

**The Ultimate Challenge - How many can you get ?**

MCR3U Lesson 5.11 More Identities

1. Prove each identity.

<p>a) <math>\frac{(1 - \sin^2 \theta)(\tan^2 \theta)}{\sin \theta} = \sin \theta</math></p> <p>c) <math>\frac{1}{\cos^2 \theta} - \tan^2 \theta = 1</math></p> <p>e) <math>\tan \theta + \frac{1}{\tan \theta} = \frac{1}{\sin \theta \cos \theta}</math></p> <p>g) <math>2 \sin \theta (1 + \cos \theta) = \frac{2 \sin^2 \theta}{1 - \cos \theta}</math></p> <p>i) <math>\frac{1 + \cos \theta}{\sin \theta} + \frac{\sin \theta}{1 + \cos \theta} = \frac{2}{\sin \theta}</math></p> <p>k) <math>\frac{1}{1 - \sin \theta} + \frac{1}{1 + \sin \theta} = \frac{2 \tan \theta}{\cos \theta \sin \theta}</math></p> <p>m) <math>\frac{1}{\tan^2 \theta} - \tan^2 \theta = \frac{1}{\sin^2 \theta} - \frac{1}{\cos^2 \theta}</math></p> <p>o) <math>\frac{1 + \sin \theta}{1 - \sin \theta} = \left( \tan \theta + \frac{1}{\cos \theta} \right)^2</math></p>	<p>b) <math>(1 + \tan^2 \theta)(1 - \sin^2 \theta) = 1</math></p> <p>d) <math>1 + \tan^2 \theta = \frac{1}{1 - \sin^2 \theta}</math></p> <p>f) <math>\sin^4 \theta - \cos^4 \theta = 2 \sin^2 \theta - 1</math></p> <p>h) <math>\frac{1 - \frac{1}{\cos^2 \theta}}{1 - \cos^2 \theta} = -\frac{1}{\cos^2 \theta}</math></p> <p>j) <math>\frac{2 \sin^2 \theta - 8 \sin \theta + 6}{6 \cos \theta - 2 \sin \theta \cos \theta} = \frac{1 - \sin \theta}{\cos \theta}</math></p> <p>l) <math>\frac{\cos \theta}{1 + \sin \theta} + \frac{1 + \sin \theta}{\cos \theta} = \frac{2}{\cos \theta}</math></p> <p>n) <math>\frac{\tan \theta}{1 + \cos \theta} + \frac{\tan \theta}{1 - \cos \theta} = \frac{2}{\sin \theta \cos \theta}</math></p>
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"Tricks"

① Factor  $(-1)$   
 $(\sin^2 \theta - 1)$   
 $= -(1 - \sin^2 \theta)$

Perfect Square  
 $(x+y)^2 = x^2 + 2xy + y^2$

② Factor  $1 - \cos^2 x$   
 $= (1 + \cos x)(1 - \cos x)$

Difference of Squares  
 $(x^2 - y^2) = (x+y)(x-y)$

③  $\sin^3 \theta = \sin \theta (\sin^2 \theta)$

Difference of Squares ~~on~~ ~~stro~~ pumped up

④  $\cos^4 \theta - \sin^4 \theta$   
 $= (\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta)$   
 $= (\cos^2 \theta - \sin^2 \theta)(1)$   
 $= \cos^2 \theta - (1 - \cos^2 \theta)$   
 $= 2\cos^2 \theta - 1$

⑤ Common Denominator  
 $\left( \sin^2 x + \frac{1}{\cos^2 x} \right)$   
 $= \frac{\sin^2 x \cos^2 x + 1}{\cos^2 x}$

$$1 \text{ a) } \frac{(1 - \sin^2 \theta)(\tan^2 \theta)}{\sin \theta} = \sin \theta$$

$$LS = \frac{(1 - \sin^2 \theta)(\tan^2 \theta)}{\sin \theta} \quad RS = \sin \theta$$

$$= \frac{(1 - \cos^2 \theta) \left( \frac{\sin^2 \theta}{\cos^2 \theta} \right)}{\sin \theta} \quad LS = RS.$$

$$= \frac{\sin^2 \theta}{\sin \theta}$$

$$= \sin \theta$$

$$b) (1 + \tan^2 \theta)(1 - \sin^2 \theta) = 1$$

$$LS = (1 + \tan^2 \theta)(1 - \sin^2 \theta) \quad RS = 1$$

$$= \left( 1 + \left( \frac{\sin \theta}{\cos \theta} \right)^2 \right) (\cos^2 \theta)$$

$$= \left( \frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta} \right) (\cos^2 \theta)$$

$$= \cos^2 \theta + \sin^2 \theta$$

$$= 1$$

$$= RS.$$

$$c) \frac{1}{\cos^2 \theta} - \tan^2 \theta = 1$$

$$LS = \frac{1}{\cos^2 \theta} - \tan^2 \theta \quad RS = 1$$

$$= \frac{1}{\cos^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$= \frac{1 - \sin^2 \theta}{\cos^2 \theta}$$

$$= \frac{\cos^2 \theta}{\cos^2 \theta}$$

$$= 1$$

$$= RS.$$

$$d) 1 + \tan^2 \theta = \frac{1}{1 - \sin^2 \theta}$$

$$LS = 1 + \tan^2 \theta \quad RS = \frac{1}{1 - \sin^2 \theta}$$

$$= \frac{\cos^2 \theta}{\cos^2 \theta} + \frac{\sin^2 \theta}{\cos^2 \theta}$$

$$= \frac{\cos^2 \theta + \sin^2 \theta}{\cos^2 \theta}$$

$$= \frac{1}{\cos^2 \theta}$$

$$= RS.$$



$$e) \quad \tan\theta + \frac{1}{\tan\theta} = \frac{1}{\sin\theta \cos\theta}$$

$$LS = \tan\theta + \frac{1}{\tan\theta} \qquad RS = \frac{1}{\sin\theta \cos\theta}$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}$$

$$= \frac{1}{\sin\theta \cos\theta}$$

$$= RS.$$

$$f) \quad \sin^4\theta - \cos^4\theta = 2 \sin^2\theta - 1$$

$$LS = \sin^4\theta - \cos^4\theta$$

$$= (\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta)$$

$$= \sin^2\theta - \cos^2\theta$$

$$= \sin^2\theta - (1 - \sin^2\theta)$$

$$= \sin^2\theta - 1 + \sin^2\theta$$

$$= 2 \sin^2\theta - 1$$

$$= RS.$$



g)  $2 \sin \theta (1 + \cos \theta) = \frac{2 \sin^3 \theta}{1 - \cos \theta}$

LS =  $2 \sin \theta (1 + \cos \theta)$     RS =  $\frac{2 \sin^3 \theta}{1 - \cos \theta}$

$$= \frac{2 \sin \theta (\sin^2 \theta)}{1 - \cos \theta}$$

$$= \frac{2 \sin \theta (1 - \cos^2 \theta)}{(1 - \cos \theta)}$$

$$= \frac{2 \sin \theta (1 + \cos \theta)(1 - \cos \theta)}{(1 - \cos \theta)}$$

$$= 2 \sin \theta (1 + \cos \theta)$$

h)  $\frac{1 - \frac{1}{\cos^2 \theta}}{1 - \cos^2 \theta} = -\frac{1}{\cos^2 \theta}$

• Common Denominator  
• Factor (-1)

LS =  $\frac{1 - \frac{1}{\cos^2 \theta}}{1 - \cos^2 \theta}$     RS =  $-\frac{1}{\cos^2 \theta}$

$$= \frac{\frac{\cos^2 \theta - 1}{\cos^2 \theta}}{1 - \cos^2 \theta}$$

$$= \frac{-1(1 - \cos^2 \theta)}{\cos^2 \theta} \times \frac{1}{1 - \cos^2 \theta}$$

$$= \frac{-1}{\cos^2 \theta}$$

$$= \text{RS}$$

i)  $1 + \cos \theta + \sin \theta = 2$

Hint: Common Den.,  
Expand  
CFE

Hint: Common Denom.  
Expand  
GCF

i)  $\frac{1 + \cos \theta}{\sin \theta} + \frac{\sin \theta}{1 + \cos \theta} = \frac{2}{\sin \theta}$

LS =  $\frac{1 + \cos \theta}{\sin \theta} + \frac{\sin \theta}{1 + \cos \theta}$

RS =  $\frac{2}{\sin \theta}$

=  $\frac{(1 + \cos \theta)(1 + \cos \theta) + (\sin \theta)(\sin \theta)}{(\sin \theta)(1 + \cos \theta)}$

LS = RS.

=  $\frac{1 + 2\cos \theta + \cos^2 \theta + \sin^2 \theta}{\sin \theta(1 + \cos \theta)}$

=  $\frac{1 + 2\cos \theta + 1}{\sin \theta(1 + \cos \theta)}$

=  $\frac{2 + 2\cos \theta}{\sin \theta(1 + \cos \theta)}$

=  $\frac{2(1 + \cos \theta)}{\sin \theta(1 + \cos \theta)}$

=  $\frac{2}{\sin \theta}$

Factor  
• GCF  
• Trinomial  
• Factor (-1)

j)  $\frac{2 \sin^2 \theta - 8 \sin \theta + 6}{6 \cos \theta - 2 \sin \theta \cos \theta} = \frac{1 - \sin \theta}{\cos \theta}$

L.S. =  $\frac{2 \sin^2 \theta - 8 \sin \theta + 6}{6 \cos \theta - 2 \sin \theta \cos \theta}$

=  $\frac{2(\sin^2 \theta - 4 \sin \theta + 3)}{2(3 \cos \theta - \sin \theta \cos \theta)}$

=  $\frac{(\sin \theta - 3)(\sin \theta - 1)}{3 \cos \theta - \sin \theta \cos \theta}$

=  $\frac{(3 - \sin \theta)(1 - \sin \theta)}{\cos \theta(3 - \sin \theta)}$  ← Factor (-1) from each binomial.

=  $\frac{1 - \sin \theta}{\cos \theta}$

= RS.



$$k) \quad \frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = \frac{2 \tan\theta}{\cos\theta \sin\theta}$$

$$\begin{aligned} \text{L.S.} &= \frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} & \text{R.S.} &= \frac{2 \tan\theta}{\cos\theta \sin\theta} \\ &= \frac{1+\sin\theta + 1-\sin\theta}{(1-\sin\theta)(1+\sin\theta)} & &= \frac{2 \sin\theta}{\cos\theta} \times \frac{1}{\cos\theta \sin\theta} \\ &= \frac{2}{1-\sin^2\theta} & &= \frac{2 \sin\theta}{\cos^2\theta \sin\theta} \\ &= \frac{2}{\cos^2\theta} & &= \frac{2}{\cos^2\theta} \end{aligned}$$

$$l) \quad \frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} = \frac{2}{\cos\theta}$$

$$\begin{aligned} \text{L.S.} &= \frac{\cos\theta}{1+\sin\theta} + \frac{1+\sin\theta}{\cos\theta} & \text{R.S.} &= \frac{2}{\cos\theta} \\ &= \frac{\cos^2\theta + (1+\sin\theta)^2}{(1+\sin\theta)(\cos\theta)} \\ &= \frac{\cos^2\theta + 1 + 2\sin\theta + \sin^2\theta}{(1+\sin\theta)(\cos\theta)} \\ &= \frac{1 + 1 + 2\sin\theta}{(1+\sin\theta)(\cos\theta)} \\ &= \frac{2(1+\sin\theta)}{(1+\sin\theta)(\cos\theta)} \\ &= \frac{2}{\cos\theta} \\ &= \text{R.S.} \end{aligned}$$



$$m \quad \frac{1}{\tan^2 \theta} - \tan^2 \theta = \frac{1}{\sin^2 \theta} - \frac{1}{\cos^2 \theta}$$

$$L.S. = \frac{1}{\tan^2 \theta} - \tan^2 \theta \quad R.S. = \frac{1}{\sin^2 \theta} - \frac{1}{\cos^2 \theta}$$

$$= \frac{\cos^2 \theta}{\sin^2 \theta} - \frac{\sin^2 \theta}{\cos^2 \theta} \quad = \frac{\cos^2 \theta - \sin^2 \theta}{(\sin^2 \theta)(\cos^2 \theta)}$$

$$= \frac{\cos^4 \theta - \sin^4 \theta}{(\sin^2 \theta)(\cos^2 \theta)}$$

$$= \frac{(\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta)}{(\sin^2 \theta)(\cos^2 \theta)}$$

$$= \frac{\cos^2 \theta - \sin^2 \theta}{(\sin^2 \theta)(\cos^2 \theta)}$$

$$= R.S.$$

$$n \quad \frac{\tan \theta}{1 + \cos \theta} + \frac{\tan \theta}{1 - \cos \theta} = \frac{2}{\sin \theta \cos \theta}$$

$$L.S. = \frac{\tan \theta}{1 + \cos \theta} + \frac{\tan \theta}{1 - \cos \theta}$$

$$= \frac{\frac{\sin \theta}{\cos \theta}}{1 + \cos \theta} + \frac{\frac{\sin \theta}{\cos \theta}}{1 - \cos \theta}$$

$$= \frac{\sin \theta}{\cos \theta(1 + \cos \theta)} + \frac{\sin \theta}{\cos \theta(1 - \cos \theta)}$$

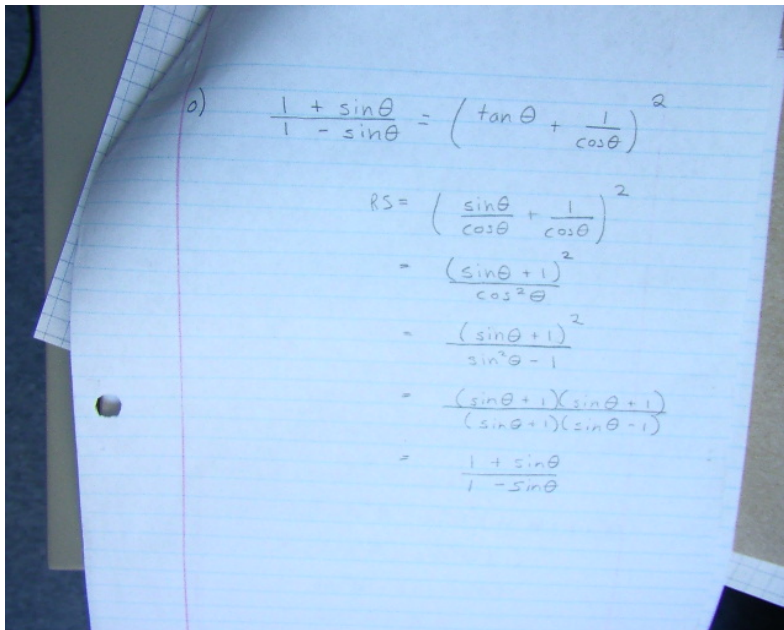
$$= \frac{\sin \theta(1 - \cos \theta) + \sin \theta(1 + \cos \theta)}{\cos \theta(1 + \cos \theta)(1 - \cos \theta)}$$

$$= \frac{\sin \theta(1 - \cos \theta + 1 + \cos \theta)}{\cos \theta(1 - \cos^2 \theta)}$$

$$= \frac{\sin \theta(2)}{\cos \theta(\sin^2 \theta)}$$

$$= \frac{2}{\sin \theta}$$

$$= R.S.$$



A photograph of a piece of lined paper with handwritten mathematical work. The paper is slightly crumpled and has a hole punch on the left side. The work shows a trigonometric identity being proven. The identity is written as  $\frac{1 + \sin\theta}{1 - \sin\theta} = \left(\tan\theta + \frac{1}{\cos\theta}\right)^2$ . Below this, the right side is simplified through several steps:  $RS = \left(\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)^2$ , then  $= \frac{(\sin\theta + 1)^2}{\cos^2\theta}$ , then  $= \frac{(\sin\theta + 1)^2}{\sin^2\theta - 1}$ , then  $= \frac{(\sin\theta + 1)(\sin\theta + 1)}{(\sin\theta + 1)(\sin\theta - 1)}$ , and finally  $= \frac{1 + \sin\theta}{1 - \sin\theta}$ .

$$\begin{aligned} b) \quad \frac{1 + \sin\theta}{1 - \sin\theta} &= \left(\tan\theta + \frac{1}{\cos\theta}\right)^2 \\ RS &= \left(\frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)^2 \\ &= \frac{(\sin\theta + 1)^2}{\cos^2\theta} \\ &= \frac{(\sin\theta + 1)^2}{\sin^2\theta - 1} \\ &= \frac{(\sin\theta + 1)(\sin\theta + 1)}{(\sin\theta + 1)(\sin\theta - 1)} \\ &= \frac{1 + \sin\theta}{1 - \sin\theta} \end{aligned}$$

## Attachments

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sinusoidal transformations.pptx