Video starts here

pause the video and do this ....

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Warm - up

Are the following functions equivalent?
Use factoring to prove it.

$f(x)=x^3+5x^2-x-5$ $\quad\quad$ $g(x)=(x+1)(x-1)(x+5)$

Warm - up

Are the following functions equivalent?
Use factoring to prove it.

$f(x)=x^3+5x^2-x-5$ $\qquad$ $g(x)=(x+1)(x-1)(x+5)$

$=x^2(x+5)-(x+5)$

$=\underline{(x^2-1)}(x+5)$

$=(x+1)(x-1)(x+5)$

Warm - up

How does the function and the graph relate to each other?

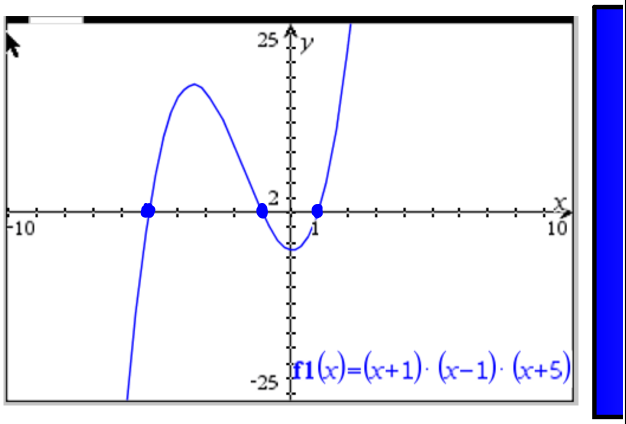
$x=-1$ $\qquad$ $x=1$ $\qquad$ $x=-5$

$f(x)=x^3+5x^2-x-5$ $\qquad$ $g(x)=(x+1)(x-1)(x+5)$

Verify by graphing

on TI-NSpire

What do you notice about the "Factors" and the zeroes of this function.



pause the video and do this ....

Factor Self Check Quiz

1) $x^2 + 3x + 2$

2) $x^3 - 5x^2 - 14x$

3) $2x^2 + 10x + 8$

4) $4x^2 - 4x - 3$

Factor Self Check Quiz,

1) $x^2 + 3x + 2$

$(x+2)(x+1)$

2) $x^3 - 5x^2 - 14x$

$x(x^2 - 5x - 14)$

$x(x-7)(x+2)$

3) $2x^2 + 10x + 8$

$2(x^2 + 5x + 4)$

$2(x+4)(x+1)$

4) $4x^2 - 4x - 3$

$= 4x^2 - 6x + 2x - 3$

$= 2x(2x-3) + (2x-3)$

$= (2x-3)(2x+1)$

Simplifying Rational Functions

Learning Goals
- define a "Rational Function"
- simplify rational expressions
- state restrictions

Rational Number - a number that can be written as a fraction

$\frac{2}{3}$ $\quad 4 = \frac{4}{1}$

Rational Expression - a fraction of two polynomials

Rational	Not Rational
$\frac{x}{x^2-4}$ $\frac{x^3-4x^2-3}{x+9}$	$\frac{x}{\sqrt{x+1}}$ $\frac{x^3+3x^{\frac{4}{5}}}{x}$

$\sqrt{} = (\ )^{\frac{1}{2}}$

Note: denominator can't equal zero

Rational Function - is a function made up of a fraction of two polynomials

$y = \frac{x}{x^2-4}$

Simplifying Rational Expressions

$$\frac{12}{3} = \frac{4}{1} = 4$$

another way of evaluating this is to ...

express the numerator and denominator as "products of prime numbers"

$$\frac{12}{3} = \frac{2(2)\cancel{(3)}}{\cancel{3}} = 4$$

- rational expressions are undefined when the denominator is 0

so how would the following "rational expressions" simplify

$$\frac{3\cancel{x}}{\cancel{x}} = 3$$

$$\frac{x\cancel{(x+1)}}{\cancel{(x+1)}} = x$$

$$\frac{30x^4y^3}{-6x^7y} = -5x^{-3}y^2 = \frac{-5y^2}{x^3}$$

Steps:

1. Factor polynomials
2. Divide by "like brackets"

ex. $\dfrac{(3x+1)\cancel{(2x-1)}}{(3x+2)\cancel{(2x-1)}}$

$= \dfrac{3x+1}{3x+2}$

$\Rightarrow$ $x=1$

$\dfrac{3(1)+1}{3(1)+2}$

$= \dfrac{4}{5}$

wrong

$\dfrac{\cancel{3x}+1}{\cancel{3x}+2} = \dfrac{1}{2}$

Worked Examples ...

1. $\dfrac{8xy^2}{4yx^2}$

$= 2x^{-1}y$

$= \dfrac{2y}{x}$

2. $\dfrac{9-6x}{3}$

$= \dfrac{\cancel{3}\cdot(3-2x)}{\cancel{3}}$

$= 3-2x$

3. $\dfrac{x-2}{x^2-5x+6}$

$= \dfrac{\cancel{x-2}}{\cancel{(x-2)}(x-3)}$

$= \dfrac{1}{x-3}$

Worked Examples ...

Please pause the video and try these on your own

4. $\frac{x^3-x}{x^2+5x+4}$

5. $\frac{10x^2-15x}{6x^2-13x+6}$

Worked Examples ...

Please pause the video and try these on your own

4. $\frac{x^3-x}{x^2+5x+4}$

5. $\frac{10x^2-15x}{6x^2-13x+6}$

$$= \frac{x(x^2-1)}{(x+4)(x+1)}$$

$$= \frac{x\cancel{(x+1)}(x-1)}{(x+4)\cancel{(x+1)}}$$

$$= \frac{x(x-1)}{x+4}$$

$$\frac{5x\cancel{(2x-3)}}{(3x-2)\cancel{(2x-3)}}$$

$$= \frac{5x}{3x-2}$$

Restrictions

- **the values of the variable in a rational function that cause the function to be undefined**
- **the values that are not in the domain of the function**
- **the x-value that makes the original denominator zero**

Every time we simplify a rational expression / function we <u>must state restrictions</u>.

State restrictions

$\frac{5}{x}$ $\quad x \neq 0$

$\frac{5}{x+2}$ $\quad x+2 \neq 0$

$\boxed{x \neq -2}$

$\frac{5}{(x+1)(x+2)}$ $\quad x+1 \neq 0 \quad x+2 \neq 0$

$x \neq -1 \quad x \neq -2$

Summarize the Steps ...

Steps
1. Factor
2. Find restrictions using denominator
3. Simplify

MCR 3U *Investigating Rational Functions* **Try On Your Own #1**

Restrictions

Restrictions are the values of the variable(s) that cause the function to be undefined.
These are the zeros of the denominator *(even if the factor cancels out)*
These are the numbers that are not in the Domain of the function.

What value(s) of x makes each denominator zero ...

a) $\frac{2}{x}$ b) $\frac{x}{x-3}$ c) $\frac{2x-3}{2x+1}$

These values are called "Restrictions".
They MUST always be stated.
AND they define the Domain

Determine the Restrictions of the following Rational Functions.

d) $\frac{2}{x^2-2x}$ e) $\frac{x+2}{x^2-4}$

What value(s) of x makes each denominator zero ...

Try On Your Own #1 - Solutions

MCR 3U *Investigating Rational Functions*

Restrictions

Restrictions are the values of the variable(s) that cause the function to be undefined.
These are the zeros of the denominator *(even if the factor cancels out)*
These are the numbers that are not in the Domain of the function.

What value(s) of x makes each denominator zero …

a) $\frac{2}{x}$ b) $\frac{x}{x-3}$ c) $\frac{2x-3}{2x+1}$

$x \neq 0$

$D=\{x\in R \mid x\neq 0\}$

$x-3\neq 0$
$x \neq 3$

$2x+1\neq 0$
$\frac{2x}{2}\neq\frac{-1}{2}$
$x\neq -\frac{1}{2}$

These values are called "Restrictions".
They MUST always be stated.
AND they define the Domain

Determine the Restrictions of the following Rational Functions.

d) $\frac{2}{x^2-2x}$ e) $\frac{x+2}{x^2-4}$ e) $\frac{x+2}{x^2-4}$

What value(s) of x makes each denominator zero …

$x^2-2x=0$
$x(x-2)=0$
$x=0$ $x-2=0$
$x=2$

$D=\{x\in R \mid x\neq 0, 2\}$

$x^2-4=0$
$(x+2)(x-2)=0$
$x=-2$ $x=2$

$D=\{x\in R \mid x\neq -2, 2\}$

Try On Your Own #2

MCR 3U *Investigating Rational Functions*

Simplifying Rational Expressions Example $\frac{x^2-4x-12}{x^2-4}$

1. Factor numerator and denominator $=\frac{(x+2)(x-6)}{(x+2)(x-2)}$
2. ***List all restrictions (values that make the denominator 0)*** $x\neq -2, x\neq 2$
3. Divide common factors in the numerator and denominator $=\frac{(x-6)}{(x-2)}$

Worked Examples: Simplify and state the restrictions.

1. $\frac{9x^2y}{3xy^2}$
2. $\frac{12-8x}{4}$
3. $\frac{x-1}{x^2-4x+3}$
4. $\frac{x^3-x}{x^2+2x+1}$
5. $\frac{12x^2-4x}{6x^2-11x+3}$
6. $\frac{(x^3+4x^2+3x)(x^2-4)}{(x^2+5x+6)(x^2-x-2)}$

Try On Your Own #2 - Solutions

MCR 3U *Investigating Rational Functions*

Simplifying Rational Expressions Example

$$\frac{x^2-4x-12}{x^2-4}$$

1. Factor numerator and denominator $=\frac{\cancel{(x+2)}(x-6)}{\cancel{(x+2)}(x-2)}$
2. *List all restrictions (values that make the denominator 0)* $x \neq -2, x \neq 2$
3. Divide common factors in the numerator and denominator $=\frac{(x-6)}{(x-2)}$

Worked Examples: Simplify and state the restrictions.

1. $\frac{9x^2y}{3xy^2}$

$= \frac{\cancel{3}\cdot 3\cdot \cancel{x}\cdot x\cdot \cancel{y}}{\cancel{3}\cdot \cancel{x}\cdot \cancel{y}\cdot y}$

$= \frac{3x}{y}, \; x\neq 0, \; y\neq 0$

2. $\frac{12-8x}{4}$

$= \frac{\cancel{4}(3-2x)}{\cancel{4}}$

$= 3-2x$

3. $\frac{x-1}{x^2-4x+3}$

$= \frac{\cancel{(x-1)}}{(x-3)\cancel{(x-1)}}$

$= \frac{1}{x-3}, \; x\neq 3, 1$

4. $\frac{x^3-x}{x^2+2x+1}$

$= \frac{x(x^2-1)}{(x+1)(x+1)}$ ← $x+1\neq 0$, $x\neq -1$

$= \frac{x\cancel{(x+1)}(x-1)}{\cancel{(x+1)}(x+1)}$

$= \frac{x(x-1)}{x+1}, \; x\neq -1$

5. $\frac{12x^2-4x}{6x^2-11x+3}$

see next page for step by step solution

$\boxed{= \frac{4x}{2x-3}, \; x\neq \frac{1}{3}, \frac{3}{2}}$

6. $\frac{(x^3+4x^2+3x)(x^2-4)}{(x^2+5x+6)(x^2-x-2)}$

---

5. $\frac{12x^2-4x}{6x^2-11x+3}$

#1 Factor Numerator and Denominator.

$= \frac{4x(3x-1)}{6x^2-9x-2x+3}$ ← M: 18, A: −11 } −9 & −2

$-11x = -9x-2x$

$= \frac{4x(3x-1)}{3x(2x-3)-1(2x-3)}$

$= \frac{4x\cancel{(3x-1)}}{(2x-3)\cancel{(3x-1)}}$

#2 Cancel any factors and state restrictions.

$3x-1\neq 0$, $3x\neq 1$, $x\neq \frac{1}{3}$

$2x-3\neq 0$, $\frac{2x}{2}\neq \frac{3}{2}$, $x\neq \frac{3}{2}$

$= \frac{4x}{2x-3}$

$\boxed{= \frac{4x}{2x-3}, \; x\neq \frac{1}{3}, \frac{3}{2}}$

#3 State Simplified Expressions with all restrictions.

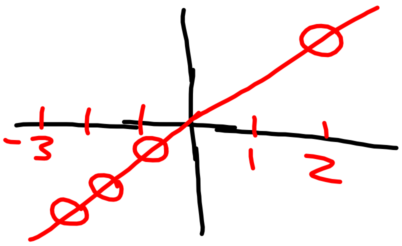
6. $$\frac{(x^3+4x^2+3x)(x^2-4)}{(x^2+5x+6)(x^2-x-2)}$$

$$= \frac{(x(x^2+4x+3))(x+2)(x-2)}{(x+3)(x+2)(x-2)(x+1)}$$

$$= \frac{x\cancel{(x+3)}\cancel{(x+1)}\cancel{(x+2)}\cancel{(x-2)}}{\cancel{(x+3)}\cancel{(x+2)}\cancel{(x-2)}\cancel{(x+1)}}$$

* Restrictions
$x \neq -3$
$x \neq -2$
$x \neq 2$
$x \neq -1$

$$= x, \quad x \neq -3, -2, -1, 2$$



Holy Line

Optional Extra Practise

pg 113 # 4-6ace, 8, 11, 15

4. Simplify. State any restrictions on the variables.

a) $\dfrac{14x^3 - 7x^2 + 21x}{7x}$ c) $\dfrac{2t(5 - t)}{5t^2(t - 5)}$ e) $\dfrac{2x^2 + 10x}{-3x - 15}$

b) $\dfrac{-5x^3y^2}{10xy^3}$ d) $\dfrac{5ab}{15a^4b - 10a^2b^2}$ f) $\dfrac{2ab - 6a}{9a - 3ab}$

5. Simplify. State any restrictions on the variables.

a) $\dfrac{a + 4}{a^2 + 3a - 4}$ c) $\dfrac{x^2 - 5x + 6}{x^2 + 3x - 10}$ e) $\dfrac{t^2 - 7t + 12}{t^3 - 6t^2 + 9t}$

b) $\dfrac{x^2 - 9}{15 - 5x}$ d) $\dfrac{10 + 3p - p^2}{25 - p^2}$ f) $\dfrac{6t^2 - t - 2}{2t^2 - t - 1}$

6. State the domain of each function. Explain how you found each answer.

a) $f(x) = \dfrac{2 + x}{x}$ d) $f(x) = \dfrac{1}{x^2 - 1}$

b) $g(x) = \dfrac{3}{x(x - 2)}$ e) $g(x) = \dfrac{1}{x^2 + 1}$

c) $h(x) = \dfrac{-3}{(x + 5)(x - 5)}$ f) $h(x) = \dfrac{x - 1}{x^2 - 1}$

4. a) $2x^2 - x + 3, x \neq 0$ d) $\dfrac{1}{a}(3a^2 - 2b), a \neq 0, \sqrt{\dfrac{2}{3}b}, b \neq 0$

b) $-\dfrac{x^2}{2y}, x \neq 0, y \neq 0$ e) $-\dfrac{2x}{3}, x \neq -5$

c) $-\dfrac{2}{5t}, t \neq 0, 5$ f) $-\dfrac{2}{3}, a \neq 0, b \neq 3$

5. a) $\dfrac{1}{a} - 1, a \neq -4, 1$ d) $\dfrac{2 + p}{5 + p}, p \neq -5, 5$

b) $-x - \dfrac{3}{5}, x \neq 3$ e) $\dfrac{t - 4}{t(t - 3)}, t \neq 0, 3$

c) $\dfrac{x - 3}{x + 5}, x \neq -5, 2$ f) $\dfrac{3t - 2}{t - 1}, t \neq -\dfrac{1}{2}, 1$

6. a) the denominator equals 0; **R**, $x \neq 0$ d) **R**, $x \neq -1, 1$

b) **R**, $x \neq 0, 2$ e) **R**

c) **R**, $x \neq -5, 5$ f) **R**, $x \neq -1, 1$

8. An isosceles triangle has two sides of length $9x + 3$. The perimeter of the triangle is $30x + 10$.

A

a) Determine the ratio of the base to the perimeter, in simplified form. State the restriction on x.

b) Explain why the restriction on x in part (a) is necessary in this situation.

11. A rectangle is six times as long as it is wide. Determine the ratio of its area to its perimeter, in simplest form, if its width is w.

15. Can two different rational expressions simplify to the same polynomial? Explain using examples.

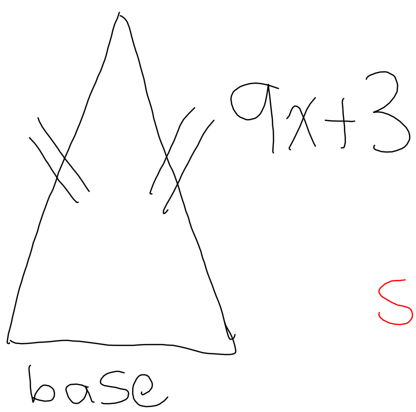
C

8. An isosceles triangle has two sides of length $9x + 3$. The perimeter of the triangle is $30x + 10$.

A

a) Determine the ratio of the base to the perimeter, in simplified form. State the restriction on x.

b) Explain why the restriction on x in part (a) is necessary in this situation.



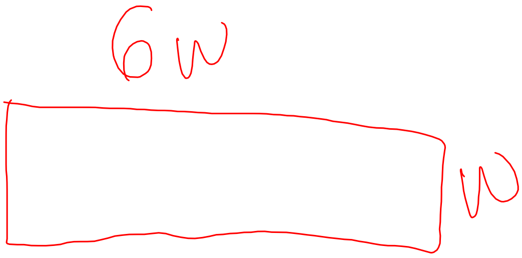
$P = 2(9x+3) + \text{base}$

$\downarrow$

solve for base

$$\text{Ratio} = \frac{\text{Perimeter}}{\text{Base}}$$

11. A rectangle is six times as long as it is wide. Determine the ratio of its area to its perimeter, in simplest form, if its width is w.



$$\text{Ratio} = \frac{\text{Area}}{\text{Perimeter}}$$

15. Can two different rational expressions simplify to the same polynomial? **C** Explain using examples.

Solution ....

15. yes; $\dfrac{(x+1)(x+2)}{(x+1)(x+3)}$ and $\dfrac{(x+4)(x+2)}{(x+4)(x+3)}$

But what do the graphs look like?

Check the Table of Values

- they are not the same !!!!!

8. **a)** $\frac{2}{5}, x > -\frac{1}{3}$

b) Because $x \leq -\frac{1}{3}$ would imply sides of length 0 or less, therefore this would not be a triangle.

11. $\frac{3w}{7}$

15. yes; $\frac{(x + 1)(x + 2)}{(x + 1)(x + 3)}$ and $\frac{(x + 4)(x + 2)}{(x + 4)(x + 3)}$