Applications of Exponential Function

Bacteria of a certain type are known to divide every hour, thus producing two bacteria for every previously existing bacterium. Suppose that 100 of these bacteria are breathed into Paul's lungs.

a. How many bacteria will there be after 5 hours? Make a table of values and sketch

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Make a table of values

t	# of bacteria	

Sketch

b. How many bacteria will there be after t hours? Make an equation.

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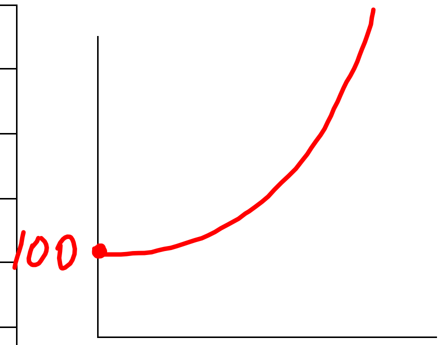
Make a table of values

t	# of bacteria	
0	100	
1	$2 \cdot 100$	
2	$2(2 \cdot 100)$	
3	$2(2 \cdot 2 \cdot 100)$	
4	$2(2 \cdot 2 \cdot 2 \cdot 100)$	
5	$2(2 \cdot 2 \cdot 2 \cdot 2 \cdot 100)$	

Sketch



b. How many bacteria will there be after t hours? Make an equation.

t	# of bacteria	
0	100	$2^0 \cdot 100$
1	$2 \cdot 100$	$2^1 \cdot 100$
2	$2(2 \cdot 100)$	$2^2 \cdot 100$
3	$2(2 \cdot 2 \cdot 100)$	$2^3 \cdot 100$
4	$2(2 \cdot 2 \cdot 2 \cdot 100)$	$2^4 \cdot 100$
5	$2(2 \cdot 2 \cdot 2 \cdot 2 \cdot 100)$	$2^5 \cdot 100$

$$2^t \cdot 100$$

$$y = ab^x$$

$$N(t) = 100 \cdot 2^t$$

General Equation

$$N(t) = a \cdot 2^{\frac{t}{d}}$$

# of hours

doubling period

doubles

# of bacteria after t hours

initial amount of bacteria

Strontium-90 has a half-life of 25 years.

a. If we start with 12mg, how much will be left after t years?
b. If we start with 12mg, how much will be left after 200 years?
c. If we start with 12mg, how many years will it take to reduce the amount to 8 mg?

How is the formula going to be different if we have half-life?

$$N(t) = a \cdot 2^{\frac{t}{d}} \quad 2 \rightarrow \frac{1}{2}$$

$$N(t) = a \cdot \left(\frac{1}{2}\right)^{\frac{t}{d}}$$

time

halfing period

remaining amount

half-life

initial amount

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$$N(t) = a\left(\tfrac{1}{2}\right)^{\frac{t}{d}}$$

a.) $N(t) = 12\left(\tfrac{1}{2}\right)^{\frac{t}{25}}$

b.) $N(t) = 12\left(\tfrac{1}{2}\right)^{\frac{200}{25}}$

$= 0.047$

c.) $8 = 12\left(\tfrac{1}{2}\right)^{\frac{t}{25}}$

$\frac{8}{12} = \left(\tfrac{1}{2}\right)^{\frac{t}{25}}$

$0.67 = (0.5)^{\frac{t}{25}}$

$0.5^{0.57} = 0.673$

$0.5^{0.58} = 0.668$

$\Downarrow$

$\frac{t}{25} = 0.58$

$t = 14.5$

A house's value will increase 5%/year. If the house was purchased for $300000.00, what will its value be 10 years later?

How is the formula going to be different if we have a percent increase?

$$N(t) = ab^{x}$$
$$= a(1+i)^{t}$$

initial value % increase time

$$N(t) = 300\,000(1+0.05)^{10}$$
$$\doteq 489\,000$$

How is the formula going to be different if we have a percent decrease?

Rhea bought a car for \$25000. The car loses ~~\$~~15% of its value per year? What is the value of the car in 5 years?

$$N(t) = ab^x$$

$$N(t) = a(1 - i)^t$$

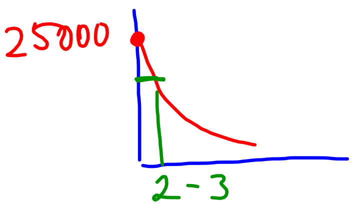
time

initial value

% ~~increase~~ decrease

$$N(t) = 25000(1-0.15)^5$$

$$= 11092.63$$



Try On Your Own

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5. In 1990, a sum of \$1000 is invested at a rate of 6% per year for 15 years.

a) What is the growth rate?

b) What is the initial amount?

c) How many growth periods are there?

d) Write an equation that models the growth of the investment, and use it to determine the value of the investment after 15 years.

6. A species of bacteria has a population of 500 at noon. It doubles every 10 h.
K The function that models the growth of the population, P, at any hour, t, is

$$P(t) = 500\left(2^{\frac{t}{10}}\right).$$

a) Why is the exponent $\frac{t}{10}$?

b) Why is the base 2?

c) Why is the multiplier 500?

d) Determine the population at midnight.

e) Determine the population at noon the next day.

f) Determine the time at which the population first exceeds 2000.

7. Which of these functions describe exponential decay? Explain.

a) $g(x) = -4(3)^x$

b) $h(x) = 0.8(1.2)^x$

c) $j(x) = 3(0.8)^{2x}$

d) $k(x) = \frac{1}{3}(0.9)^{\frac{x}{2}}$

9. **A** A student records the internal temperature of a hot sandwich that has been left to cool on a kitchen counter. The room temperature is 19 °C. An equation that models this situation is

$$T(t) = 63(0.5)^{\frac{t}{10}} + 19$$

where T is the temperature in degrees Celsius and t is the time in minutes.

a) What was the temperature of the sandwich when she began to record its temperature?

b) Determine the temperature, to the nearest degree, of the sandwich after 20 min.

c) How much time did it take for the sandwich to reach an internal temperature of 30 °C?

15. **T** A town has a population of 8400 in 1990. Fifteen years later, its population grew to 12 500. Determine the average annual growth rate of this town's population.

Try On Your Own

Solutions

5. a) The growth rate is 6%.
b) The initial amount is \$1000.
c) There are 15 growth periods.
d) An equation that models the growth of the investment is $V(n) = 1000(1.06)^n$.

The value of the investment after 15 years is $V(15) = 1000(1.06)^{15}$, or \$2396.56.

6. a) The exponent $\frac{t}{10}$ reflects the doubling period of 10 hours.
b) The base 2 represents the population doubling in number (100% growth rate).
c) 500 is the initial population.
d) $P(12) = 500(2^{\frac{12}{10}})$
$= 1149$ bacteria
e) $P(24) = 500(2^{\frac{24}{10}})$
$= 2639$ bacteria
f) The time at which the population first exceeds 2000 will be the first hour t such that
$2000 = 500(2^{\frac{t}{10}})$
$2^{\frac{t}{10}} = \frac{2000}{500}$
$= 4$
Since 2^2 is 4, t is 20. So the population will exceed 2000 at 20 hours from the starting time, or at 8:00 A.M

Try On Your Own

Solutions

7. c and d; both c and d have bases between 0 and 1, so they represent exponential decay.

9. a) The initial temperature of the sandwich was $T(0)$, which is equal to $63 + 19$, or 82°C.

b)
$$\begin{aligned} T(20) &= 63(0.5)^{\frac{20}{10}} + 19 \\ &= 63(0.5)^2 + 19 \\ &= 35°\text{C} \end{aligned}$$

c) The number of minutes t it took for the sandwich to reach an internal temperature of 30°C is the first t such that

$$30 = 63(0.5)^{\frac{t}{10}} + 19, \text{ or when}$$

$$\begin{aligned} (0.5)^{\frac{t}{10}} &= \frac{30 - 19}{63} \\ &= \frac{11}{63} \end{aligned}$$

The closest value of t satisfying this equation is $t = 25$, so it took about 25 minutes for the sandwich to reach an internal temperature of 30°C.

15.
$$\begin{aligned} 8400(1 + n)^{15} &= 12\,500 \\ (1 + n)^{15} &= \frac{12\,500}{8400} \\ &= \frac{125}{84} \\ 1 + n &= \sqrt[15]{\frac{125}{8}} \\ &= 1.027 \end{aligned}$$

So the average annual growth n of this town's population is about 2.7%.