

"Tricks"

① Factor  $(-1)$

$$\frac{\sin^2 \theta - 1}{\cos^2 \theta}$$

$$= - \frac{(1 - \sin^2 \theta)}{\cos^2 \theta}$$

$$= - \frac{\cancel{\cos^2 \theta}}{\cancel{\cos^2 \theta}}$$

② Factor  $1 - \cos^2 x$

$$= (1 + \cos x)(1 - \cos x)$$

③  $\sin^3 \theta = \sin \theta (\sin^2 \theta)$

Perfect Square.  
 $(x+y)^2 = x^2 + 2xy + y^2$

Difference of Squares  
 $(x^2 - y^2) = (x+y)(x-y)$

Difference of Squares ~~or~~ "pumped up"

④  $\cos^4 \theta - \sin^4 \theta$

$$= (\cos^2 \theta - \sin^2 \theta)(\cos^2 \theta + \sin^2 \theta)$$

$$= (\cos^2 \theta - \sin^2 \theta)(1)$$

$$= \cos^2 \theta - (1 - \cos^2 \theta)$$

$$= 2\cos^2 \theta - 1$$

⑤ Common Denominator

$$\left( \sin^2 x + \frac{1}{\cos^2 x} \right)$$

$$= \frac{\sin^2 x \cos^2 x + 1}{\cos^2 x}$$

$$a) 1 - \sin^2 x = \frac{\sin^2 x}{\tan^2 x}$$

The image shows a handwritten proof of the identity  $1 - \sin^2 x = \frac{\sin^2 x}{\tan^2 x}$ . The left side (LS) is simplified to  $\cos^2 x$ . The right side (RS) is simplified by expressing  $\tan^2 x$  as  $\frac{\sin^2 x}{\cos^2 x}$ , then multiplying the numerator and denominator by  $\cos^2 x$  to cancel the  $\sin^2 x$  terms, resulting in  $\cos^2 x$ . A vertical red line separates the two sides, and the text "LS = RS" is written at the bottom.

$$\begin{aligned} a.) \quad 1 - \sin^2 x &= \frac{\sin^2 x}{\tan^2 x} \\ 1 - \sin^2 x &= \frac{\sin^2 x}{\frac{\sin^2 x}{\cos^2 x}} \\ = \cos^2 x &= \cancel{\sin^2 x} \cdot \frac{\cos^2 x}{\cancel{\sin^2 x}} \\ &= \cos^2 x \end{aligned}$$

LS = RS

$$b) \frac{\sin^2 x - \cos^2 x}{\sin x - \cos x} = \sin x \cos x \left( \frac{1}{\sin x} + \frac{1}{\cos x} \right)$$

$$b.) \frac{\sin^2 x - \cos^2 x}{\sin x - \cos x} = \frac{(\sin x + \cos x)(\cancel{\sin x} - \cancel{\cos x})}{\cancel{\sin x} - \cancel{\cos x}} = \sin x + \cos x$$

$$\sin x \cos x \left( \frac{1}{\sin x} + \frac{1}{\cos x} \right) = \cancel{\sin x} \cos x \left( \frac{\cos x + \sin x}{\cancel{\sin x} \cancel{\cos x}} \right) = \cos x + \sin x$$

$$LS = RS$$

$$c) \frac{\sin x}{1 - \cos x} - \frac{1}{\tan x} = \frac{1}{\sin x}$$

Handwritten solution for the identity:

$$\begin{aligned}
 c.) \quad & \frac{\sin x}{1 - \cos x} - \frac{1}{\tan x} && \frac{1}{\sin x} \\
 = & \frac{\sin x}{1 - \cos x} - \frac{1}{\frac{\sin x}{\cos x}} \\
 = & \frac{\sin x}{1 - \cos x} - \frac{\cos x}{\sin x} \\
 = & \frac{\sin^2 x - \cos x + \cos^2 x}{(1 - \cos x)(\sin x)} \\
 = & \frac{\overbrace{\sin^2 x + \cos^2 x}^= 1} - \cos x}{(1 - \cos x)(\sin x)} \\
 = & \frac{1 - \cancel{\cos x}}{(1 - \cancel{\cos x})(\sin x)} \\
 = & \frac{1}{\sin x}
 \end{aligned}$$

$$d) \sin^2 x + \tan^2 x = \sec^2 x - \cos^2 x$$

The image shows a handwritten proof of the identity  $\sin^2 x + \tan^2 x = \sec^2 x - \cos^2 x$ . The proof is split into two columns by a vertical line. The left column (LS) shows the left side of the equation being simplified. The right column (RS) shows the right side of the equation being simplified. At the bottom, it states "LS = RS".

Left Side (LS):

$$\begin{aligned} d.) \quad & \sin^2 x + \tan^2 x \\ &= \sin^2 x + \frac{\sin^2 x}{\cos^2 x} \\ &= \frac{\sin^2 x \cos^2 x + \sin^2 x}{\cos^2 x} \\ &= \frac{\sin^2 x (\cos^2 x + 1)}{\cos^2 x} \end{aligned}$$

Right Side (RS):

$$\begin{aligned} & \sec^2 x - \cos^2 x \\ &= \frac{1}{\cos^2 x} - \cos^2 x \\ &= \frac{1 - \cos^4 x}{\cos^2 x} \\ &= \frac{(1 + \cos^2 x)(1 - \cos^2 x)}{\cos^2 x} \\ &= \frac{(1 + \cos^2 x)(\sin^2 x)}{\cos^2 x} \end{aligned}$$

LS = RS

$$e) \frac{\cos x}{1 - \sin x} - \sec x = \tan x$$

Handwritten proof of the identity:

$$\begin{aligned}
 e) \quad & \frac{\cos x}{1 - \sin x} - \sec x && \tan x \\
 & = \frac{\cos x}{1 - \sin x} - \frac{1}{\cos x} && = \frac{\sin x}{\cos x} \\
 & = \frac{\cos^2 x - 1 + \sin x}{(1 - \sin x)(\cos x)} && \begin{aligned} & \xrightarrow{\text{arrow}} \frac{-1(\sin x)(\sin x - 1)}{(1 - \sin x)(\cos x)} \\ & = \frac{\sin x(-\cancel{\sin x} + 1)}{(1 - \cancel{\sin x})(\cos x)} \\ & = \frac{\sin x}{\cos x} \end{aligned} \\
 & = \frac{-1(\overbrace{-\cos^2 x + 1}^{\sin^2 x} - \sin x)}{(1 - \sin x)(\cos x)} && \\
 & = \frac{-1(\sin^2 x - \sin x)}{(1 - \sin x)(\cos x)} && \therefore LS = RS
 \end{aligned}$$

# Homework

Pg. 310 # 10, 11, 12

## Attachments

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sinusoidal transformations.pptx