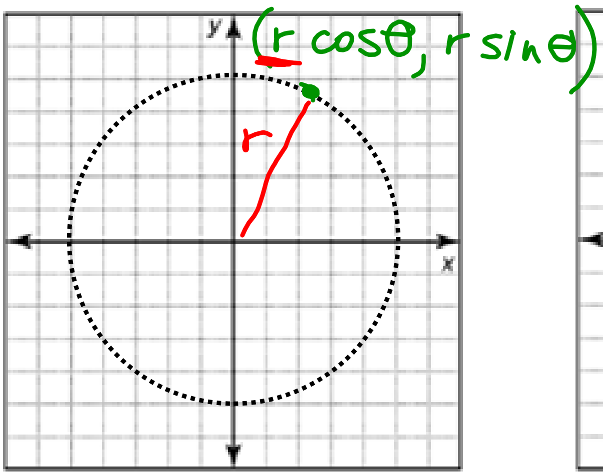
$\sqrt{3^2 + 4^2} = r$

$\sqrt{9 + 16} = r$

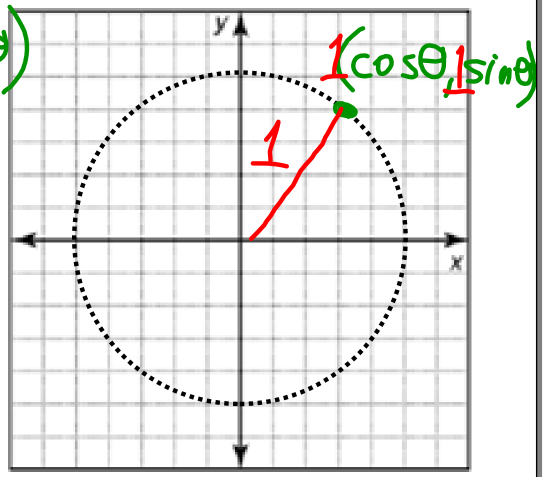
$\sqrt{25} = r$

$5 = r$

Polar Coordinates



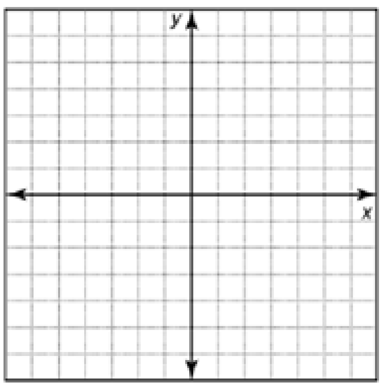
Unit Circle



Try On Your Own ...

Determine the exact values of $\sin\theta$, $\cos\theta$, and $\tan\theta$.

c) P(-2, -5)



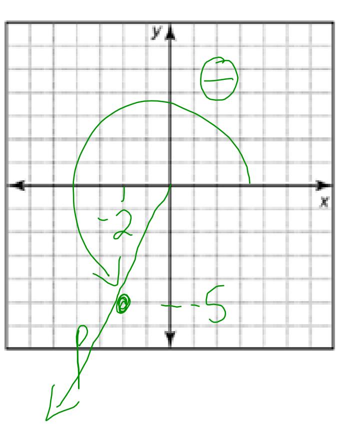
You may need to use $\sqrt{}$

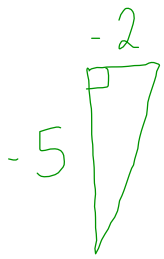
Try On Your Own ...

Determine the exact values of $\sin\theta$, $\cos\theta$, and $\tan\theta$.

c) P(-2, -5)

$x = -2$
$y = -5$





$x^2 + y^2 = r^2$
$(-2)^2 + (-5)^2 = r^2$
$4 + 25 = r^2$
$29 = r^2$
$\sqrt{29} = r$

$\sin\theta = \frac{y}{r} = \frac{-5}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}} = \frac{-5\sqrt{29}}{29}$

$\cos\theta = \frac{x}{r} = \frac{-2}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}} = \frac{-2\sqrt{29}}{29}$

$\tan\theta = \frac{y}{x} = \frac{-5}{-2}$

$\tan\theta = \frac{5}{2}$

2. $\angle\theta$ is in standard position with its terminal arm in the stated quadrant, and $0 < \theta < 360°$. A trigonometric ratio is given. Find the exact values of the other two trigonometric ratios.

$\sin\theta = \frac{4}{5}$, Quadrant II

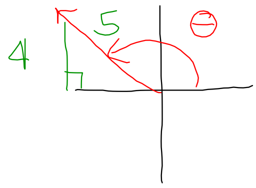
Would a diagram be helpful?

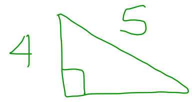
2. $\angle\theta$ is in standard position with its terminal arm in the stated quadrant, and $0 < \theta < 360°$. A trigonometric ratio is given. Find the exact values of the other two trigonometric ratios.

$\sin\theta = \frac{4}{5}$, Quadrant II

1. sketch θ

2. label Pythagorean Triangle





$y = 4$
$r = 5$
$x^2 + y^2 = r^2$
$x^2 + 4^2 = 5^2$
$x^2 = 25 - 16$
$\sqrt{x^2} = \pm\sqrt{9}$
$x = -3$

Quadrant II

$\cos\theta = \frac{x}{r}$

$\boxed{\cos\theta = \frac{-3}{5}}$

$\tan\theta = \frac{y}{x}$

$\boxed{\tan\theta = \frac{4}{-3}}$

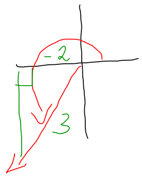
3. $\angle\theta$ is in standard position with its terminal arm in the stated quadrant, and $0 < \theta < 360°$. A trigonometric ratio is given. Find the exact values of the other two trigonometric ratios.

$\cos\theta = -\frac{2}{3}$, Quadrant III

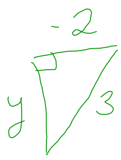
3. $\angle\theta$ is in standard position with its terminal arm in the stated quadrant, and $0 < \theta < 360°$. A trigonometric ratio is given. Find the exact values of the other two trigonometric ratios.

$\cos\theta = -\frac{2}{3}$, Quadrant III

1. sketch θ



2. label Pythagorean Triangle



$x = -2$
$r = 3$
$x^2 + y^2 = r^2$
$y^2 = 3^2 - (-2)^2$
$y^2 = 13$
$y = \pm\sqrt{13}$

Quadrant III

$\therefore y = -\sqrt{13}$

$\sin\theta = \frac{y}{r}$
$= \frac{-\sqrt{13}}{3}$

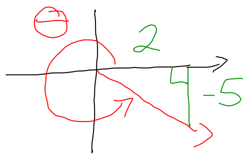
$\tan\theta = \frac{y}{x}$
$= \frac{-\sqrt{13}}{-2}$
$= \frac{\sqrt{13}}{2}$

4. $\tan\theta = -\frac{5}{2}$, Quadrant IV

4. $\tan\theta = -\frac{5}{2}$, Quadrant IV

1. sketch θ

2. label Pythagorean Triangle



$x = 2$

$y = -5$

$x^2 + y^2 = r^2$

$2^2 + (-5)^2 = r^2$

$29 = r^2$

$\pm\sqrt{29} = r$

$\sqrt{29} = r$

r is always positive.

$\sin\theta = \frac{y}{r}$

$= \frac{-5}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}}$

$= -\frac{5\sqrt{29}}{29}$

$\cos\theta = \frac{x}{r}$

$= \frac{2}{\sqrt{29}} \cdot \frac{\sqrt{29}}{\sqrt{29}}$

$= \frac{2\sqrt{29}}{29}$

5. Find θ where the terminal arm ends at point (-5, 3)

a. use cosine

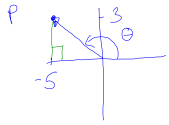
b. use sine

5. Find θ where the terminal arm ends at point (-5, 3)

a. use cosine

b. use sine

x = -5
y = 3



$x^2 + y^2 = r^2$

$(-5)^2 + 3^2 = r^2$

$34 = r^2$

$\sqrt{34} = r$

$\cos\theta = \frac{x}{r} = \frac{-5}{\sqrt{34}} = \frac{-5\sqrt{34}}{34}$

$\sin\theta = \frac{y}{r} = \frac{3}{\sqrt{34}} = \frac{3\sqrt{34}}{34}$

Calculate θ using $\cos^{-1}$ and $\sin^{-1}$ on Nspire.

$\cos^{-1}\left(\frac{-5\cdot\sqrt{34}}{34}\right)$	149.036
$\sin^{-1}\left(\frac{3\cdot\sqrt{34}}{34}\right)$	30.9638

θ = 149° this is correct

θ = 31° this is incorrect.

Remember there are always two answers for the Principal Angle θ between $0 < \theta < 360$

The Nspire gave us the answer for $\sin^{-1}$ in Q I BUT our angle is in Q II. This the Relative Acute Angle!

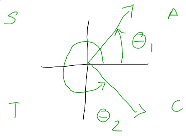
so $\theta = 180 - 31$
$= 149°$

the same answer from $\cos^{-1}$!!

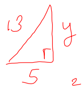
6. Determine x, y, r, and θ , where cos θ = 5/13

6. Determine x, y, r, and θ , where $\cos\theta = 5/13$

$\cos\theta = \frac{5}{13} = \frac{x}{r}$



There are 2 possible answers, θ can be in QI or QIV



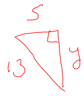
$y^2 = r^2 - x^2$

$y^2 = 13^2 - 5^2$

$y^2 = 144$

$y = \pm\sqrt{144}$

$y = \pm 12$



$x = 5$

$r = 13$

$y = \pm 12$

To find θ .... $\cos\theta = \frac{5}{13}$

$\cos\beta = \frac{5}{13}$

$\cos^{-1}\left(\frac{5}{13}\right)$ 67.3801

$\theta_1 = \beta$

$\doteq 67°$

$\theta_2 = 360 - \beta$

$= 360 - 67°$

$= 293°$

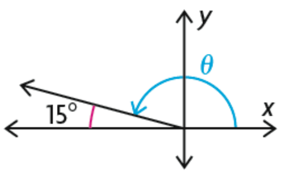
Try On Your Own #2

p. 299 #5ab, 8abc

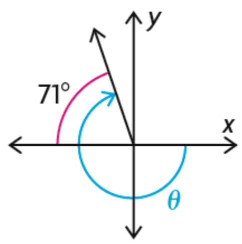
No Calculators - Trig Tables Only

5. i) For each angle θ, predict which primary trigonometric ratios are positive.
ii) Determine the primary trigonometric ratios to the nearest hundredth.

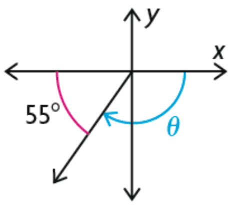
a)



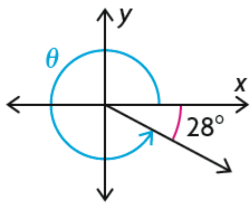
c)



b)



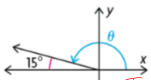
d)



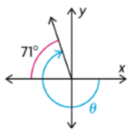
No Calculators - Trig Tables Only

5. i) For each angle θ, predict which primary trigonometric ratios are positive.
ii) Determine the primary trigonometric ratios to the nearest hundredth.

a)



c)



Ratios from Tables

$\beta = 15^\circ$

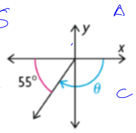
$\theta = 165^\circ$

$\sin 165^\circ = +0.2588$

$\cos 165^\circ = -0.9659$

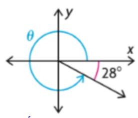
$\tan 165^\circ = -0.2679$

b) S A



T C

d)



$\beta = 55^\circ$

$\theta = -(180 - 55)$

$= -125^\circ$

$\sin(-125) = -0.8192$

$\cos(-125) = -0.5736$

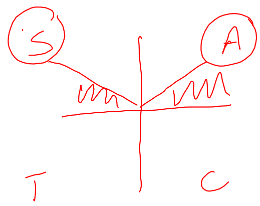
$\tan(-125) = +1.4281$

No Calculators - Trig Tables Only

8. Use each trigonometric ratio to determine all values of θ, to the nearest degree if $0° \leq \theta \leq 360°$.

a) $\sin\theta = 0.4815$
b) $\tan\theta = -0.1623$
c) $\cos\theta = -0.8722$
d) $\cot\theta = 8.1516$
e) $\csc\theta = -2.3424$
f) $\sec\theta = 0$

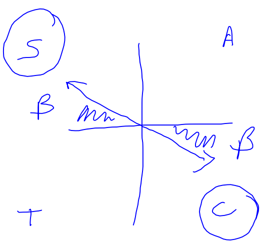
a) $\sin\theta = 0.4815$ $\qquad \theta_1 = 29°$
$\sin\beta = 0.4815$ $\qquad \theta_2 = 180 - 29$
$\beta \doteq 29°$ $\qquad = 151°$



8. Use each trigonometric ratio to determine all values of θ, to the nearest degree if $0° \leq \theta \leq 360°$.

a) $\sin\theta = 0.4815$
b) $\tan\theta = -0.1623$
c) $\cos\theta = -0.8722$
~~**d)** $\cot\theta = 8.1516$~~
~~**e)** $\csc\theta = -2.3424$~~
~~**f)** $\sec\theta = 0$~~

b) $\tan\theta = -0.1623$
$\tan\beta = 0.1623$
$\beta \doteq 9°$



$\theta_1 = 180 - 9$
$= 171°$

$\theta_2 = 360 - 9$
$= 351°$

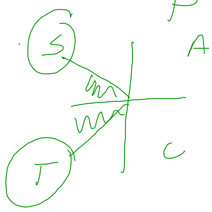
8. Use each trigonometric ratio to determine all values of θ, to the nearest degree if $0° \leq \theta \leq 360°$.

a) $\sin\theta = 0.4815$
b) $\tan\theta = -0.1623$
c) $\cos\theta = -0.8722$
d) $\cot\theta = 8.1516$
e) $\csc\theta = -2.3424$
f) $\sec\theta = 0$

c) $\cos\theta = -0.8722$

$\cos\beta = 0.8722$

$\beta \doteq 29°$



$\theta_1 = 180 - 29$
$= 151°$

$\theta_2 = 180 + 29$
$= 209°$

Additional Practice

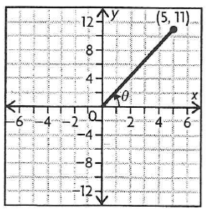
pg 299 # 2, 6, 10, 12

2. Each point lies on the terminal arm of angle θ in standard position.

i) Draw a sketch of each angle θ.
ii) Determine the value of r to the nearest tenth.
iii) Determine the primary trigonometric ratios for angle θ.
iv) Calculate the value of θ to the nearest degree.

a) $(5, 11)$ **b)** $(-8, 3)$ **c)** $(-5, -8)$ **d)** $(6, -8)$

2. a) i)



ii) $r^2 = x^2 + y^2$

$r^2 = 5^2 + 11^2$

$r^2 = 146$

$r = \sqrt{146}$

$= 12.1$

Not Exact, Leave as radical!

iii) $\sin\theta = \frac{y}{r}$

$\sin\theta = \frac{11}{12.1}$

$\cos\theta = \frac{x}{r}$

$\cos\theta = \frac{5}{12.1}$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{11}{5}$

iv) $\tan\theta = \frac{11}{5}$

$\theta = \tan^{-1}\frac{11}{5}$

$= 66°$

b, c, d answers see textbook.

see text book for answers

6. Angle θ is a principal angle that lies in quadrant 2 such that $0° \leq \theta \leq 360°$.
K Given each trigonometric ratio,

i) determine the exact values of x, y, and r
ii) sketch angle θ in standard position
iii) determine the principal angle θ and the related acute angle β to the nearest degree

a) $\sin\theta = \frac{1}{3}$

~~**d)**~~ $\cancel{\csc\theta = 2.5}$

~~**b)**~~ $\cancel{\cot\theta = -\frac{4}{3}}$

e) $\tan\theta = -1.1$

c) $\cos\theta = -\frac{1}{4}$

~~**f)**~~ $\cancel{\sec\theta = -3.5}$

see text book for answers

10. Given each point $P(x, y)$ lying on the terminal arm of angle θ,

i) state the value of θ, using both a counterclockwise and a clockwise rotation

ii) determine the primary trigonometric ratios

a) $P(-1, -1)$

b) $P(0, -1)$

c) $P(-1, 0)$

d) $P(1, 0)$

see text book for answers

12. Given $\cos\theta = -\frac{5}{12}$, where $0° \leq \theta \leq 360°$,

a) in which quadrant could the terminal arm of θ lie?

b) determine all possible primary trigonometric ratios for θ.

c) evaluate all possible values of θ to the nearest degree.

Attachments

Unit Circle copy.gsp