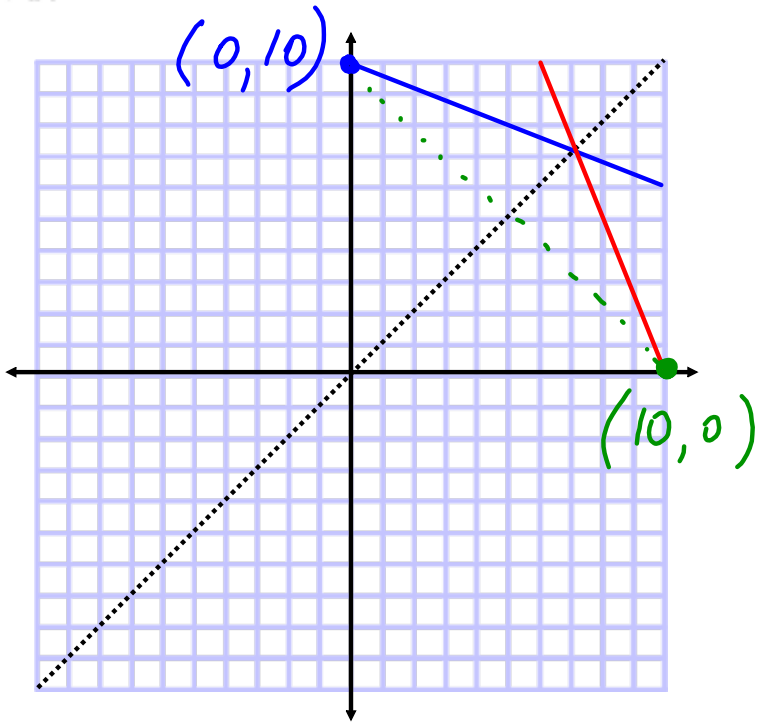
PLEASE WATCH THE VIDEO OF THE FOLDING ACTIVITY FIRST

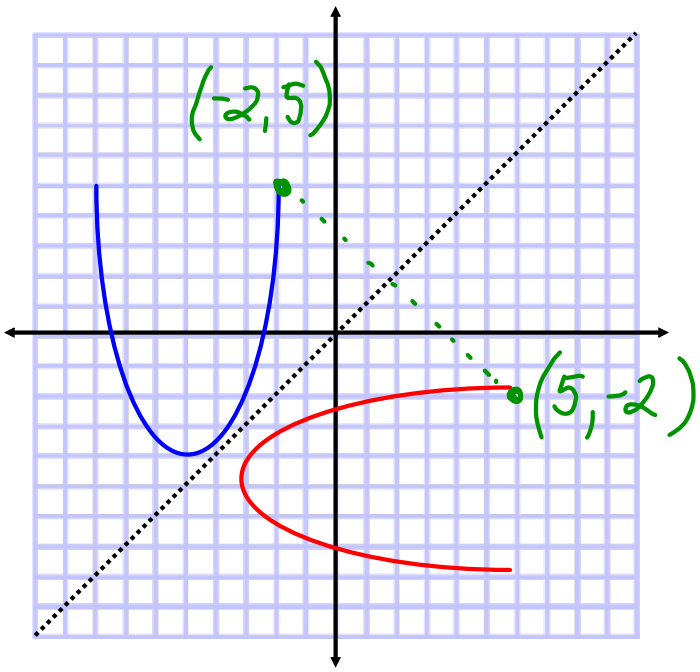
Inverse Functions

Learning Goals
- determine the inverse of a function algebraically
- investigate properties of inverse functions

$h(x) = -\frac{2}{5}x + 10$



$g(x) = (x+5)^2 - 4$



What do you notice about the coordinates of the new points?

they are reversed

How are the graphs different / same?

same --- shape

different --- orientation

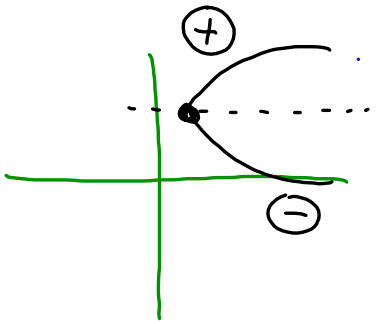
What do you notice about the equations of the new lines?

Straight line

$y = -\frac{2}{5}x + 10$ $\quad y^{-1} = -\frac{5}{2}x + 25$

Parabola

$y = (x+5)^2 - 4$ $\quad y^{-1} = \pm\sqrt{x+4} - 5$



Inverse - is the mirror image of the original function over the line $y=x$

- denoted by $f^{-1}(x)$

$\sin\theta = 0.5$

$\sin^{-1}$

Finding the inverse

Method 1

1. Exchange x and y
2. Solve for y

ex. $y=2x-8$

$x = 2y - 8$

$x + 8 = 2y$

$\frac{x}{2} + 4 = y$

Method 2

1. List all operations in order
2. Apply the opposite operations in opposite order

ex. $y=3x^2-5$

1. Square
2. $\times 3$
3. -5 ⊛

1. $+5$
2. $\div 3$
3. $\sqrt{\ }$

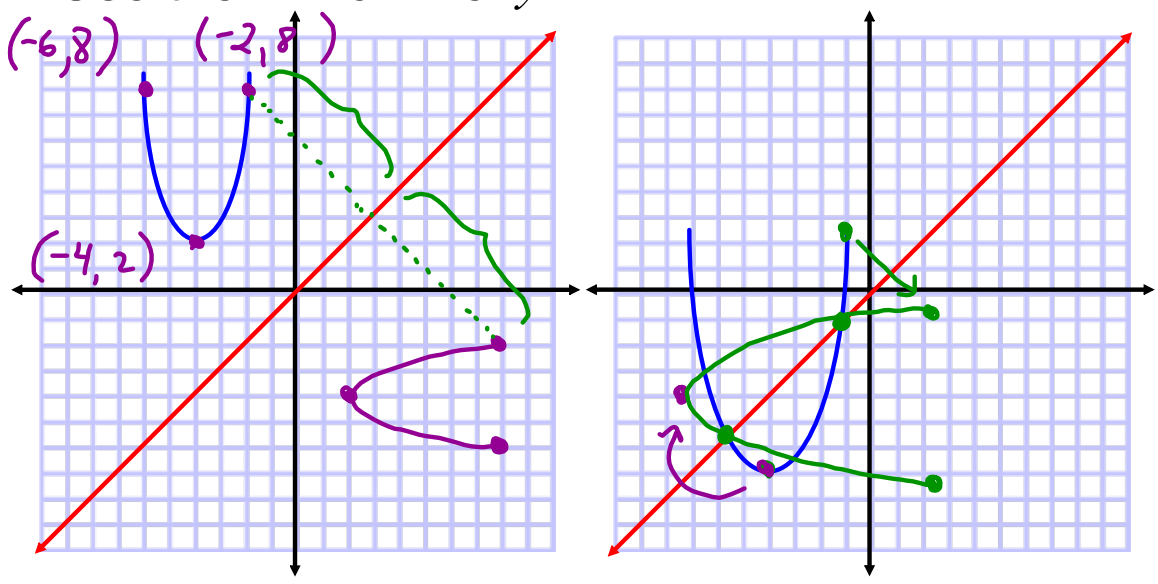
$$y^{-1} = \pm\sqrt{\frac{x+5}{3}}$$

Method 3 - On a graph

Reverse coordinates and graph

OR

Use the mirror line $y=x$



Find the inverse

$y=2-3\sqrt{x}$

$x = 2 - 3\sqrt{y}$

$x - 2 = -3\sqrt{y}$

$\frac{x-2}{-3} = \sqrt{y}$

$\left(\frac{x-2}{-3}\right)^2 = y^{-1}$

$y = -\frac{1}{4}x^2 - 2$

$x = -\frac{1}{4}y^2 - 2$

$x + 2 = -\frac{1}{4}y^2$

$-4x - 8 = y^2$

$\pm\sqrt{-4x-8} = y^{-1}$

That's the end of the '**Directed Lesson**' and now you will be **"Trying it On Your Own"** with a twist.

Try On Your Own #1 works best if you can print the pdf (3 pages). Work through each page of examples and then check your answers - don't just go to the solutions.

In the process you will be recalling how to graph by hand:

Linear Functions using a Table Of Values (Grade 9) and $y = mx + b$

Quadratic Functions using Parent Functions and transformations.

In the process you will be recalling Algebra Skills:

Balancing of Equations (Grade 9)

Solving Quadratic Equations (Grade 11)

Once you have completed the Try On Your Own #1
the final slide will ask you to ...

Time to Go Back and Wonder ....

What do you notice about

the transformations from

the original to the inverse?

Compare the values of a, k ,d and c

Are they 'opposites'?

Try On Your Own #1

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Inverse Functions

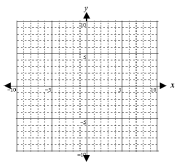
Properties of Inverse Relations
Given a function f :

- f^{-1} is the name for the inverse relation
- if $(a,b) \in f$, then $(b,a) \in f^{-1}$ (Read: if the point (a, b) is a solution of the function f , then the point (b, a) is a solution of the inverse relation f^{-1})
- The domain of f is the range of f^{-1}, and the range of f is the domain of f^{-1}
- The graph of $y = f^{-1}(x)$ is a reflection of $y = f(x)$ about the line $y = x$
- To determine the equation of the inverse, interchange x and y, then solve for y.

#1

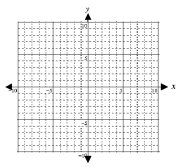
a) Given f(x) = 3x - 6 create a Table of Values, and graph.

x	$y=3x-6$
-1	
0	
1	
2	
3	



b) Create a table of values for the ***inverse*** and graph the inverse relation.

x	y



c)Determine the equation of the inverse.

a) Evaluate the following using the Table of Values

i) $f(-1)$ ii) $f^{-1}(-9)$ iii) $f(2)$ iv) $f^{-1}(0)$

b) Evaluate the following using Algebra

v) $f^{-1}(a-4)$

Try On Your Own #1 - solutions

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Inverse Functions

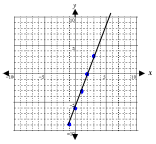
Properties of Inverse Relations
Given a function f :

- f^{-1} is the name for the inverse relation
- if $(a,b) \in f$, then $(b,a) \in f^{-1}$ (Read: if the point (a, b) is a solution of the function f , then the point (b, a) is a solution of the inverse relation f^{-1})
- The domain of f is the range of f^{-1}, and the range of f is the domain of f^{-1}
- The graph of $y = f^{-1}(x)$ is a reflection of $y = f(x)$ about the line $y = x$
- To determine the equation of the inverse, interchange x and y, then solve for y.

#1

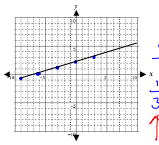
a) Given f(x) = 3x - 6 create a Table of Values, and graph.

x	$y=3x-6$
-1	$3(-1)-6=-9$
0	$3(0)-6=-6$
1	$3(1)-6=-3$
2	$3(2)-6=0$
3	$3(3)-6=3$



b) Create a table of values for the ***inverse*** and graph the inverse relation.

x	y
-9	-1
-6	0
-3	1
0	2
3	3



c)Determine the equation of the inverse.

$y = 3x - 6$

inverse...

$x = 3y - 6$

$x + 6 = 3y - 6 + 6$

$\frac{x+6}{3} = \frac{3y}{3}$

$\frac{x}{3} + \frac{6}{3} = y$

$\frac{1}{3}x + 2 = y$

slope, y intercept

a) Evaluate the following using the Table of Values

i) $f(-1) = -9$ ii) $f^{-1}(-9) = -1$ iii) $f(2) = 0$ iv) $f^{-1}(0) = 2$ "back to where you started from"

b) Evaluate the following using Algebra

v) $f^{-1}(a-4)$

from * $f^{-1}(x) = \frac{1}{3}x + 2$

$f^{-1}(a-6) = \frac{1}{3}(a-6)+4$

$= \frac{1}{3}a - \frac{6}{3} + 4$

$= \frac{1}{3}a - 2 + 4$

$= \frac{1}{3}a + 2$

Try On Your Own #1

MCR 3U

#2

Determine the inverse of each function algebraically.

a) $f(x) = -2 - 3x$ b) $f(x) = -\frac{1}{4}x - 2$

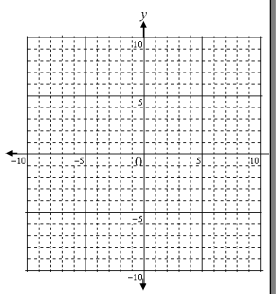
#3

1) Graph ***$f(x) = (x-2)^2 + 6$***
Graph the Parent Function and transformations

2) Identify five points

3) Graph the Inverse by swapping x and y.

4) Determine the equation of the inverse from the graph, using your knowledge of the Square Root function and transformations *(hint - there will be two equations)*



5) Now determine the Inverse Relation using only algebra.

Try On Your Own #1 - answers

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#2

Determine the inverse of each function algebraically.

a) $f(x) = -2 - 3x$ b) $f(x) = -\frac{1}{4}x - 2$

$-\frac{x+2}{3} = y$ $-4x - 8 = y$

#3

1) Graph $f(x) = (x-2)^2 + 6$

Graph the Parent Function and transformations

2) Identify five points

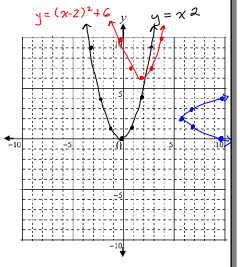
3) Graph the Inverse by swapping x and y.

4) Determine the equation of the inverse from the graph, using your knowledge of the Square Root function and transformations *(hint - there will be two equations)*

$y = \sqrt{x-6} + 2$

$y = -\sqrt{x-6} + 2$

5) Now determine the Inverse Relation using only algebra.



$\sqrt{x-6} + 2 = y$ $-\sqrt{x-6} + 2 = y$

Try On Your Own #1 - solutions

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#2

Determine the inverse of each function algebraically.

a) $f(x) = -2 - 3x$ b) $f(x) = -\frac{1}{4}x - 2$

$y = -2 - 3x$

$x = -2 - 3y$

$x + 2 = -2 - 3y + 2$

$\frac{x+2}{-3} = \frac{-3y}{-3}$

$-\frac{x+2}{3} = y$

$y = -\frac{1}{4}x - 2$

$x = -\frac{1}{4}y - 2$

$x + 2 = -\frac{1}{4}y - 2 + 2$

$-4(x+2) = \left(-\frac{1}{4}y\right) \cdot -4$

$-4x - 8 = y$

#3

1) Graph $f(x) = (x-2)^2 + 6$

Graph the Parent Function and transformations Right 2

$y = x^2$, Horizontal Translation, Vertical Translation up 6

2) Identify five points

3) Graph the Inverse by swapping x and y.

4) Determine the equation of the inverse from the graph, using your knowledge of the ~~Square Root function~~ and transformations *(hint - there will be two equations)*

$f(x) = \sqrt{x}$ Right 6, up 2 $y = \sqrt{x-6} + 2$

and

$f(x) = -\sqrt{x}$ (Reflection x) $y = -\sqrt{x-6} + 2$

5) Now determine the Inverse Relation using only algebra.

$y = (x-2)^2 + 6$

Inverse $x = (y-2)^2 + 6$

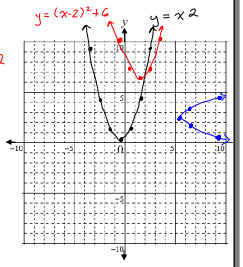
$x - 6 = (y-2)^2 + 6 - 6$

$\pm\sqrt{x-6} = \sqrt{(y-2)^2}$

$\pm\sqrt{x-6} = y - 2$

$\sqrt{x-6} = y - 2$ $-\sqrt{x-6} = y - 2$

$\sqrt{x-6} + 2 = y$ $-\sqrt{x-6} + 2 = y$



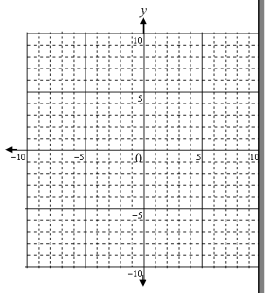
Try On Your Own #1

MCR 3U

#4

1) Graph $f(x) = -2(x+5)^2 + 2$

Graph the Parent Function and transformations



2) Identify five points

3) Graph the Inverse by swapping x and y.

4) Determine the equation of the inverse from the graph, using your knowledge of the Square Root function and Transformations.
(hint - there will be two equations)

5) Now determine the Inverse Relation using only algebra.

#5

1) Given $f(x) = -(x+5)^2 - 7$, determine the Inverse algebraically.

2) Graph the original function and the inverse using graphing technology

3) 'Sketch' the graphs of the original function and the inverse

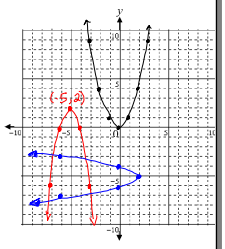
Try On Your Own #1 - answers

MCR 3U

#4

1) Graph $f(x) = -2(x+5)^2 + 2$

Graph the Parent Function and transformations



2) Identify five points

3) Graph the Inverse by swapping x and y.

4) Determine the equation of the inverse from the graph, using your knowledge of the Square Root function and Transformations.
(hint - there will be two equations)

5) Now determine the Inverse Relation using only algebra.

$y = \sqrt{-\frac{1}{2}(x-2)} - 5$ and $y = -\sqrt{-\frac{1}{2}(x-2)} - 5$

$y = \sqrt{-\frac{1}{2}(x-2)} - 5$ and $y = -\sqrt{-\frac{1}{2}(x-2)} - 5$

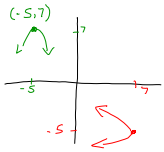
#5

1) Given $f(x) = -(x+5)^2 - 7$, determine the Inverse algebraically.

2) Graph the original function and the inverse using graphing technology

3) 'Sketch' the graphs of the original function and the inverse

$y = \sqrt{-(x+7)} - 5$ and $y = -\sqrt{-(x+7)} - 5$



Try On Your Own #1 - solutions

MCR 3U

#4

1) Graph $f(x) = -2(x+5)^2 + 2$

Graph the Parent Function and transformations

$y = x^2$ Reflection on x axis → $a = 2$
Vertical Stretch (x2)
Horizontal Translation Left 5 $d = -5$
Vertical Translation Up 2 → $c = 2$

2) Identify five points

3) Graph the Inverse by swapping x and y.

4) Determine the equation of the inverse from the graph, using your knowledge of the Square Root function and Transformations.

(hint - there will be two equations)

$y = \sqrt{x}$
$y = -\sqrt{x}$

Reflection on y axis → $k = -\frac{1}{2}$
Horizontal stretch x2
Horizontal Translation Right 2 → $d = 2$
Vertical Translation Down 5 → $c = -5$

$y = a\sqrt{k(x-d)} + c$

$y = \sqrt{-\frac{1}{2}(x-2)} - 5$ and $y = -\sqrt{-\frac{1}{2}(x-2)} - 5$

5) Now determine the Inverse Relation using only algebra.

$y = -2(x+5)^2 + 2$

Inverse

$x = -2(y+5)^2 + 2$

$\frac{x-2}{-2} = \frac{-2(y+5)^2}{-2}$

$\sqrt{-\frac{1}{2}(x-2)} = \sqrt{(y+5)^2}$

$\sqrt{-\frac{1}{2}(x-2)} = y+5$

$y = \sqrt{-\frac{1}{2}(x-2)} - 5$ and $y = -\sqrt{-\frac{1}{2}(x-2)} - 5$

#5

1) Given $f(x) = -(x+5)^2 - 7$, determine the Inverse algebraically.

2) Graph the original function and the inverse using graphing technology

3) 'Sketch' the graphs of the original function and the inverse

$y = -(x+5)^2 - 7$

Inverse

$x = -(y+5)^2 - 7$

$\frac{x+7}{-1} = \frac{-1(y+5)^2}{-1}$

$\sqrt{-1(x+7)} = \sqrt{(y+5)^2}$

$\sqrt{-1(x+7)} = y+5$

$\sqrt{-(x+7)} - 5 = y$

$y = a\sqrt{k(x-d)} + c$

$y = \sqrt{-(x+7)} - 5$ and $y = -\sqrt{-(x+7)} - 5$

$k = -1$ → Reflection on y
$d = -7$ → H.T. Left 7
$c = -5$ → V.T. Down 5

(-5,7)

Time to Go Back and Wonder ....

What do you notice about

the transformations from

the original to the inverse?

Compare the values of a, k ,d and c

Are they 'opposites'?

Try On Your Own #2

pg. 46 # 4ab, 9ace, 16

4. For each linear function, interchange x and y. Then solve for y to determine the inverse.

a) $y = 4x - 3$

b) $y = 2 - \frac{1}{2}x$

c) $3x + 4y = 6$

d) $2y - 10 = 5x$

9. a) Determine f^{-1} for the linear function $f(x) = 5x - 2$.
b) Graph f and f^{-1} on the same axes.
c) Explain how you can tell that f^{-1} is also a linear function.
d) State the coordinates of any points that are common to both f and f^{-1}.
e) Compare the slopes of the two lines.

16. The ordered pair (1, 5) belongs to a function f. Explain why the ordered pair (2, 1) cannot belong to f^{-1}.

4. a) $x = 4y - 3$

$x + 3 = 4y$

$y = \frac{x + 3}{4}$

b) $x = 2 - \frac{y}{2}$

$x + 2 = \frac{-y}{2}$

$y = 2(x - 2)$

9. a) $x = 5f^{-1}(x) - 2$

$x + 2 = 5f^{-1}(x)$

$f^{-1}(x) = \frac{x + 2}{5}$

c) The graph of $f^{-1}(x)$ is a straight line, so it is linear.

e) The slopes of the two lines are reciprocals of each other.

16. If $(2, 1)$ belonged to f^{-1}, then $(1, 2)$ would belong to f, and since $(1, 5)$ also belongs to f, then f would not be a function. So, since f is a function, $(2, 1)$ could not belong to f^{-1}.

Try On Your Own #3

pg. 160 # 3, 5

3. Given $f(x) = 2x^2 - 1$, determine the equation of the inverse.

5. **a)** Sketch the graph of $f(x) = 3(x - 2)^2 - 2$.
b) Sketch the graph of its inverse on the same axes.
c) Determine the equation of the inverse.

3. $f(x) = 2x^2 - 1$: set $y = 2x^2 - 1$, then switch x and y, and solve for y.

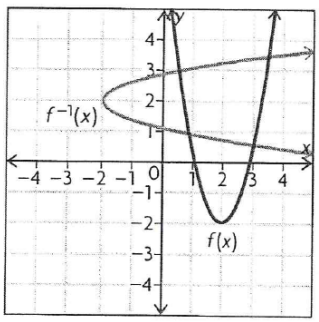
$$x = 2y^2 - 1$$
$$x + 1 = 2y^2$$
$$y^2 = \frac{x + 1}{2}$$
$$y = \pm\sqrt{\frac{x + 1}{2}}$$

So I get

$$f^{-1}(x) = \pm\sqrt{\frac{x + 1}{2}}$$

5. a)–b) $f(x) = 3(x - 2)^2 - 2$: this will stretch the graph of $y = x^2$ by a factor of 3, shift it right by 2 units, then down by 2 units.

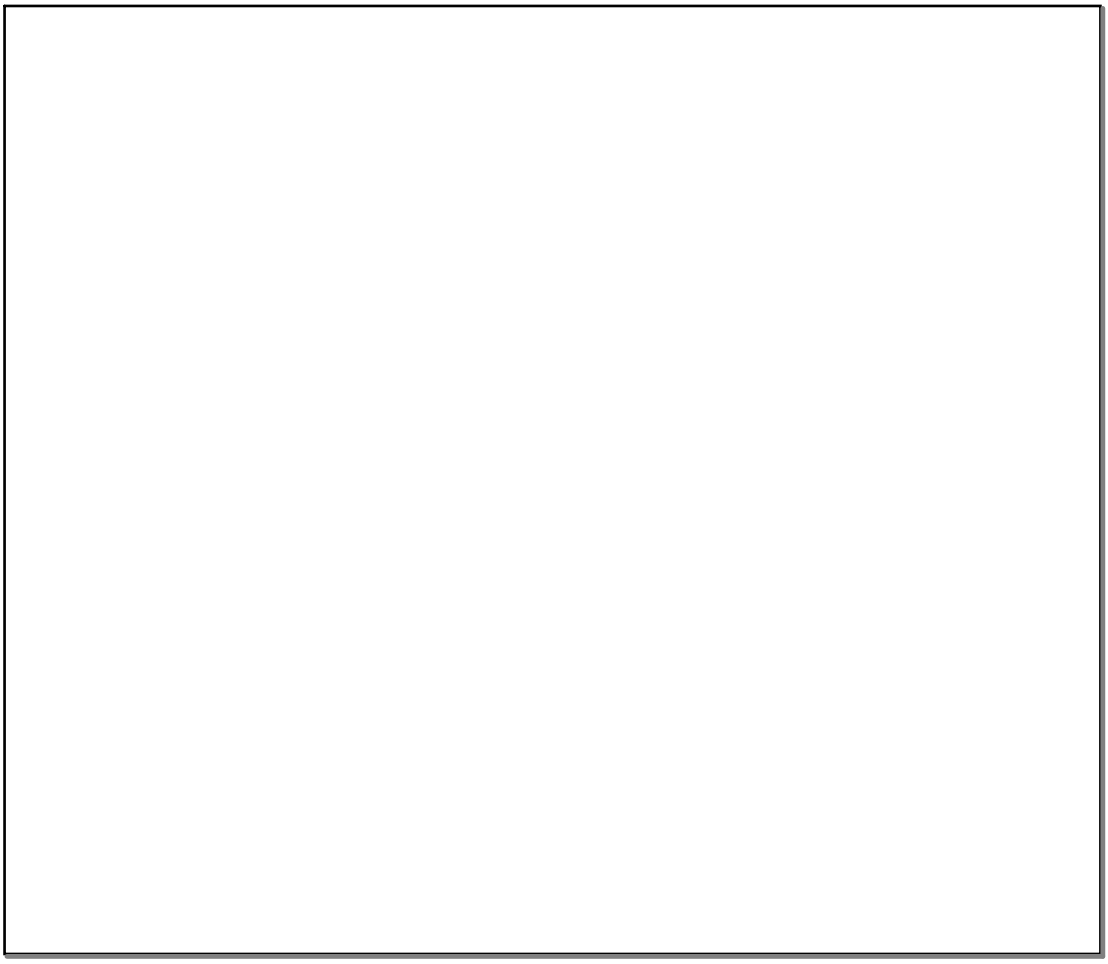
To get the graph of the inverse, $y = f^{-1}(x)$, I reflect the graph of $y = f(x)$ in the line $y = x$.



$a = 3$
$d = 2$
$c = -2$

$k = \frac{1}{3}$
$d = -2$
$c = 2$

$f^{-1}(x) = \pm\sqrt{\frac{1}{3}(x+2)} + 2$



Attachments

http://www.mathplayground.com/functionmachine.html