The Ultimate Challenge - How many can you get ?

MCR3U Lesson 5.11 More Identities

1. Prove each identity.

a) $\dfrac{(1-\sin^2\theta)(\tan^2\theta)}{\sin\theta} = \sin\theta$

b) $(1+\tan^2\theta)(1-\sin^2\theta) = 1$

c) $\dfrac{1}{\cos^2\theta} - \tan^2\theta = 1$

d) $1+\tan^2\theta = \dfrac{1}{1-\sin^2\theta}$

e) $\tan\theta + \dfrac{1}{\tan\theta} = \dfrac{1}{\sin\theta\cos\theta}$

f) $\sin^4\theta - \cos^4\theta = 2\sin^2\theta - 1$

g) $2\sin\theta(1+\cos\theta) = \dfrac{2\sin^3\theta}{1-\cos\theta}$

h) $\dfrac{1-\dfrac{1}{\cos^2\theta}}{1-\cos^2\theta} = -\dfrac{1}{\cos^2\theta}$

i) $\dfrac{1+\cos\theta}{\sin\theta} + \dfrac{\sin\theta}{1+\cos\theta} = \dfrac{2}{\sin\theta}$

j) $\dfrac{2\sin^2\theta - 8\sin\theta + 6}{6\cos\theta - 2\sin\theta\cos\theta} = \dfrac{1-\sin\theta}{\cos\theta}$

k) $\dfrac{1}{1-\sin\theta} + \dfrac{1}{1+\sin\theta} = \dfrac{2\tan\theta}{\cos\theta\sin\theta}$

l) $\dfrac{\cos\theta}{1+\sin\theta} + \dfrac{1+\sin\theta}{\cos\theta} = \dfrac{2}{\cos\theta}$

m) $\dfrac{1}{\tan^2\theta} - \tan^2\theta = \dfrac{1}{\sin^2\theta} - \dfrac{1}{\cos^2\theta}$

n) $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} = \dfrac{2}{\sin\theta\cos\theta}$

o) $\dfrac{1+\sin\theta}{1-\sin\theta} = \left(\tan\theta + \dfrac{1}{\cos\theta}\right)^2$

"Tricks"

① Factor (-1)

$\sin^2\theta - 1$

$= -(1 - \sin^2\theta)$

Perfect Square.

$(x+y)^2 = x^2 + 2xy + y^2$

Difference of Squares

$(x^2 - y^2) = (x+y)(x-y)$

② Factor $1 - \cos^2 x$

$= (1+\cos x)(1-\cos x)$

③ $\sin^3\theta = \sin\theta(\sin^2\theta)$

Difference of Squares ~~on steroids~~ "pumped up"

④ $\cos^4\theta - \sin^4\theta$

$= (\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)$

$= (\cos^2\theta - \sin^2\theta)(1)$

$= \cos^2\theta - (1-\cos^2\theta)$

$= 2\cos^2\theta - 1$

⑤ Common Denominator

$\left(\sin^2 x + \dfrac{1}{\cos^2 x}\right)$

$= \dfrac{\sin^2 x\cos^2 x + 1}{\cos^2 x}$

1 a) $\dfrac{(1 - \sin^2\theta)(\tan^2\theta)}{\sin\theta} = \sin\theta$

$LS = \dfrac{(1-\sin^2\theta)(\tan^2\theta)}{\sin\theta}$ $\qquad RS = \sin\theta$

$= \dfrac{\cos^2\theta \left(\dfrac{\sin^2\theta}{\cos^2\theta}\right)}{\sin\theta}$ $\qquad LS = RS.$

$= \dfrac{\sin^2\theta}{\sin\theta}$

$= \sin\theta$

b) $(1 + \tan^2\theta)(1 - \sin^2\theta) = 1$

$LS = (1+\tan^2\theta)(1-\sin^2\theta)$ $\qquad RS = 1$

$= \left(1 + \left(\dfrac{\sin\theta}{\cos\theta}\right)^2\right)(\cos^2\theta)$

$= \left(\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta}\right)(\cos^2\theta)$

$= \cos^2\theta + \sin^2\theta$

$= 1$

$= RS.$

c) $\frac{1}{\cos^2\theta} - \tan^2\theta = 1$

$LS = \frac{1}{\cos^2\theta} - \tan^2\theta$ $\qquad RS = 1$

$= \frac{1}{\cos^2\theta} - \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{1-\sin^2\theta}{\cos^2\theta}$

$= \frac{\cos^2\theta}{\cos^2\theta}$

$= 1$

$= RS.$

d) $1 + \tan^2\theta = \frac{1}{1-\sin^2\theta}$

$LS = 1 + \tan^2\theta$ $\qquad RS = \frac{1}{1-\sin^2\theta}$

$= \frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$ $\qquad = \frac{1}{\cos^2\theta}$

$= \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}$

$= \frac{1}{\cos^2\theta}$

$= RS.$

e) $\tan\theta + \dfrac{1}{\tan\theta} = \dfrac{1}{\sin\theta\cos\theta}$

$LS = \tan\theta + \dfrac{1}{\tan\theta}$ $\qquad RS = \dfrac{1}{\sin\theta\cos\theta}$

$= \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$

$= \dfrac{1}{\sin\theta\cos\theta}$

$= RS.$

f) $\sin^4\theta - \cos^4\theta = 2\sin^2\theta - 1$

$LS = \sin^4\theta - \cos^4\theta$

$= (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)$

$= \sin^2\theta - \cos^2\theta$

$= \sin^2\theta - (1 - \sin^2\theta)$

$= \sin^2\theta - 1 + \sin^2\theta$

$= 2\sin^2\theta - 1$

$= RS.$

g) $2\sin\theta(1+\cos\theta) = \dfrac{2\sin^3\theta}{1-\cos\theta}$

$LS = 2\sin\theta(1+\cos\theta)$

$RS = \dfrac{2\sin^3\theta}{1-\cos\theta}$

$= \dfrac{2\sin\theta(\sin^2\theta)}{1-\cos\theta}$

$= \dfrac{2\sin\theta(1-\cos^2\theta)}{(1-\cos\theta)}$

$= \dfrac{2\sin\theta(1+\cos\theta)(1-\cos\theta)}{(1-\cos\theta)}$

$= 2\sin\theta(1+\cos\theta)$

- Common Denominator
- Factor (-1)

h) $\dfrac{1-\frac{1}{\cos^2\theta}}{1-\cos^2\theta} = -\dfrac{1}{\cos^2\theta}$

$LS = \dfrac{1-\frac{1}{\cos^2\theta}}{1-\cos^2\theta}$

$RS = -\dfrac{1}{\cos^2\theta}$

$= \dfrac{\frac{\cos^2\theta-1}{\cos^2\theta}}{1-\cos^2\theta}$

$= \dfrac{-1(1-\cos^2\theta)}{\cos^2\theta} \times \dfrac{1}{1-\cos^2\theta}$

$= \dfrac{-1}{\cos^2\theta}$

$= RS$

Hint: Common Den,
Expand
CCF

i) $\dfrac{1+\cos\theta}{\ldots} + \dfrac{\sin\theta}{\ldots} = \dfrac{2}{\ldots}$

Hint: Common Den.
Expand
GCF

i) $\frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta} = \frac{2}{\sin\theta}$

$$LS = \frac{1+\cos\theta}{\sin\theta} + \frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{(1+\cos\theta)(1+\cos\theta) + (\sin\theta)(\sin\theta)}{(\sin\theta)(1+\cos\theta)}$$

$$= \frac{1 + 2\cos\theta + \cos^2\theta + \sin^2\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{1 + 2\cos\theta + 1}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2 + 2\cos\theta}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2(1+\cos\theta)}{\sin\theta(1+\cos\theta)}$$

$$= \frac{2}{\sin\theta}$$

$$RS = \frac{2}{\sin\theta}$$

$LS = RS$.

- Factor
- GCF
- Trinomial
- Factor (-1)

j. $\frac{2\sin^2\theta - 8\sin\theta + 6}{6\cos\theta - 2\sin\theta\cos\theta} = \frac{1-\sin\theta}{\cos\theta}$

$$L.S. = \frac{2\sin^2\theta - 8\sin\theta + 6}{6\cos\theta - 2\sin\theta\cos\theta}$$

$$= \frac{2(\sin^2\theta - 4\sin\theta + 3)}{2(3\cos\theta - \sin\theta\cos\theta)}$$

$$= \frac{(\sin\theta - 3)(\sin\theta - 1)}{3\cos\theta - \sin\theta\cos\theta}$$

$$= \frac{(3-\sin\theta)(1-\sin\theta)}{\cos\theta(3-\sin\theta)}$$ ← Factor (-1) from each binomial.

$$= \frac{1-\sin\theta}{\cos\theta}$$

$= RS.$

k) $\dfrac{1}{1-\sin\theta} + \dfrac{1}{1+\sin\theta} = \dfrac{2\tan\theta}{\cos\theta\sin\theta}$

$$L.S. = \frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta}$$
$$= \frac{1+\sin\theta+1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)}$$
$$= \frac{2}{1-\sin^2\theta}$$
$$= \frac{2}{\cos^2\theta}$$

$$RS = \frac{2\tan\theta}{\cos\theta\sin\theta}$$
$$= \frac{2\sin\theta}{\cos\theta} \times \frac{1}{\cos\theta\sin\theta}$$
$$= \frac{2\sin\theta}{\cos^2\theta\sin\theta}$$
$$= \frac{2}{\cos^2\theta}$$

l) $\dfrac{\cos\theta}{1+\sin\theta} + \dfrac{1+\sin\theta}{\cos\theta} = \dfrac{2}{\cos\theta}$

$$LS = \frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} \qquad RS = \frac{2}{\cos\theta}$$
$$= \frac{\cos^2\theta + (1+\sin\theta)^2}{(1+\sin\theta)(\cos\theta)}$$
$$= \frac{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}{(1+\sin\theta)(\cos\theta)}$$
$$= \frac{1+1+2\sin\theta}{(1+\sin\theta)(\cos\theta)}$$
$$= \frac{2(1+\sin\theta)}{(1+\sin\theta)(\cos\theta)}$$
$$= \frac{2}{\cos\theta}$$
$$= RS$$

m. $\dfrac{1}{\tan^2\theta} - \tan^2\theta = \dfrac{1}{\sin^2\theta} - \dfrac{1}{\cos^2\theta}$

$$
\begin{aligned}
LS &= \frac{1}{\tan^2\theta} - \tan^2\theta \\
&= \frac{\cos^2\theta}{\sin^2\theta} - \frac{\sin^2\theta}{\cos^2\theta} \\
&= \frac{\cos^4\theta - \sin^4\theta}{(\sin^2\theta)(\cos^2\theta)} \\
&= \frac{(\cos^2\theta - \sin^2\theta)(\cos^2\theta + \sin^2\theta)}{(\sin^2\theta)(\cos^2\theta)} \\
&= \frac{\cos^2\theta - \sin^2\theta}{(\sin^2\theta)(\cos^2\theta)} \\
&= R.S.
\end{aligned}
$$

$$
\begin{aligned}
R.S. &= \frac{1}{\sin^2\theta} - \frac{1}{\cos^2\theta} \\
&= \frac{\cos^2\theta - \sin^2\theta}{(\sin^2\theta)(\cos^2\theta)}
\end{aligned}
$$

n. $\dfrac{\tan\theta}{1+\cos\theta} + \dfrac{\tan\theta}{1-\cos\theta} = \dfrac{2}{\sin\theta\cos\theta}$

$$
\begin{aligned}
LS &= \frac{\tan\theta}{1+\cos\theta} + \frac{\tan\theta}{1-\cos\theta} \\
&= \frac{\frac{\sin\theta}{\cos\theta}}{1+\cos\theta} + \frac{\frac{\sin\theta}{\cos\theta}}{1-\cos\theta} \\
&= \frac{\sin\theta}{\cos\theta(1+\cos\theta)} + \frac{\sin\theta}{\cos\theta(1-\cos\theta)} \\
&= \frac{\sin\theta(1-\cos\theta) + \sin\theta(1+\cos\theta)}{\cos\theta(1+\cos\theta)(1-\cos\theta)} \\
&= \frac{\sin\theta(1 - \cos\theta + 1 + \cos\theta)}{\cos\theta(1-\cos^2\theta)} \\
&= \frac{\sin\theta(2)}{\cos\theta(\sin^2\theta)} \\
&= \frac{2}{\sin\theta} \\
&= RS
\end{aligned}
$$

o) $\dfrac{1+\sin\theta}{1-\sin\theta} = \left(\tan\theta + \dfrac{1}{\cos\theta}\right)^2$

$$RS = \left(\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)^2$$

$$= \frac{(\sin\theta + 1)^2}{\cos^2\theta}$$

$$= \frac{(\sin\theta + 1)^2}{\sin^2\theta - 1}$$

$$= \frac{(\sin\theta + 1)(\sin\theta + 1)}{(\sin\theta + 1)(\sin\theta - 1)}$$

$$= \frac{1+\sin\theta}{1-\sin\theta}$$

Attachments

sinusoidal transformations.pptx