Applications

Sinusoidal Functions

Open your notebook to

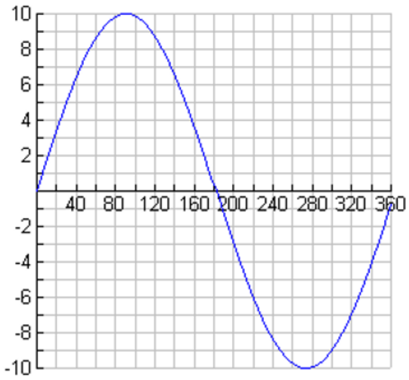
Cycle 3 Day 9 and

Cycle 4 day 7

Interpreting Sinusoidal Functions

You will be finding the equation of each graph today.

Warm - Up #1 - From Cycle 3 Day 9



Min ____________

Max ____________

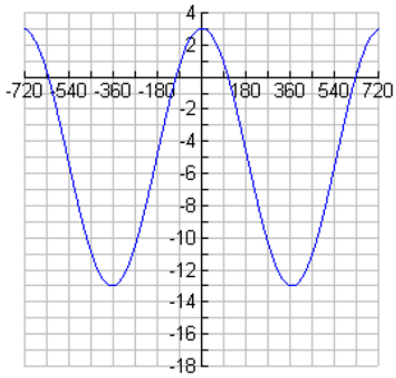
Axis ____________

Amplitude ____________

Domain ____________

Range ____________

Period ____________



Min ____________

Max ____________

Axis ____________

Amplitude ____________

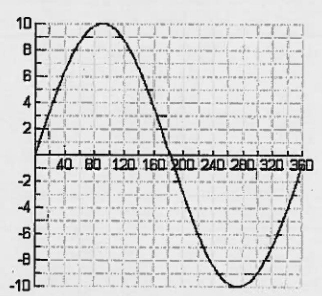
Domain ____________

Range ____________

Period ____________

Now determine the equation

Warm - up



Min -10

Max 10

Axis $y = 0$

Amplitude 10

Domain $x \in \mathbb{R}$

Range $-10 \le y \le 10$

Period = 360°

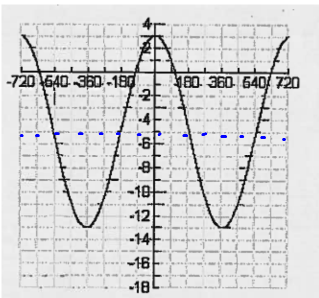
$a = 10$

$K = \frac{360}{360} = 1$

$d = 0$

$c = 0$

$y = 10 \sin x$



Min -13

Max 3

Axis $y = -5$

Amplitude 8

Domain $x \in \mathbb{R}$

Range $-13 \le y \le 3$

Period = 720°

$a = 8$

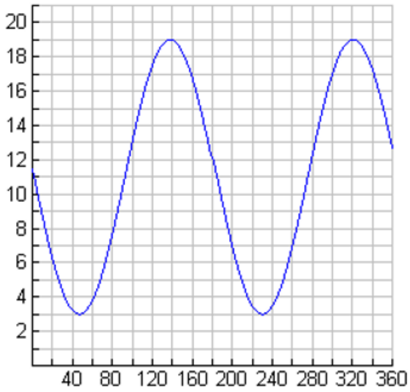
$K = \frac{360}{720} = \frac{1}{2}$

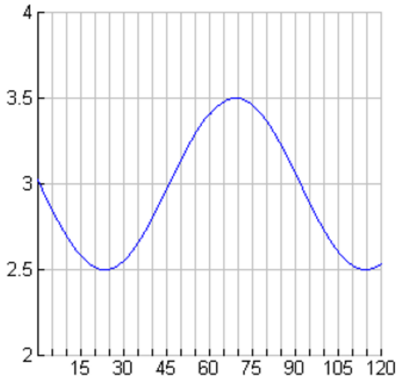
$d = 0$ (cosine)

$c = -5$

$y = 8 \cos \frac{1}{2}x - 5$

Warm - up #2 - from Cycle 3 Day 9

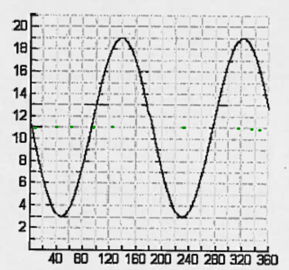




Min ____________
Max ____________
Axis ____________
Amplitude ____________
Domain ____________
Range ____________
Period ____________

Min ____________
Max ____________
Axis ____________
Amplitude ____________
Domain ____________
Range ____________
Period ____________

Now determine the equation



Min 3
Max 19
Axis $y = 11$
Amplitude 8
Domain $x \in \mathbb{R}$
Range $3 \le y \le 19$

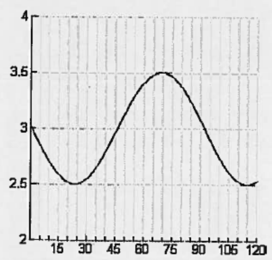
Period = 180°

$a = 8$
$k = \frac{360}{180} = 2$

$d = 0$
sine curve reflected on x axis

$c = 11$

$y = -8\sin 2x + 11$



Min 2.5
Max 3.5
Axis $y = 3$
Amplitude 0.5
Domain $x \in \mathbb{R}$
Range $2.5 \le y \le 3.5$

Period = 90°

use sine curve reflected on x axis.

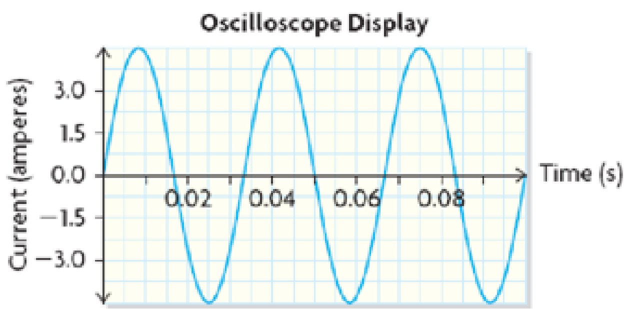
$a = -0.5$
$k = \frac{360}{90} = 4$

$d = 0$
$c = 3$
$y = -0.5\sin 4x + 3$

Turn on the video

Warm - up #3 - from Cycle 3 Day 9

4. An oscilloscope hooked up to an AC (alternating current) circuit shows a sine curve on its display:

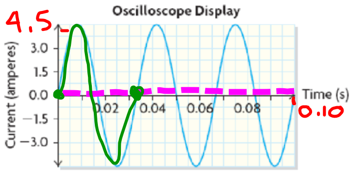


a) What is the period of the function?
b) What is the equation of the axis of the function?
c) What is the amplitude of the function?
d) State the units of measure for each of your answers above.

Now determine the equation

Warm - up #4 - from Cycle 3 Day 9

4. An oscilloscope hooked up to an AC (alternating current) circuit shows a sine curve on its display:



a) What is the period of the function?
b) What is the equation of the axis of the function?
c) What is the amplitude of the function?
d) State the units of measure for each of your answers above.

3cycles → 0.10sec
1 cycle → $\frac{0.10}{3}$
= 0.033 s

axis $y = 0$
amperes

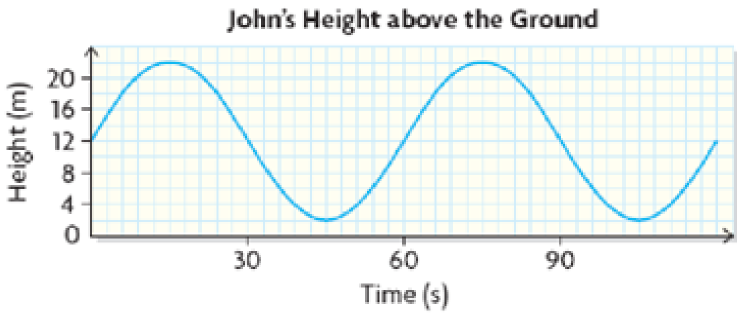
Amplitude
$a = 4.5$
amperes

$$f(x) = a \cdot \sin(k(x-d)) + c$$
$$= 4.5 \sin(10800(x-0)) + 0$$

$$k = \frac{360}{\text{period}} = \frac{360}{0.033} = 10800$$

Warm - up #5 - from Cycle 3 Day 9

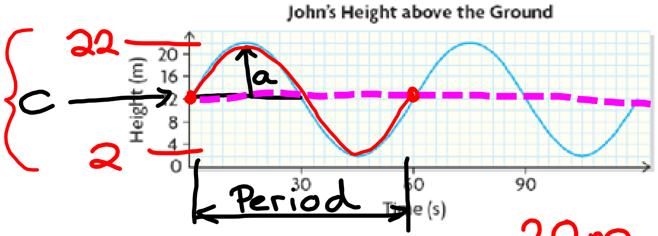
9. The graph shows John's height above the ground as a function of time as he rides a Ferris wheel.



a) What is the diameter of the Ferris wheel?
b) What is John's initial height above the ground?
c) At what height did John board the Ferris wheel?
d) How high above the ground is the axle on the wheel?

Now determine the equation

9. The graph shows John's height above the ground as a function of time as he rides a Ferris wheel.



$a = 10$
$c = 12$

a) What is the ~~diameter~~ of the Ferris wheel? 20 m
b) What is John's initial height above the ground? 12 m
c) At what height did John board the Ferris wheel? 12 m
d) How high above the ground is the axle on the wheel? 12 m

$$f(x) = 10 \cdot \sin\left(6\left(x - 0\right)\right) + 12$$

$$k = \frac{360}{\text{period}} = \frac{360}{60} = 6$$

Try on your own...

6. Sketch a height-versus-time graph of the sinusoidal function that models each situation. Draw at least three cycles. Assume that the first point plotted on each graph is at the lowest possible height.

a) A Ferris wheel with a radius of 7 m, whose axle is 8 m above the ground, and that rotates once every 40 s

b) A water wheel with a radius of 3 m, whose centre is at water level, and that rotates once every 15 s

c) A bicycle tire with a radius of 40 cm and that rotates once every 2 s

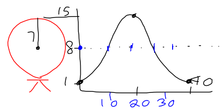
d) A girl lying on an air mattress in a wave pool that is 3 m deep, with waves 0.5 m in height that occur at 7 s intervals

Now determine the equation

P. 371 # 6 - Sketch the Graphs

6. Sketch a height-versus-time graph of the sinusoidal function that models each situation. Draw at least three cycles. Assume that the first point plotted on each graph is at the lowest possible height. → use $y = -\cos x$

a) A Ferris wheel with a radius of 7 m, whose axle is 8 m above the ground, and that rotates once every 40 s



$a = -7$

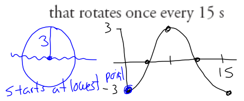
$k = \frac{360}{40} = 9$

$d = 0$

$c = 8$

$f(x) = -7\cos 9x + 8$

b) A water wheel with a radius of 3 m, whose centre is at water level, and that rotates once every 15 s



$a = -3$

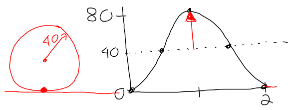
$k = \frac{360}{15} =$

$d = 0$

$c = 0$

$f(x) = -3\cos 24x$

c) A bicycle tire with a radius of 40 cm and that rotates once every 2 s



$a = -40$

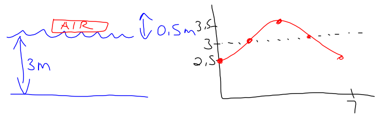
$k = \frac{360}{2} = 180$

$d = 0$

$c = 40$

$f(x) = -40\cos 180x + 40$

d) A girl lying on an air mattress in a wave pool that is 3 m deep, with waves 0.5 m in height that occur at 7 s intervals



$a = -0.5$

$k = \frac{360}{7}$

$d = 0$

$c = 3$

$f(x) = -0.5\cos\left(\frac{360}{7}x\right) + 3$

MCR3U ***6.3 Interpreting Sinusoidal Functions***

Example # 1

The height above the ground of a rider on a Ferris wheel can be modelled by

$h(x) = 25\sin(x - 90)^{\circ} + 27$, where $h(x)$ is the height, in metres, and x is the angle, in degrees, the rider has rotated from the boarding position.

Draw a picture of the Ferris Wheel's motion below ,

Now graph on Nspire, adjusting window settings accordingly.

Answer the following questions from your NSpire Graph ….

Trace to determine the height of the rider when x = 30°

Height at 30°______________________

Trace to determine the angle when $h(x)$ = 40m

Angle at 40m _____________________

MCR3U ***6.3 Interpreting Sinusoidal Functions***

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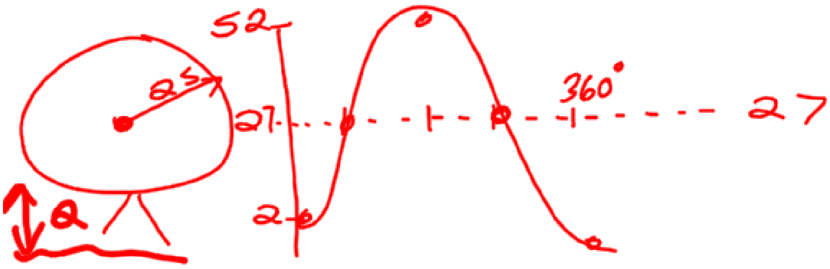
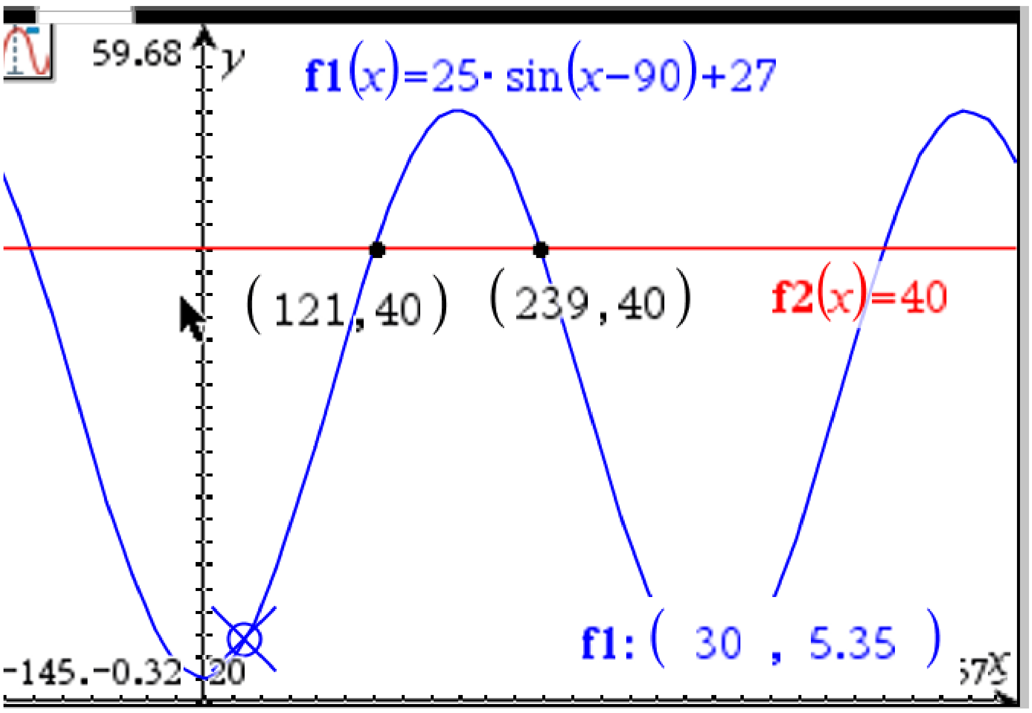


Now graph on Nspire, adjusting window settings accordingly.

Answer the following questions from your NSpire Graph ....

Trace to determine the height of the rider when x = 30°
Height at 30° 5.35 m

Trace to determine the angle when $h(x)$ = 40m
Angle at 40m 121° and 239°



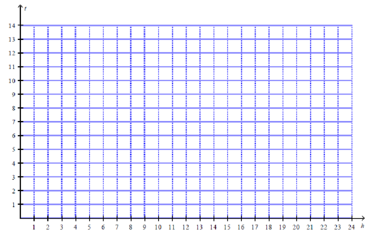
MCR3U ***Applications of Sinusoidal Function***

Tides in the Bay

The depth of water in a bay varies according to the tides. A pole is placed in the water to measure the water's depth. At high tide (midnight) the water at the pole is 12 m deep. At low tide the water at the pole is 2 m deep. Assume the tides run in a 12 hour sinusoidal cycle.

Create a model

Graph the Height vs Time on the graph provided.
Create a sinusoidal model.



a) How deep is the water at the pole at 3:00 am.

b) How deep is the water at the pole at 4:45 pm.

c) At what time during the day will the depth of the water be 10 m?

now what ???

we will do tommorrow in class.

MCR3U ***Applications of Sinusoidal Function***

Tides in the Bay

The depth of water in a bay varies according to the tides. A pole is placed in the water to measure the water's depth. At high tide (midnight) the water at the pole is 12 m deep. At low tide the water at the pole is 2 m deep. Assume the tides run in a 12 hour sinusoidal cycle.

Create a model

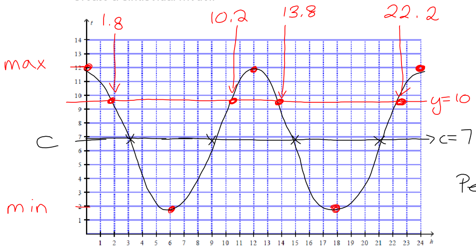
Graph the Height vs Time on the graph provided.
Create a sinusoidal model.

$C = \frac{max + min}{2}$

$= \frac{12+2}{2}$

$= \frac{14}{2}$

$\boxed{C = 7}$



$\boxed{a = 5}$

Period = 12

$K = \frac{360}{12}$

$\boxed{K = 30}$

$f(x) = 5\cos 30x + 7$

a) How deep is the water at the pole at 3:00 am.

7 m

b) How deep is the water at the pole at 4:45 pm.
An algebraic solution is required.

4:45 pm = 16:45 ⇒ $\boxed{16.75 \text{ hours}}$ $f(16.75) = 5\cos(30(16.75)) + 7 \doteq 3$ m

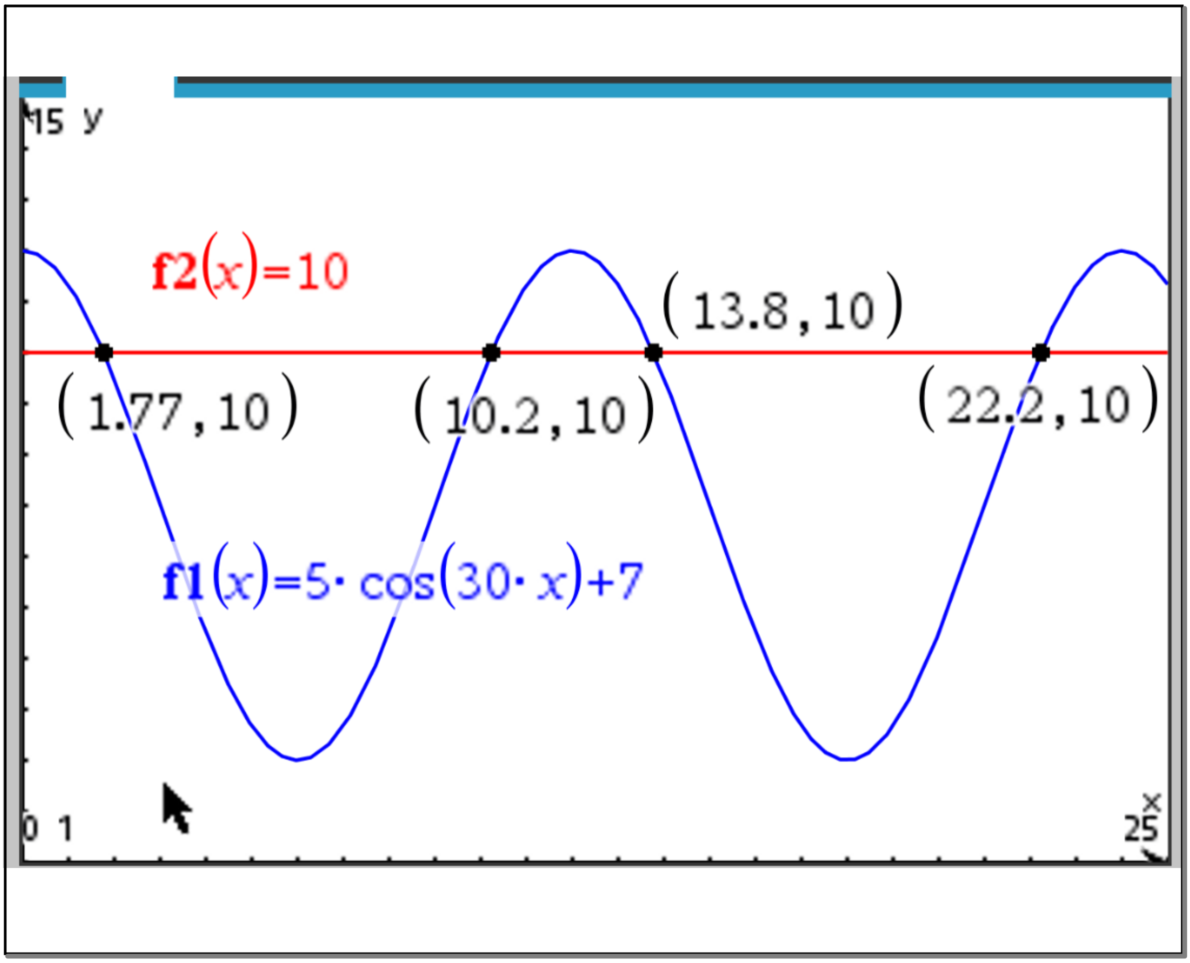
c) At what time during the day will the depth of the water be 10 m?
An algebraic solution is required.

$x = 1.8 \rightarrow$ 1:48 am $\quad .8(60)$

$x = 10.2 \rightarrow$ 10:12 am $\quad .2(60)$

$x = 13.8 \rightarrow$ 1:48 pm $\quad 13-2$

$x = 22.2 \rightarrow$ 10:12 pm $\quad 22-12$



Try On Your Own

P. 372 #10

P. 399 #4, 6

10. The height, $h(t)$, of a basket on a water wheel at time t can be modelled by $h(t) = 2\sin(12t) + 1.5°$, where t is in seconds and $h(t)$ is in metres.

a) Using graphing technology in DEGREE mode and the WINDOW settings shown, graph $h(t)$ and sketch the graph.

b) How long does it take for the wheel to make a complete revolution? Explain how you know.

c) What is the radius of the wheel? Explain how you know.

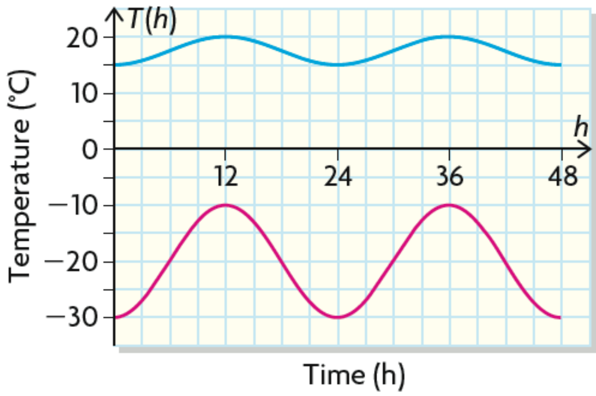
d) Where is the centre of the wheel located in terms of the water level? Explain how you know.

e) Calculate $h(10)$, and explain what it represents in terms of the situation.

4. The interior and exterior temperatures of an igloo were recorded over a 48 h period. The data were collected and plotted, and two curves were drawn through the appropriate points.

K

Interior and Exterior Temperatures



a) How are these curves similar? Explain how each of them might be relat to this situation.

b) Describe the domain and range of each curve.

c) Assuming that the curves can be represented by sinusoidal functions, determine the equation of each function.

6. Milton is floating in an inner tube in a wave pool. He is 1.5 m from the bottom of the pool when he is at the trough of a wave. A stopwatch starts timing at this point. In 1.25 s, he is on the crest of the wave, 2.1 m from the bottom of the pool.

a) Determine the equation of the function that expresses Milton's distance from the bottom of the pool in terms of time.

b) What is the amplitude of the function, and what does it represent in this situation?

c) How far above the bottom of the pool is Milton at $t = 4$ s?

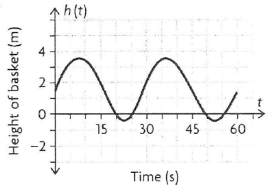
d) If data are collected for only 40 s, how many complete cycles of the sinusoidal function will there be?

e) If the period of the function changes to 3 s, what is the equation of this new function?

Try On Your Own - Solutions

P. 372 #10

10. a)



b) The time it takes for the wheel to complete one revolution is the period of the graph, and the period is the time it takes the graph to go through one complete cycle. Since the graph goes through one complete cycle between 0 s and 30 s, for example, the period is $30 - 0$ or 30 s. Therefore, the wheel completes one revolution in 30 s.

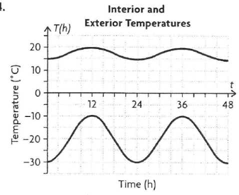
c) The radius of the wheel is the amplitude of the graph, and the amplitude is half the distance between the maximum and minimum values. Since the minimum is -0.5 and the maximum is 3.5, the amplitude is $\frac{3.5 - (-0.5)}{2}$ or 2 m. Therefore, the radius of the wheel is 2 m.

d) Where the centre of the wheel is located in terms of the water level is the equation of the axis of the graph, and the equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is -0.5 and the maximum is 3.5, the equation of the axis is $y = \frac{3.5 + (-0.5)}{2}$ or $y = 1.5$ m. Therefore, the centre of the wheel is located 1.5 m above the water.

Try On Your Own - Solutions

P. 399 #4,

4.



a) They have the same period (24) and the same horizontal translation (12), but different amplitudes (2.5 and 10) and different equations of the axis ($T = 17.5$ and $T = -20$). The top curve is probably the interior temperature (higher, with less fluctuation), while the bottom curve is probably the exterior temperature.

b) The domain is all possible values of t. Since both curves cover a 48 h period, the domain for both curves is $\{t \in \mathbf{R} \mid 0 \le t \le 48\}$. The range is all possible values of T. Since T oscillates between 15 and 20 for the top curve, the range for the top curve is $\{T \in \mathbf{R} \mid 15 \le T \le 20\}$. Since T oscillates between -30 and -10 for the bottom curve, the range for the bottom curve is $\{T \in \mathbf{R} \mid -30 \le T \le -10\}$.

c) i) Since the amplitude for the top curve is 2.5, a in the function $y = a\cos(k(x - d)) + c$ is 2.5. Since the period is 24 h, k in the function $y = a\cos(k(x - d)) + c$ is $\frac{360}{24}$ or 15.

Since the equation of the axis is $y = 17.5$, c in the function $y = a\cos(k(x - d)) + c$ is 17.5. Since the function would have crossed the y-axis at its maximum had it not been shifted 12 units to the right, d in the equation $y = a\cos(k(x - d))$ is 12. Therefore, the equation for the top curve is $T = 2.5\cos[15(h - 12)]° + 17.5$.

ii) Since the amplitude for the bottom curve is 10, a in the function $y = a\cos(k(x - d)) + c$ is 10. Since the period is 24 h, k in the function $y = a\cos(k(x - d)) + c$ is $\frac{360}{24}$ or 15.

Since the equation of the axis is $y = -20$, c in the function $y = a\cos(k(x - d)) + c$ is -20. Since the function would have crossed the y-axis at its maximum had it not been shifted 12 units to the right, d in the equation $y = a\cos(k(x - d))$ is 12. Therefore, the equation for the top curve is $T = 10\cos[15(h - 12)]° - 20$.

Try On Your Own - Solutions

P. 399 #6,

6. a) The amplitude is half the distance between the maximum and minimum values. Since the minimum is 1.5 m and the maximum is 2.1 m, the amplitude is $\frac{2.1 - 1.5}{2}$ or 0.3 m. Since the amplitude is 0.3 m, a in the equation $y = a\cos(k(x - d)) + c$ is 0.3. The period is the change in t that occurs as the function goes through one complete cycle. Since Milton goes through one complete cycle between $t = 0$ s and $t = 2.5$ s, for example, the period is $2.5 - 0$ or 2.5 s. Since the period is 2.5 s, k in the equation $y = a\cos(k(x - d)) + c$ is $\frac{360}{2.5}$ or 144. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 1.5 m and the maximum is 2.1 m, the equation of the axis is $y = \frac{2.1 + 1.5}{2}$ or $y = 1.8$.

144. The equation of the axis is the equation of the horizontal line that is halfway between the maximum and the minimum. Since the minimum is 1.5 m and the maximum is 2.1 m, the equation of the axis is $y = \frac{2.1 + 1.5}{2}$ or $y = 1.8$.
Since the equation of the axis is $y = 1.8$, c in the equation $y = a\cos(k(x - d)) + c$ is 1.8. Since the function crosses the y-axis at its minimum value, the equation for it should use the cosine function and be reflected in its axis. Therefore, the sign of a in the equation $y = a\cos(k(x - d))$ is negative. Therefore, the equation for the function is $d = -0.3\cos(144t)^\circ + 1.8$.

b) The amplitude is half the distance between the maximum and minimum values. Since the minimum is 1.5 m and the maximum is 2.1 m, the amplitude is $\frac{2.1 - 1.5}{2}$ or 0.3 m. In this situation it represents the height of the crest relative to the normal water level.

c) Since the equation for this situation is $d = -0.3\cos(144t)^\circ + 1.8$, at $t = 4$ s, Milton's height above the bottom of the pool is

$$\begin{aligned} d &= -0.3\cos(144 \times 4)^\circ + 1.8 \\ &= -0.3\cos(576)^\circ + 1.8 \\ &= -0.3 \times -0.8 + 1.8 \\ &= 0.2 + 1.8 \\ &= 2 \text{ m} \end{aligned}$$

d) If data are collected for only 40 s, there will be $40 \div 2.5$ or 16 complete cycles.

e) If the period of the function changes to 3 s, k in the equation $y = a\cos(k(x - d)) + c$ is $\frac{360}{3}$ or 120. Therefore, the equation becomes $d = -0.3\cos(120t)^\circ + 1.8$.

Attachments

Unit Circle copy.gsp

Unit Circle Functions .gsp

Desktop (create shortcut).DeskLink

MCR3U - Page 363 #5.tns

sinusoidal transformations.pptx