Trigonometric Identities

Learning Goal

- prove simple trigonometric identities

9. Show that $\tan 30° + \frac{1}{\tan 30°} = \frac{1}{\sin 30° \cos 30°}$.

LS	RS
$\tan 30° + \frac{1}{\tan 30°}$	$\frac{1}{\sin 30° \cos 30°}$
$\frac{\sqrt{3}}{3} + \frac{1}{\frac{\sqrt{3}}{3}}$	$\frac{1}{\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)}$
$= \frac{\sqrt{3}}{3} + \frac{3}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$	$= \frac{1}{\frac{\sqrt{3}}{4}}$
$= \frac{\sqrt{3}}{3} + \frac{3\sqrt{3}}{3}$	$= \frac{4}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$
$= \frac{4\sqrt{3}}{3}$	$= \frac{4\sqrt{3}}{3}$

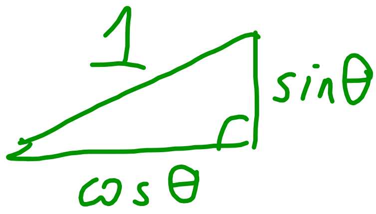
Fundamental Trigonometric Identities

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$ from unit circle

$\sin^2\theta + \cos^2\theta = 1$ from Pythagorean Theorem

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$



$\sin^2\theta = (\sin\theta)^2$

$\csc\theta = \dfrac{1}{\sin\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\cos\theta}{\sin\theta}$

Proofs

1. Separate sides.
2. Change everything to sin and cos.
3. Simplify or transform each side until they match.
    - use identities
    - find common denominators
    - factor

9. Show that $\tan 30° + \frac{1}{\tan 30°} = \frac{1}{\sin 30° \cos 30°}$.

Is this true for other angles ? What about ALL angles ?

Replace 30 ° with θ

LS	RS
$\tan\theta + \frac{1}{\tan\theta}$	$\frac{1}{\sin\theta\ \cos\theta}$
$= \frac{\sin\theta}{\cos\theta} + \frac{1}{\frac{\sin\theta}{\cos\theta}}$	
$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$	
$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta \cdot \sin\theta}$	
$= \frac{1}{\cos\theta \cdot \sin\theta}$	

$\therefore LS = RS$

Factor

$\sin^2\theta + 2\sin\theta + 1$

$\sin\theta = x$

$x^2 + 2x + 1$

$(x+1)(x+1)$

$(\sin\theta + 1)^2$

$6\cos^2\theta - 7\cos\theta - 5$

$\cos\theta = x$

$6x^2 - 7x - 5$

$= (3x-5)(2x+1)$

$(3\cos\theta - 5)(2\cos\theta + 1)$

Simplify

$\sin\theta \cot\theta - \sin\theta \cos\theta$

$= \frac{\cancel{\sin\theta}}{1} \cdot \frac{\cos\theta}{\cancel{\sin\theta}} - \sin\theta\cos\theta$

$= \underline{\cos\theta} - \sin\theta\,\underline{\cos\theta}$

$= \cos\theta\,(1 - \sin\theta)$

Try On Your Own

Prove (use a formal proof)

$$\cos\theta + \sin\theta \tan\theta = \frac{1}{\cos\theta}$$

Prove (use a formal proof)

$$\cos\theta + \sin\theta\tan\theta = \frac{1}{\cos\theta}$$

$$= \frac{\cos\theta}{1} + \sin\theta\,\frac{\sin\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta}{\cos\theta} + \frac{\sin^2\theta}{\cos\theta}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos\theta}$$

$$= \frac{1}{\cos\theta}$$

$$\frac{1}{\cos\theta}$$

$\therefore$ LS = RS

$$1 + \cot^2\theta = \csc^2\theta$$

$1 + \cot^2\theta = \csc^2\theta$

$1 + \frac{\cos^2\theta}{\sin^2\theta}$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin^2\theta}$

$= \frac{1}{\sin^2\theta}$

$\frac{1}{\sin^2\theta}$

LS = RS

$$\frac{1}{1+\cos\theta} + \frac{1}{1-\cos\theta} = \frac{2}{\sin^2\theta}$$

$$\frac{1}{1+\cos\theta}+\frac{1}{1-\cos\theta}=\frac{2}{\sin^2\theta}$$

$$=\frac{(1-\cos\theta)+(1+\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$$

$$=\frac{1-\cancel{\cos}\theta+1+\cancel{\cos}\theta}{1-\cancel{\cos}\theta+\cancel{\cos}\theta-\cos^2\theta}$$

$$=\frac{2}{1-\cos^2\theta}$$

$$=\frac{2}{\sin^2\theta}$$

$$\frac{2}{\sin^2\theta}$$

LS = RS

Try On Your Own - Extra

a) $\sin\theta\, cot\theta = cos\theta$

b) $\cot\theta\sec\theta=\csc\theta$

c) $\dfrac{\sin\theta}{\csc\theta}+\dfrac{\cos\theta}{\sec\theta}=1$

d) $\dfrac{1+\sin\theta}{1-\sin\theta}=\dfrac{\csc\theta+1}{\csc\theta-1}$

a.

$\sin\theta \cot\theta = \cos\theta$

$\cancel{\sin}\theta \cdot \frac{\cos\theta}{\cancel{\sin}}$ | $\cos\theta$

$= \cos\theta$

$LS = RS$

b.

$\cot\theta \sec\theta = \csc\theta$

$\frac{\cancel{\cos\theta}}{\sin\theta} \cdot \frac{1}{\cancel{\cos\theta}}$ | $\frac{1}{\sin\theta}$

$= \frac{1}{\sin\theta}$

$LS = RS$

c.

$$\frac{\sin\theta}{\csc\theta} + \frac{\cos\theta}{\sec\theta} = 1$$

$$= \frac{\sin\theta}{\frac{1}{\sin\theta}} + \frac{\cos\theta}{\frac{1}{\cos\theta}}$$

$$= \sin\theta \cdot \frac{\sin\theta}{1} + \cos\theta \cdot \frac{\cos\theta}{1}$$

$$= \sin^2\theta + \cos^2\theta$$

$$= 1$$

$$1$$

LS = RS

d.

$$\frac{1+\sin\theta}{1-\sin\theta} = \frac{\csc\theta + 1}{\csc\theta - 1}$$

$$\frac{1+\sin\theta}{1-\sin\theta}$$

$$\frac{\frac{1}{\sin\theta} + 1}{\frac{1}{\sin\theta} - 1}$$

$$= \frac{1+\sin\theta}{\sin\theta} \div \frac{1-\sin\theta}{\sin\theta}$$

$$= \frac{1+\sin\theta}{\cancel{\sin\theta}} \times \frac{\cancel{\sin\theta}}{1-\sin\theta}$$

$$= \frac{1+\sin\theta}{1-\sin\theta}$$

LS = RS

Homework

pg 310 # 2, 3, 5, 8, 11, 12

Attachments

sinusoidal transformations.pptx

Unit Circle Functions .gsp