Warm-Up

Simplify and State Restrictions

1/ $\dfrac{7a^2b^3}{21a^4b}$

2/ $\dfrac{2t(5-t)}{5t^2(t-5)}$

3/ $\dfrac{4a^2b - 2ab^2}{(2a-b)^2}$

4/ $\dfrac{x^2 - 7xy + 10y^2}{x^2 + xy - 6y^2}$

Simplify...state restrictions

1/ $\dfrac{7a^2b^3}{21a^4b}$

$= \dfrac{7 \cdot a \cdot a \cdot b \cdot b \cdot b}{7 \cdot 3 \cdot a \cdot a \cdot a \cdot a \cdot b}$

$= \dfrac{b^2}{3a^2}, \quad a \neq 0 \quad b \neq 0$

Simplify...state restrictions

2/

$$\frac{2t(5-t)}{5t^2(t-5)}$$

$(5-t)$
$(-t+5)$
$(-1)(t-5)$

$$= \frac{-2t \quad (t-5)}{5t^2 t (t-5)}$$

$$= \frac{-2}{5t} \; ; t \neq 0 \qquad t-5 \neq 0$$
$$t \neq 5$$

Simplify...state restrictions

3/

$$\frac{4a^2b - 2ab^2}{(2a-b)^2}$$

$$= \frac{2ab(2a-b)}{(2a-b)(2a-b)}$$

$$= \frac{2ab}{2a-b} \qquad 2a-b \neq 0$$
$$2a \neq b$$
$$a \neq \frac{b}{2}$$

Simplify...state restrictions

4/

$$\frac{x^2 - 7xy + 10y^2}{x^2 + xy - 6y^2}$$

$$\left[\begin{array}{c} x^2 - 7x + 10 \\ (x-5)(x-2) \end{array}\right.$$

$$= \frac{(x-5y)(x-2y)}{(x+3y)(x-2y)}$$

$$\rightarrow (x-5y)(x-2y)$$

$$x^2 - 2xy - 5xy + 10y^2$$

$$= \frac{x-5y}{x+3y}$$

$$x+3y \neq 0 \qquad x-2y \neq 0$$
$$x = -3y \qquad x \neq 2y$$

Multiplying and Dividing
Rational Expressions

Learning Goal
  - multiply and divide rational expressions

Multiplying fractions

$$\frac{3}{5}\left(\frac{2}{7}\right) = \frac{6}{35}$$

Multiplying:
- top with top
- bottom with bottom

$$\frac{\cancel{3}}{5}\left(\frac{2}{\cancel{3}}\right) = \frac{6}{15} = \frac{2}{5}$$

Rule:

**Cancel out numbers / expressions
that are the same on top and bottom**

$$\frac{5x^2 y^4}{12 x^4 y}\left(\frac{2 x^5 y^7}{3xy}\right)$$

Restrictions: $x \neq 0$
$\qquad\qquad\qquad\qquad y \neq 0$

$$\frac{5x^2 y^{\cancel{4}3}}{_6\cancel{\textcircled{12}} \cancel{x^4} \cancel{y}}\left(\frac{\textcircled{2} \cancel{x^5} y^{\cancel{7}6}}{3\cancel{x}\cancel{y}}\right)$$

$$= \frac{5 x^2 y^9}{18}$$

Steps:
1. Factor
2. Find restrictions
3. Simplify $\longrightarrow$ cancel things out
4. Multiply

$$\frac{(x^2-4)}{(x+6)^2} \left(\frac{x^2+9x+18}{2(2-x)} \right)$$

Restrictions: $\begin{array}{c} x+6 \neq 0 \\ x \neq -6 \end{array}$ $\begin{array}{c} 2-x \neq 0 \\ 2 \neq x \end{array}$

$$= \frac{\boxed{(x-2)}(x+2)}{(x+6)(x+6)} \quad \frac{(x+3)(x+6)}{2\boxed{(2-x)}} \longrightarrow -1(-2+x)$$

$$= \frac{(x-2)(x+2)}{(x+6)} \quad \frac{(x+3)}{(2)(-1)(-2+x)}$$

$$= \frac{(x+2)(x+3)}{-2(x+6)}$$

$$\frac{3h(h^2+2)}{h^2} \left(\frac{5h^3}{2h^2+4} \right)$$

$$= \frac{3h(h^2+2)}{h^2} \quad \frac{5h}{2(h^2+2)}$$

Restrictions: $h^2+2 \neq 0$

$h^2 \neq 0$ $h^2 \neq -2$

$h \neq 0$

$$= \frac{15h^2}{2}$$

Try this on your own...

$$\frac{a^2 - 2a - 3}{2a(a-3)^3} \left(\frac{a^2(a-3)}{4a(a+1)^2} \right)$$

Restrictions:

Restrictions: $a \neq 0$
$a \neq 3$
$a \neq -1$

$$= \frac{(a-3)(a+1)}{2a(a-3)} \left(\frac{a^2(a-3)}{4a(a+1)} \right)$$

$$= \frac{1}{2(a-3)(4)(a+1)}$$

$$= \frac{1}{8(a-3)(a+1)}$$

Dividing fractions

$$\frac{3}{5} \div \frac{3}{7}$$

$$= \frac{\cancel{3}}{5}\left(\frac{7}{\cancel{3}}\right)$$

$$= \frac{7}{5}$$

Dividing:
- 1st ⟶ stays the same
- ÷ change to x
- 2nd ⟶ reciprocal

$$\frac{21a - 3a^2}{16a + 4a^2} \div \frac{14 - 9a + a^2}{12 + 7a + a^2}$$

$$= \frac{3a(7-a)}{4a(4+a)} \div \frac{(7-a)(2-a)}{(3+a)(4+a)}$$

Restrictions:
$a \neq 0 \qquad a \neq -4 \qquad a \neq -3$

$$= \frac{3a\cancel{(7-a)}}{4a\cancel{(4+a)}} \times \frac{\cancel{(3+a)}\cancel{(4+a)}}{\cancel{(7-a)}(2-a)}$$

Restrictions:
$a \neq 7$
$a \neq 2$

$$= \frac{3(3+a)}{4(2-a)}$$

given $f(x) = \dfrac{a^2 - b^2}{5a - 5b}$ Try on your own...

g(x) = $\dfrac{(a+b)^3}{2}$

find $\dfrac{f(x)}{g(x)}$ and state restrictions

$$\frac{(a-b)(a+b)}{5(a-b)} \div \frac{(a+b)^3}{2}$$

↑
$a - b \neq 0$
$a \neq b$

$$= \frac{(a-b)(a+b)}{5(a-b)} \times \frac{2}{(a+b)^3}$$

↑
$a + b \neq 0$
$a \neq -b$

$$= \frac{2}{5(a+b)^2}$$

Try On Your Own #1 ...

MCR 3U   **Multiplying and Dividing Rational Expressions**

Try On Your Own #1

1. $\frac{1}{2} \times \frac{2}{3}$ $\frac{1}{2} \times \frac{2}{3}$ 2. $\frac{2x}{3} \times \frac{1}{x}$ 3. $\frac{3x^2}{7} \times \frac{7}{x}$

4. $\frac{(x-2)(x+2)}{(x+3)} \times \frac{(x+3)}{(x+2)}$ 5. $\frac{x+2}{x+1}$

6. $\frac{1}{2} \div \frac{1}{7}$ 7. $\frac{2}{3} \div \frac{1}{3}$ 8. $\frac{2}{3} \div \frac{3}{5}$

9. $\frac{x+2}{x+1} \div \frac{x+3}{x+1}$

Try On Your Own #1 - Solutions ...

MCR 3U   **Multiplying and Dividing Rational Expressions**

1. $\frac{1}{2} \times \frac{2}{3}$ $1 = 2 \times \frac{1}{2} \times \frac{2}{3} \div \frac{2}{3} = 1$ $f(x) = \frac{2x}{3} \times \frac{1}{x}$ 3. $\frac{3x^2}{7} \times \frac{7}{x}$

$= \frac{2}{6}$ $= \frac{1}{3}$ $f(x) = \frac{2}{3}, x \neq 0$ $= 3x, x \neq 0$

$= \frac{1}{3}$ $y = \frac{2}{3}$ $y = \frac{2}{3}$

4. $\frac{(x-2)(x+2)}{(x+3)} \times \frac{(x+3)}{(x+2)}$ 5. $\frac{(x+2)}{(x+1)}, x \neq -1$

$= x - 2, x \neq -2, -3$

be careful, the x's do not cancel out

6. $\frac{1}{2} \div \frac{1}{7}$ 7. $\frac{2}{3} \div \frac{1}{3}$ 8. $\frac{2}{3} \div \frac{3}{5}$

$= \frac{1}{2} \times \frac{7}{1}$ $= \frac{2}{3} \times \frac{3}{1}$ $= \frac{2}{3} \times \frac{5}{3}$

$= \frac{7}{2}$ $= 2$ $= \frac{10}{9}$

add #10 and be careful of restrictions ....

9. $\frac{(x+2)}{(x+1)} \div \frac{(x+3)}{(x+1)}$ **10.** $\frac{(x-2)}{(x+3)} \times \frac{(x+3)}{(x+4)} \div \frac{(x-2)}{(x-5)}, x \neq -3, -4, 5$

$= \frac{(x+2)}{(x+1)} \times \frac{(x+1)}{(x+3)}$ $= \frac{(x-2)}{(x+4)} \times \frac{(x-5)}{(x-2)}, x \neq 2$

$= \frac{(x+2)}{(x+3)}, x \neq -1, -3$ $= \frac{(x-5)}{(x+4)}, x \neq -4, -3, 2, 5$

$= \frac{2}{3}$

Try On Your Own #2 ...

MCR 3U   **Multiplying and Dividing Rational Expressions**

Multiplying
- Factor numerators and denominators
- State all restrictions (values that make the denominator equal 0)
- Divide out factors
- Multiply numerators and denominators
- Simplify and state all restrictions

Example: Simplify and state all restrictions

1. $\dfrac{4}{x^2+7x+12} \times \dfrac{3x^2+12x}{2} =$

2. $\dfrac{x^2-5x+6}{3x} \times \dfrac{9x^2}{x^2+2x-8} =$

Try On Your Own #2 - Solutions ...

MCR 3U   **Multiplying and Dividing Rational Expressions**

Multiplying
- Factor numerators and denominators
- State all restrictions (values that make the denominator equal 0)
- Divide out factors
- Multiply numerators and denominators
- Simplify and state all restrictions

Example: Simplify and state all restrictions

1. $\dfrac{4}{x^2+7x+12} \times \dfrac{3x^2+12x}{2} =$

$= \dfrac{\overset{2}{\cancel{4}}}{(x+3)(x+4)} \times \dfrac{3x(x+4)}{\cancel{2}}$, $\quad x+3 \neq 0$
$\qquad\qquad\qquad\qquad\qquad\qquad\quad x \neq -3$

$= \dfrac{6x}{x+3}$, $x \neq -4,-3$ $\qquad x+4 \neq 0$
$\qquad\qquad\qquad\qquad\qquad\qquad\quad x \neq -4$

2. $\dfrac{x^2-5x+6}{3x} \times \dfrac{9x^2}{x^2+2x-8} =$

$= \dfrac{(x-3)(x-2)}{3x} \times \dfrac{\overset{3x}{\cancel{9x^2}}}{(x+4)(x-2)}$ $\quad\begin{array}{l}3x \neq 0\\ \boxed{x \neq 0}\end{array}$
$\qquad\qquad\qquad\qquad\qquad\qquad\quad \begin{array}{l}x+4 \neq 0\\ \boxed{x \neq -4}\end{array}$

$= \dfrac{3x(x-3)}{x+4}$, $x \neq -4,0,2$ $\quad\begin{array}{l}x-2 \neq 0\\ \boxed{x \neq 2}\end{array}$

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Try On Your Own #3 ...

MCR 3U **Multiplying and Dividing Rational Expressions**

Dividing
- Factor numerators and denominators
- State all restrictions
- Flip and multiply (change ÷ to x and use reciprocal of second expression)
- State new restrictions
- Divide out factors
- Multiply numerators and denominators
- Simplify and state all restrictions

Example: Simplify

1.
$$\frac{2x^2 + x - 6}{x^2 - 9} \div \frac{x^2 + 4x + 4}{5x^3 - 5x^2 - 30x} =$$

2.
$$\frac{x^2 - 4}{x^2 + 5x + 6} \div \frac{x^2 - x - 2}{x^3 + 4x^2 + 3x} =$$

Try On Your Own #3 - Solutions ...

MCR 3U **Multiplying and Dividing Rational Expressions**

Dividing
- Factor numerators and denominators
- State all restrictions
- Flip and multiply (change ÷ to x and use reciprocal of second expression)
- State new restrictions
- Divide out factors
- Multiply numerators and denominators
- Simplify and state all restrictions

Example: Simplify

1.
$$\frac{2x^2 + x - 6}{x^2 - 9} \div \frac{x^2 + 4x + 4}{5x^3 - 5x^2 - 30x} =$$

$$2x^2 + x - 6$$
$$2x^2 + 4x - 3x - 6$$
$$2x(x+2) - 3(x+2)$$
$$(x+2)(2x-3)$$

$$5x^3 - 5x^2 - 30x$$
$$5x(x^2 - x - 6)$$
$$5x(x-3)(x+2)$$

$$= \frac{(2x-3)(x+2)}{(x+3)(x-3)} \div \frac{(x+2)(x+2)}{5x(x-3)(x+2)}$$

$$= \frac{(2x-3)(x+2)}{(x+3)(x-3)} \times \frac{5x(x-3)}{(x+2)}$$

$$= \frac{5x(2x-3)}{x+3} \quad \to x \neq -3, -2, 0, 3$$

$$x+3 \neq 0 \quad x-3 \neq 0$$
$$x \neq -3 \quad x \neq 3$$
$$5x \neq 0 \quad x+2 \neq 0$$
$$x \neq 0 \quad x \neq -2$$

Try On Your Own #3 - Solutions ...

2.

$$\frac{x^2 - 4}{x^2 + 5x + 6} \div \frac{x^2 - x - 2}{x^3 + 4x^2 + 3x} =$$

$$= \frac{(x+2)(x-2)}{(x+3)(x+2)} \div \frac{(x-2)(x+1)}{x(x^2+4x+3)}$$

$$= \frac{(x+2)(x-2)}{(x+3)(x+2)} \div \frac{(x-2)(x+1)}{x(x+3)(x+1)}$$

$x \neq 0$
$x \neq -3$
$x \neq -1$
~~$x \neq -3$~~
~~$x \neq -2$~~

$$= \frac{(x+2)(x-2)}{(x+3)(x+2)} \times \frac{x(x+3)(x+1)}{(x-2)(x+1)}$$

$x \neq 2$
~~$x \neq -1$~~

$$= x, \quad x \neq -3, -2, -1, 0, 2$$

Try On Your Own #4 ...

Extra Practice

MCR 3U   **Multiplying and Dividing Rational Expressions**

At the Boards

Multiplying
- Factor numerators and denominators
- State all restrictions (values that make the denominator equal 0)
- Divide out factors
- Multiply numerators and denominators
- Simplify and state all restrictions

Example: Simplify and state all restrictions

1. $\dfrac{6}{x^2 + 5x + 6} \times \dfrac{2x^2 + 6x}{3}$

2. $\dfrac{x^2 - 4x - 5}{4x^2} \times \dfrac{12x}{x^2 - 6x - 7}$

Dividing
- Factor numerators and denominators
- **State all restrictions**
- Flip and multiply (change ÷ to x and use reciprocal of second expression)
- State new restrictions
- Divide out factors
- Multiply numerators and denominators
- Simplify and state all restrictions

Example: Simplify
1.
$\dfrac{3x^2 + 4x - 4}{x^2 - 3x - 4} \div \dfrac{x^2 + 4x + 4}{3x^3 - 6x^2 - 24x}$

Try On Your Own #4 - Answers ...

MCR 3U   **Multiplying and Dividing Rational Expressions**

At the Boards

Multiplying
- Factor numerators and denominators
- State all restrictions (values that make the denominator equal 0)
- Divide out factors
- Multiply numerators and denominators
- Simplify and state all restrictions

Example: Simplify and state all restrictions

1. $\dfrac{6}{x^2+5x+6} \times \dfrac{2x^2+6x}{3}$ $= \dfrac{4x}{x+2}$, $x \neq -2$
$x \neq -3$

2. $\dfrac{x^2-4x-5}{4x^2} \times \dfrac{12x}{x^2-6x-7}$ $= \dfrac{3(x-5)}{x(x-7)}$, $x \neq 0$
$x \neq -1$
$x \neq 7$

Dividing
- Factor numerators and denominators
- **State all restrictions**
- Flip and multiply (change ÷ to x and use reciprocal of second expression)
- State new restrictions
- Divide out factors
- Multiply numerators and denominators
- Simplify and state all restrictions

Example: Simplify

1. $\dfrac{3x^2+4x-4}{x^2-3x-4} \div \dfrac{x^2+4x+4}{3x^3-6x^2-24x}$ $= \dfrac{3x(3x-2)}{x+1}$, $x \neq 4$
$x \neq -2$
$x \neq -1$
$x \neq 0$

Optional Extra Practice ···

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