## Warm - up #1

Determine the equation of a parabola that

a. has zeros 5 and -2 and goes through (1,36)

b. has a vertex of (4, -5) and goes through (7, -23)

## Warm - Up #1 - Solution

Determine the equation of a parabola that

a. has zeros 5 and -2 and goes through (1,36)

b. has a vertex of (4, -5) and goes through (7, -23)

a., $f(x) = -3(x-5)(x+2)$

b., $-23 = a(7-4)^2 - 5$

$-23 = a(3)^2 - 5$

$-23 = 9a - 5$

$-18 = 9a$

$-2 = a$

$\therefore f(x) = -2(x-4)^2 - 5$

# Warm - Up #2

Determine the equation of a parabola that

has zeros $\sqrt{3}, -2\sqrt{3}$ and goes through $(3\sqrt{3}, 90)$

verify your answer by graphing with technology.

Warm - up #2

Determine the equation of a parabola that

has zeros $\sqrt{3}, -2\sqrt{3}$ and goes through $(3\sqrt{3}, 90)$

verify your answer by graphing with technology.

$f(x) = a(x-s)(x-t)$

$f(x) = a(x-\sqrt{3})(x+2\sqrt{3})$

$90 = a(3\sqrt{3}-\sqrt{3})(3\sqrt{3}+2\sqrt{3})$

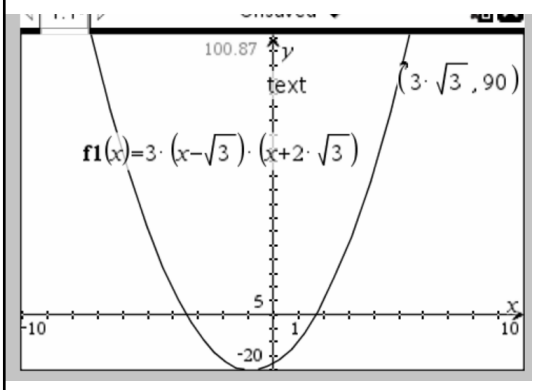
$90 = a(2\sqrt{3})(5\sqrt{3})$

$90 = a(10 \cdot 3)$

$\frac{90}{30} = \frac{30a}{30}$

$3 = a$

$\therefore f(x) = 3(x-\sqrt{3})(x+2\sqrt{3})$

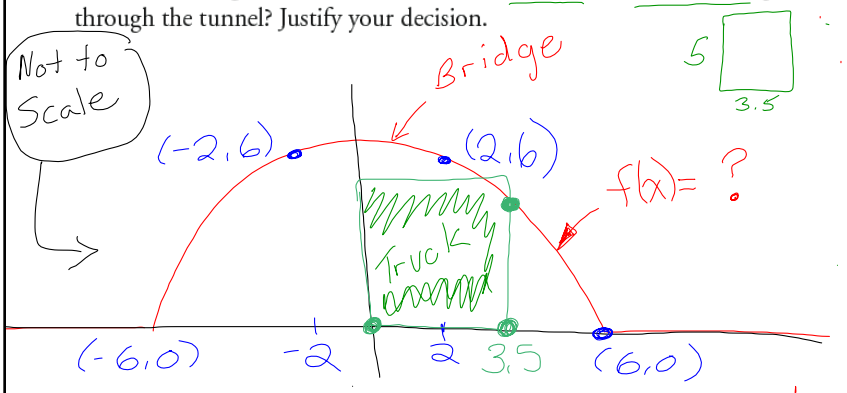


pg 192 # Try On Your Own #2 from Lesson 3

**10.** A tunnel with a parabolic arch is 12 m wide. If the height of the arch 4 m from the left edge is 6 m, can a truck that is 5 m tall and 3.5 m wide pass through the tunnel? Justify your decision.

Is it wise to drive down the middle of a tunnel ?


**10.** A tunnel with a parabolic arch is 12 m wide. If the height of the arch 4 m from the left edge is 6 m, can a truck that is 5 m tall and 3.5 m wide pass through the tunnel? Justify your decision.




Determine an equation to model the bridge

$f(x) = a(x-s)(x-t)$

Determine the Height of the bridge 3.5 m from the middle, if the height is more than 5m the truck can fit.

is $f(3.5) > 5$ ?

*Turn on the Video*

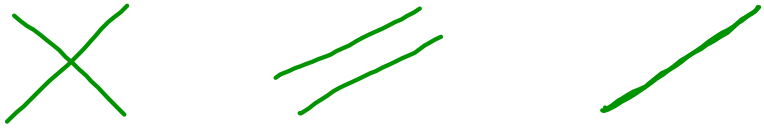
# Linear - Quadratic Systems

**Learning Goal**

- **Find the point(s) of intersection of a linear-quadratic system**
- **Find the number of POI of a linear-quadratic system**

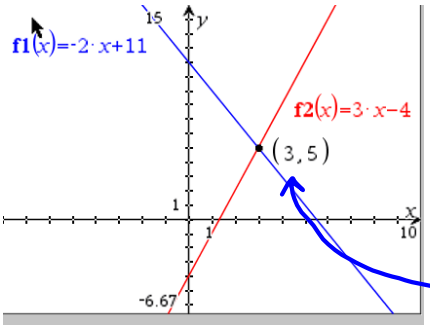
## Recall Linear Systems from Grade 10

**How many times can two lines intersect?**



**How did you find the intersection point?**

**y = -2x +11 and y = 3x - 4**




$-2x + 11 = 3x - 4$

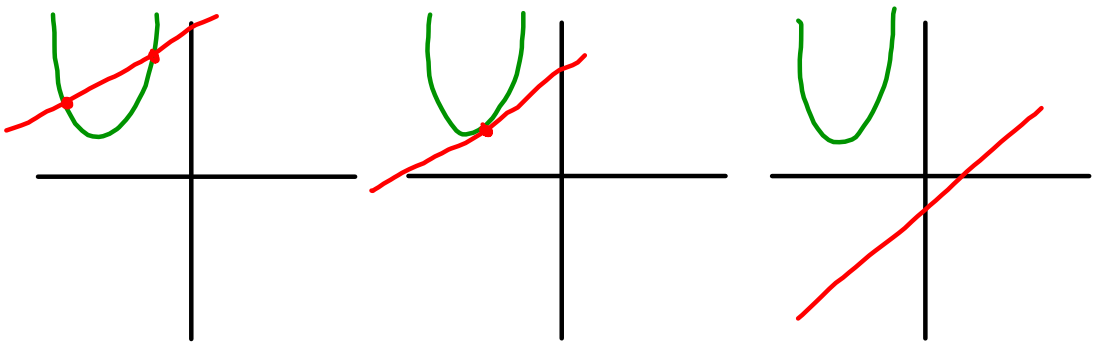
$15 = 5x$

$3 = x$

## Now ... Think Quadratic and Linear Systems

**How many times can a parabola and a line intersect?**

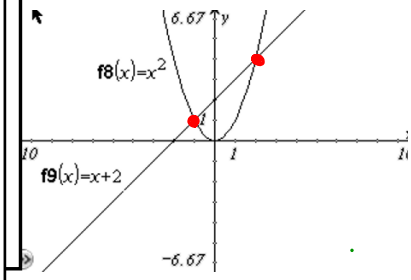
**How do you solve for the intersection points.**



**Determine using algebra ...**
**the Points of Intersection between**

**$f(x) = x^2$ and $g(x) = x + 2$**

First graph it ...




$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2$ $\quad$ $x = -1$

$f(2) = 2^2$

$= 4$

$(2, 4)$

$f(-1) = (-1)^2$

$= 1$

$(-1, 1)$

**Pause the video and try on your own...**

**Determine using algebra ...**
**the Points of Intersection between**

**$f(x) = x^2+1$ and $g(x) = 2x + 4$**

**Pause the video and try on your own...**

**Determine using algebra ...**
**the Points of Intersection between**

**$f(x) = x^2+1$ and $g(x) = 2x + 4$**

$f(x) = g(x)$

$x^2 + 1 = 2x + 4$

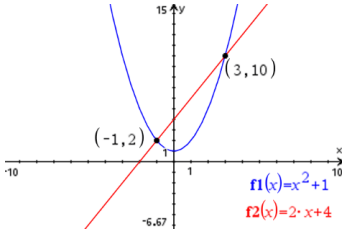
$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

$x = 3$ $\quad$ $x = -1$

$f(3) = 3^2 + 1 = 9 + 1 = 10$ $\quad$ $(3, 10)$

$f(-1) = (-1)^2 + 1 = 1 + 1 = 2$ $\quad$ $(-1, 2)$




**Determine using algebra ...**

**the *number* of points of intersection**
**of the quadratic and linear functions.**

$f(x) = 3x^2 + 12x + 14$ $\quad$ $g(x) = 2x - 8$

1. Set equations equal to each other.

$3x^2 + 12x + 14 = 2x - 8$

2. Move everything to one side.

$3x^2 + 10x + 22 = 0$

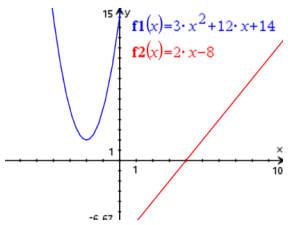
3. Use the discriminant to determine number of solutions.

$b^2 - 4ac$

$10^2 - 4(3)(22)$

$= 100 - 264$

$= -263$ $\longrightarrow$ no answer
no POI



**Determine the value of "k"**

**so that there is one Point of Intersection**

$f(x) = 3x^2 + 12x + 14 \qquad g(x) = 4x + k$

1. Set equations equal to each other.

$3x^2 + 12x + 14 = 4x + k$

2. Move everything to one side.

$3x^2 + 8x + 14 - k = 0$

3. Use the discriminant to find "k".

$b^2 - 4ac$

$= 8^2 - 4(3)(14 - k)$

$= 64 - 12(14 - k)$

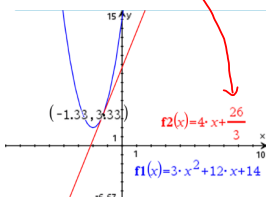
$\stackrel{?}{=} 64 - 168 + 12k$

$0 = -104 + 12k$

$-12k = -104$

$k = \frac{+104}{12}$

$k = \frac{26}{3}$




# Set the Plan and then do ...

***Solve by Algebra***

Adam has decided to celebrate his birthday by going skydiving. He loves to freefall so he will wait for some time before opening his parachute.

*His height after jumping from the airplane during the freefall can be modelled by the quadratic function $h_1 = -4.9t^2 + 5500$, where t is the time in seconds and h(t) is the height above the ground, in metres, t seconds after jumping out.

*After he releases his parachute, he begins falling at a constant rate. His height above the ground can be modelled by the linear function $h_2 = -5t + 4500$.

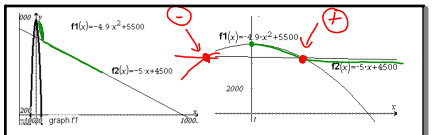
How long after jumping out of the airplane did Adam release his parachute?

How high was he at that time?

## Plan

Draw a graph

Find point of intersection (t, h)




$-4.9x^2 + 5500 = -5x + 4500$

$-4.9x^2 + 5x + 1000 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-5 \pm \sqrt{25 - 4(-4.9)(1000)}}{2(-4.9)}$

$= \dfrac{-5 \pm \sqrt{19625}}{-9.8}$

$= \dfrac{-5 \pm 140.09}{-9.8}$

$-13.8$ ↑ back in time

$14.8$ ∴ he opened the parachute at 14.8 sec

$f(14.8) = -5(14.8) + 4500$

$= 4426$

∴ he was at a height of 4426 m.

# Set the Plan and then do ...

**The height of a baseball, after it is tossed out of a window is modelled by the function**

$$h(t) = -5t^2 + 20t + 15$$

**where t is the time since it was tossed in seconds and h is the height in metres.**

**A boy shoots at the baseball with a paintball gun. He shoots at the exact same time as the baseball is thrown. The trajectory of the paintball is given by the function**

$$g(t) = 3t + 3$$

**where t is the time since it was shot in seconds and g is the height in metres.**

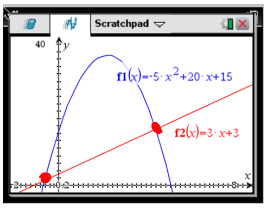
**Will the paintball hit the baseball?**

**If so, when? At what height will the baseball be?**

## Plan

Draw a graph

Find intersection point




$-5x^2 + 20x + 15 = 3x + 3$

$-5x^2 + 17x + 12 = 0$

$5x^2 - 17x - 12 = 0$

$5x^2 - 20x + 3x - 12 = 0$

$5x(x-4) + 3(x-4) = 0$

$(x-4)(5x+3) = 0$

$x = 4 \qquad x = \frac{-3}{5}$

↑ can't go back in time

-60, -20, 3, -17

$f(4) = 3(4) + 3$

$= 15$

∴ the paint is going to hit at 4 sec at a height of 15 m

## Try On Your Own

pg 198 # 4ab, 8-11 p. 186 #14

**4.** Determine the point(s) of intersection of each pair of functions.

K **a)** $f(x) = -2x^2 - 5x + 20, g(x) = 6x - 1$

**b)** $f(x) = 3x^2 - 2, g(x) = x + 7$

**c)** $f(x) = 5x^2 + x - 2, g(x) = -3x - 6$

**d)** $f(x) = -4x^2 - 2x + 3, g(x) = 5x + 4$

**8.** Determine the value of $k$ such that $g(x) = 3x + k$ intersects the quadratic function $f(x) = 2x^2 - 5x + 3$ at exactly one point.

**9.** Determine the value(s) of $k$ such that the linear function $g(x) = 4x + k$ does not intersect the parabola $f(x) = -3x^2 - x + 4$.

**10.** A daredevil jumps off the CN Tower and falls freely for several seconds before A releasing his parachute. His height, $h(t)$, in metres, $t$ seconds after jumping can be modelled by

$h_1(t) = -4.9t^2 + t + 360$ before he released his parachute; and
$h_2(t) = -4t + 142$ after he released his parachute.

How long after jumping did the daredevil release his parachute?

**11.** A quadratic function is defined by $f(x) = 3x^2 + 4x - 2$. A linear function T is defined by $g(x) = mx - 5$. What value(s) of the slope of the line would make it a tangent to the parabola?

**14.** If $f(x) = x^2 - 6x + 14$ and $g(x) = -x^2 - 20x - k$, determine T the value of $k$ so that there is exactly one point of intersection between the two parabolas.

Try On Your Own - Solutions

pg 198 # 4ab, 8-11,14

**4.** Determine the point(s) of intersection of each pair of functions.

K **a)** $f(x) = -2x^2 - 5x + 20, g(x) = 6x - 1$

**b)** $f(x) = 3x^2 - 2, g(x) = x + 7$

**c)** $f(x) = 5x^2 + x - 2, g(x) = -3x - 6$

**d)** $f(x) = -4x^2 - 2x + 3, g(x) = 5x + 4$

b) $f(x) = g(x)$

$3x^2 - 2 = x + 7$

$3x^2 - x - 9 = 0$

doesn't factor ∴ solve using Q.F.

$a = 3 \quad b = -1 \quad c = -9$

$x = \frac{1 \pm \sqrt{(-1)^2 - 4(3)(-9)}}{2(3)}$

$x = 1.91$ or $x = -1.57$

Then use $g(x)$ to find "y value"

$g(x) = x + 7$

$g(1.91) = 1.91 + 7 = 8.91$

$g(-1.57) = -1.57 + 7 = 5.43$

Therefore POI's

$(1.91, 8.91) \quad (-1.57, 5.43)$

**8.** Determine the value of $k$ such that $g(x) = 3x + k$ intersects the quadratic function $f(x) = 2x^2 - 5x + 3$ at exactly one point.

$f(x) = g(x)$

$2x^2 - 5x + 3 = 3x + k$

$2x^2 - 5x - 3x + 3 - k = 0$

$2x^2 - 8x + 3 - k = 0$

one intersection point $\Rightarrow$ one solution ∴ $b^2 - 4ac = 0$

$a = 2 \quad b = -8 \quad c = 3 - k$

$b^2 - 4ac = 0$

$(-8)^2 - 4(2)(3-k) = 0$

$64 - 8(3-k) = 0$

$64 - 24 + 8k = 0$

$40 + 8k = 0$

$\frac{8k}{8} = \frac{-40}{8}$

$k = -5$ ✓

**9.** Determine the value(s) of $k$ such that the linear function $g(x) = 4x + k$ does not intersect the parabola $f(x) = -3x^2 - x + 4$.

$f(x) = g(x)$

$-3x^2 - x + 4 = 4x + k$

$-3x^2 - 5x + 4 - k = 0$

→ If they don't intersect, then no solution to equation.

→ This will happen if Discriminant $< 0$

$a = -3$, $b = -5$, $c = 4 - k$

$b^2 - 4ac < 0$

$(-5)^2 - 4(-3)(4-k) < 0$

$25 + 12(4-k) < 0$

$25 + 48 - 12k < 0$

$73 - 12k < 0$

$73 - 73 - 12k < 0 - 73$

$-12k < -73$

"divide by a negative" switch the sign. ☺

$\frac{-12k}{-12} > \frac{-73}{-12}$

$k > \frac{73}{12}$ ✓

**10.** A daredevil jumps off the CN Tower and falls freely for several seconds before releasing his parachute. His height, $h(t)$, in metres, $t$ seconds after jumping can be modelled by

$h_1(t) = -4.9t^2 + t + 360$ before he released his parachute; and
$h_2(t) = -4t + 142$ after he released his parachute.

How long after jumping did the daredevil release his parachute?

→ Find the Intersection Point !

$-4.9t^2 + t + 360 = -4t + 142$

$-4.9t^2 + 5t + 218 = 0$

Q.F.

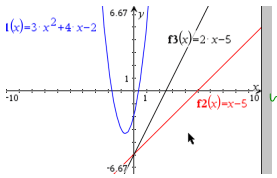
$t = \frac{-5 \pm \sqrt{5^2 - 4(-4.9)(218)}}{2(-4.9)}$

$t = -6.18$ OR $7.199$

∴ Opened @ 7.2 seconds

Time can't be negative ☺

**11.** A quadratic function is defined by $f(x) = 3x^2 + 4x - 2$. A linear function is defined by $g(x) = mx - 5$. What value(s) of the slope of the line would make it a tangent to the parabola?




$f(x) = g(x)$

$3x^2 + 4x - 2 = mx - 5$

$3x^2 + 4x - mx - 2 + 5 = 0$

$3x^2 + (4-m)x + 3 = 0$

Tangent $\Rightarrow$ "Touches just once"

$\Rightarrow$ one POI

$\Rightarrow$ one solution

$\Rightarrow b^2 - 4ac = 0$

$a = 3 \quad b = 4 - m \quad c = 3$

$b^2 - 4ac = 0$

$(4-m)^2 - 4(3)(3) = 0$

$(4-m)^2 - 36 = 0$

$(4-m)^2 = 36$

$\sqrt{(4-m)^2} = \pm\sqrt{36}$

$4 - m = \pm 6$

OR

$(4-m)^2 - 36 = 0$

$(4-m)(4-m) - 36 = 0$

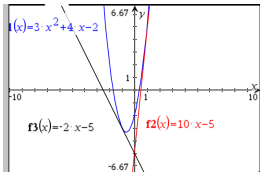
$16 - 4m - 4m + m^2 - 36 = 0$

$m^2 - 8m - 20 = 0$

$(m-10)(m+2) = 0$

$m = 10$ OR $m = -2$

now verify graphically . . . .




**14.** If $f(x) = x^2 - 6x + 14$ and $g(x) = -x^2 - 20x - k$, determine the value of $k$ so that there is exactly one point of intersection between the two parabolas.

$f(x) = g(x)$

$x^2 - 6x + 14 = -x^2 - 20x - K$

$2x^2 + 14x + 14 + K = 0$

one intersection point $\Rightarrow b^2 - 4ac = 0$

$a = 2 \quad b = 14 \quad c = 14 + K$

$b^2 - 4ac = 0$

$14^2 - 4(2)(14+K) = 0$

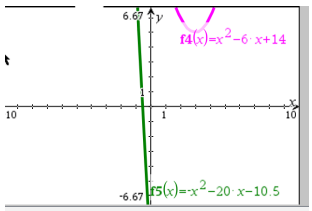
$196 - 8(14+K) = 0$

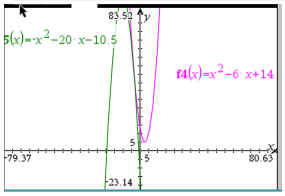
$196 - 112 - 8K = 0$

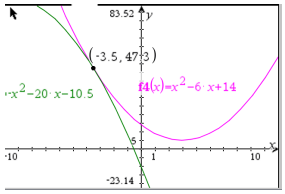
$84 - 8K = 0$

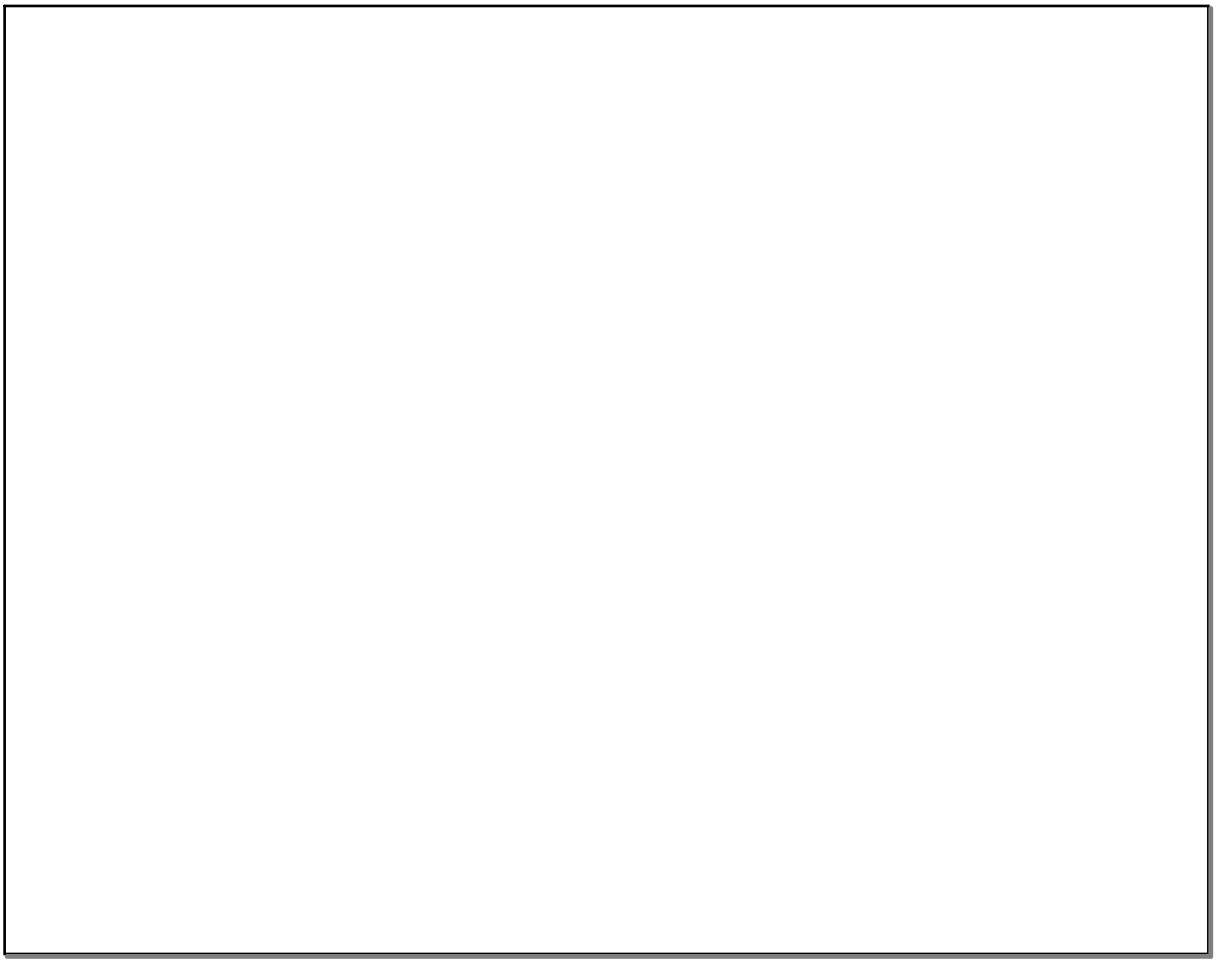
$\frac{8K}{8} = \frac{84}{8}$

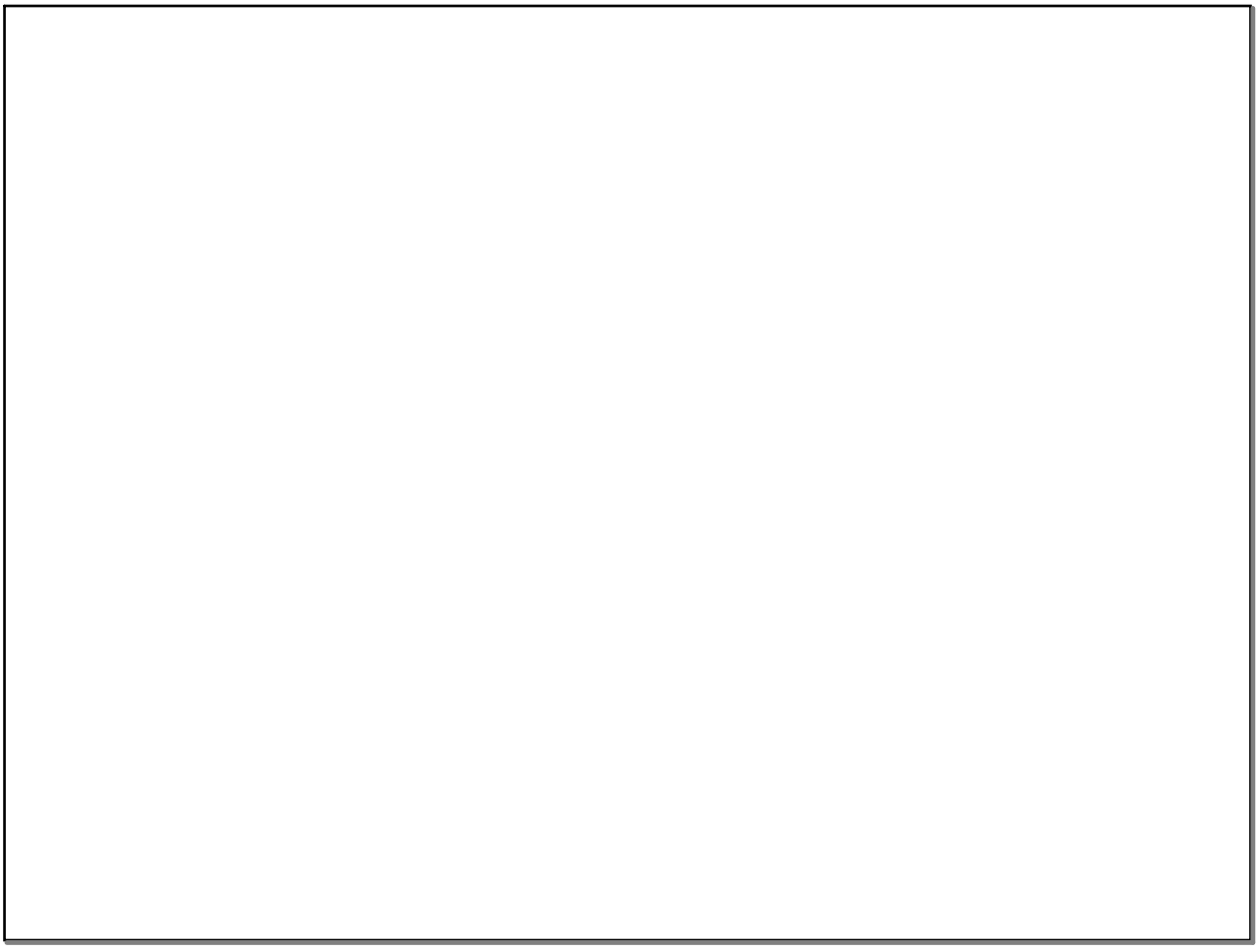
$K = 10.5$







Attachments

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