

You will need the following for this Lesson ...

- *Handout (print it if possible)*
- *Your Unit Circle*
- *Trigonometric Ratio Chart*

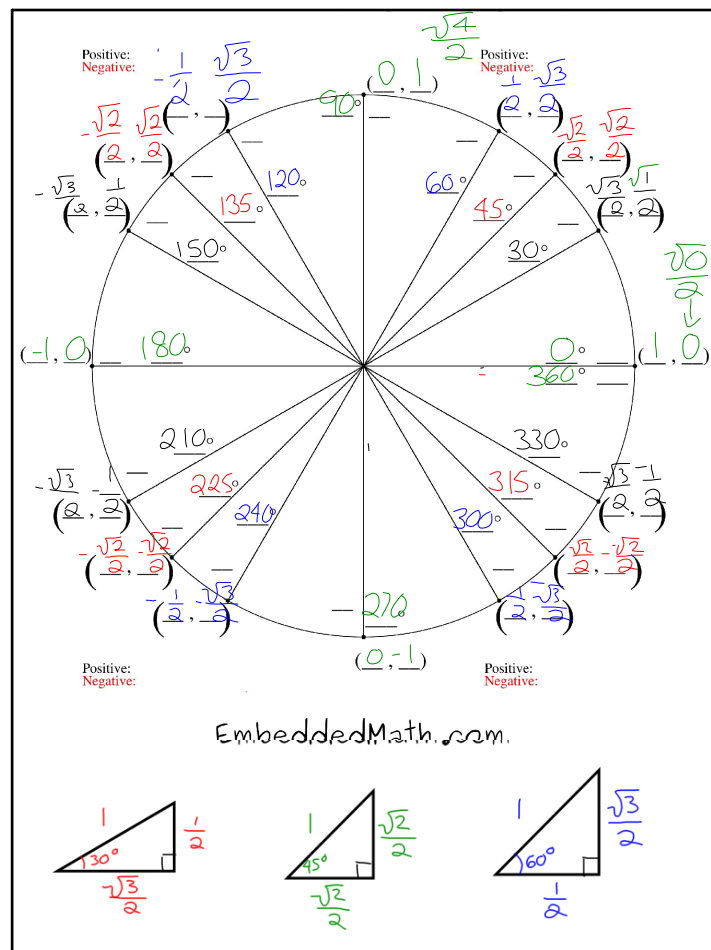
Trigonometric Ratios The CAST Rule

Learning Goals

- be able to determine the exact trigonometric ratio of any angle between 0 and 360

$m^\circ \angle A$	$\sin A$	$\cos A$	$\tan A$	$m^\circ \angle A$	$\sin A$	$\cos A$	$\tan A$
1	0.0175	0.9998	0.0175	46	0.7193	0.6947	1.0355
2	0.0349	0.9994	0.0349	47	0.7314	0.6820	1.0724
3	0.0523	0.9986	0.0524	48	0.7431	0.6691	1.1106
4	0.0698	0.9976	0.0699	49	0.7547	0.6561	1.1504
5	0.0872	0.9962	0.0875	50	0.7660	0.6428	1.1918
6	0.1045	0.9945	0.1051	51	0.7771	0.6293	1.2349
7	0.1219	0.9925	0.1228	52	0.7880	0.6157	1.2799
8	0.1392	0.9903	0.1405	53	0.7986	0.6018	1.3270
9	0.1564	0.9877	0.1584	54	0.8090	0.5878	1.3764
10	0.1736	0.9848	0.1763	55	0.8192	0.5736	1.4281
11	0.1908	0.9816	0.1944	56	0.8290	0.5592	1.4826
12	0.2079	0.9781	0.2126	57	0.8387	0.5446	1.5399
13	0.2250	0.9744	0.2309	58	0.8480	0.5299	1.6003
14	0.2419	0.9703	0.2493	59	0.8572	0.5150	1.6643
15	0.2588	0.9659	0.2679	60	0.8660	0.5000	1.7321
16	0.2756	0.9613	0.2867	61	0.8746	0.4848	1.8040
17	0.2924	0.9563	0.3057	62	0.8829	0.4695	1.8807
18	0.3090	0.9511	0.3249	63	0.8910	0.4540	1.9626
19	0.3256	0.9455	0.3443	64	0.8988	0.4384	2.0503
20	0.3420	0.9397	0.3640	65	0.9063	0.4226	2.1445
21	0.3584	0.9336	0.3839	66	0.9135	0.4067	2.2460
22	0.3746	0.9272	0.4040	67	0.9205	0.3907	2.3559
23	0.3907	0.9205	0.4245	68	0.9272	0.3746	2.4751
24	0.4067	0.9135	0.4452	69	0.9336	0.3584	2.6051
25	0.4226	0.9063	0.4663	70	0.9397	0.3420	2.7475
26	0.4384	0.8988	0.4877	71	0.9455	0.3256	2.9042
27	0.4540	0.8910	0.5095	72	0.9511	0.3090	3.0777
28	0.4695	0.8829	0.5317	73	0.9563	0.2924	3.2709
29	0.4848	0.8746	0.5543	74	0.9613	0.2756	3.4874
30	0.50	0.8660	0.5774	75	0.9659	0.2588	3.7321
31	0.5150	0.8572	0.6009	76	0.9703	0.2419	4.0108
32	0.5299	0.8480	0.6249	77	0.9744	0.2250	4.3315
33	0.5446	0.8387	0.6494	78	0.9781	0.2079	4.7046
34	0.5592	0.8290	0.6745	79	0.9816	0.1908	5.1446
35	0.5736	0.8192	0.7002	80	0.9848	0.1736	5.6713
36	0.5878	0.8090	0.7265	81	0.9877	0.1564	6.3138
37	0.6018	0.7986	0.7536	82	0.9903	0.1392	7.1154
38	0.6157	0.7880	0.7813	83	0.9925	0.1219	8.1443
39	0.6293	0.7771	0.8098	84	0.9945	0.1045	9.5144
40	0.6428	0.7660	0.8391	85	0.9962	0.0872	11.4301
41	0.6561	0.7547	0.8693	86	0.9976	0.0698	14.3007
42	0.6691	0.7431	0.9004	87	0.9986	0.0523	19.0811
43	0.6820	0.7314	0.9325	88	0.9994	0.0349	28.6363
44	0.6947	0.7193	0.9657	89	0.9998	0.0175	57.2900
45	0.7071	0.7071	1	90	1	0	Undefined

Trigonometric Ratios



Warm up #1

Pause the video and Try on your own

Use your Unit Circle to determine ...

$$\sin 30^\circ = \quad \cos 60^\circ = \quad \tan 45^\circ =$$

$$\sin 150^\circ = \quad \cos 120^\circ = \quad \tan 135^\circ =$$

$$\sin 210^\circ = \quad \cos 240^\circ = \quad \tan 225^\circ =$$

$$\sin 330^\circ = \quad \cos 300^\circ = \quad \tan 315^\circ =$$

What do you notice?

What do you wonder?

Warm up #1

Use your Unit Circle to determine ...

$$\sin 30^\circ = \frac{1}{2} \quad \cos 60^\circ = \frac{\sqrt{3}}{2} \quad \tan 45^\circ = 1$$

$$\sin 150^\circ = \frac{1}{2} \quad \cos 120^\circ = -\frac{\sqrt{3}}{2} \quad \tan 135^\circ = -1$$

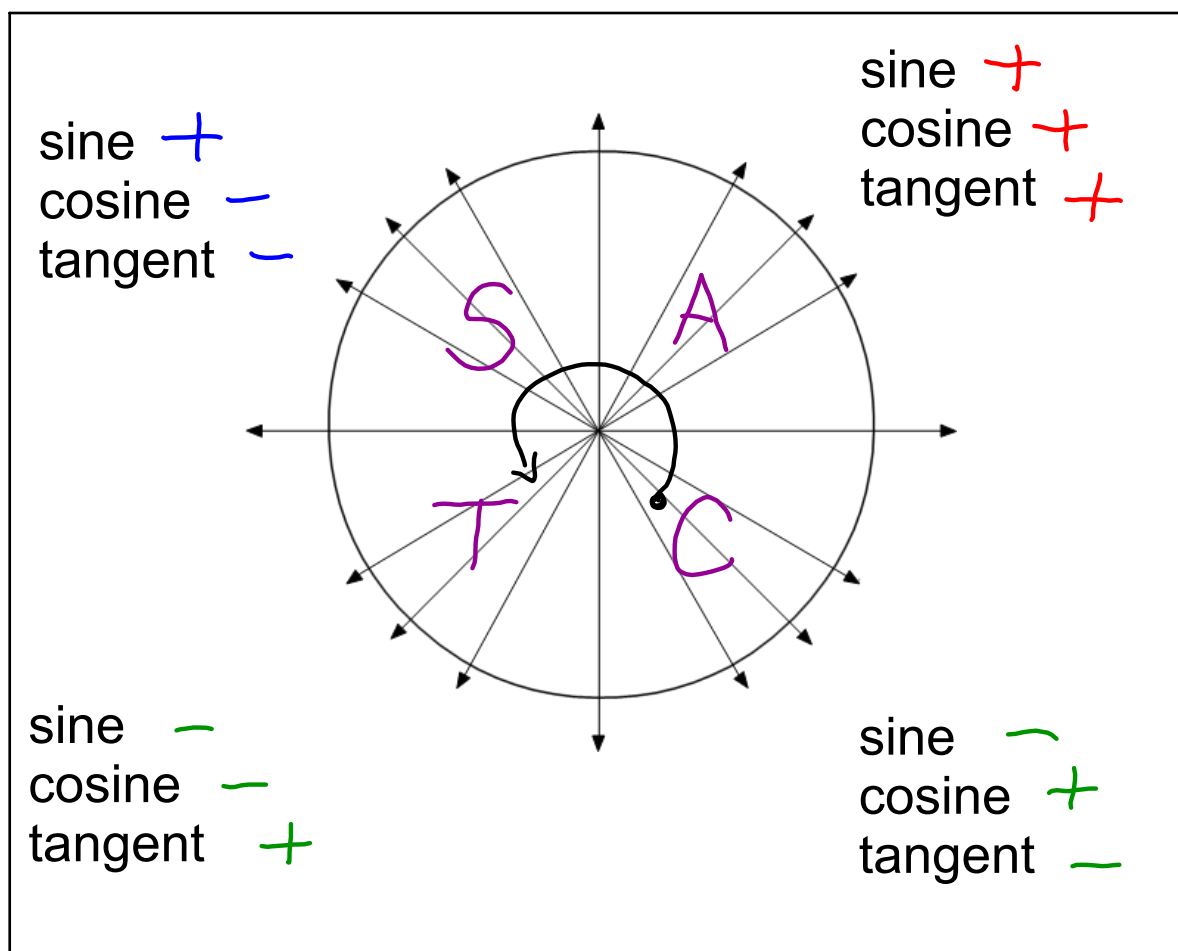
$$\sin 210^\circ = -\frac{1}{2} \quad \cos 240^\circ = -\frac{\sqrt{3}}{2} \quad \tan 225^\circ = 1$$

$$\sin 330^\circ = -\frac{1}{2} \quad \cos 300^\circ = \frac{\sqrt{3}}{2} \quad \tan 315^\circ = -1$$

What do you notice?

What do you wonder?

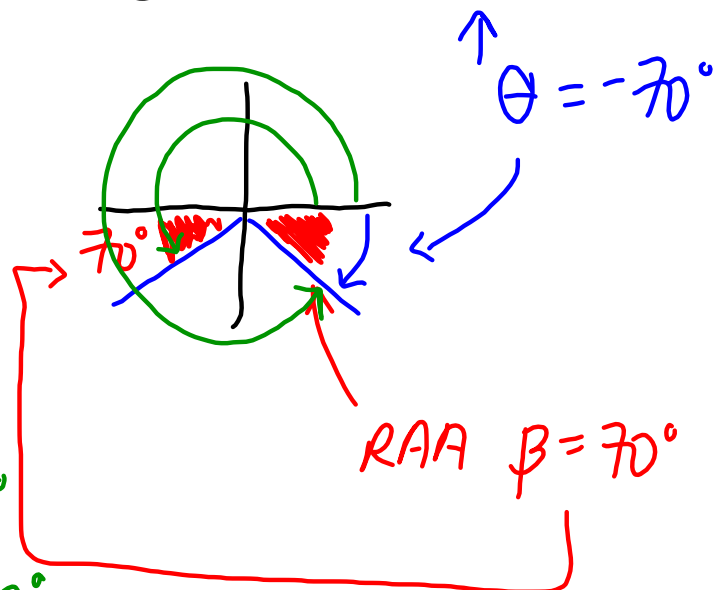
THE CAST RULE



Complete these on a sheet of paper as the Lesson continues.

#1 Find all possible angles for $\sin \theta = -0.94$

S	A
T	C

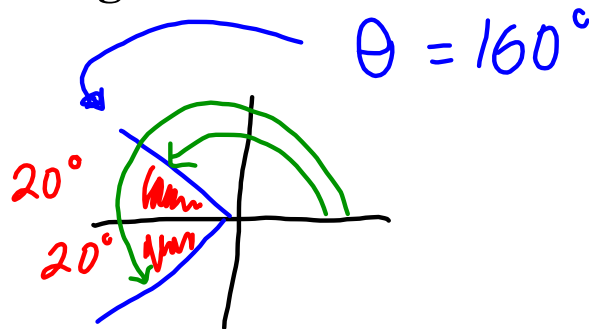


$$180^\circ + 70^\circ = 250^\circ$$

$$360^\circ - 70^\circ = 290^\circ$$

#2 Find all possible angles for $\cos \theta = -0.94$

✓S	A
✓T	C

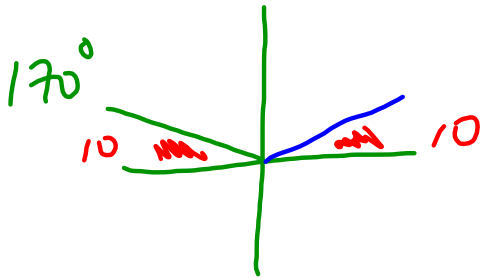


$$180^\circ + 20^\circ = 200^\circ$$

#3 Use the Related Acute Angle to ...

Find an equivalent expression for $\sin 170^\circ$

$$\sin 170^\circ = \sin 10^\circ$$

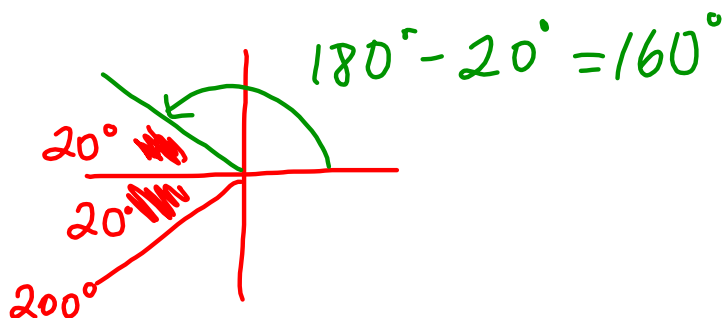


10	.1736
11	1908



#4 Use the Related Acute Angle to ...

Find an equivalent expression for $\cos 200^\circ$



20	.3420	.9397
21	3584	0226

This is the end of the video

Do the Try on your own questions for practice

= make sure that you draw your quadrants so you can visualize where things are positive or negative

Try On Your Own #1 - No Calculators - only trig table

Pause the Video

If you have a printer - Print the Note

If you don't have a printer you MUST copy this as a note.

MCR 3U

Using the CAST Rule and the Related Acute Angle
Trig Table - No Calculators

Use each trigonometric ratio to determine the Related Acute Angle.
Then use this to determine all values of θ , to the nearest degree if $0^\circ < \theta < 360^\circ$.

a) $\sin \theta = -0.3256$

b) $\cos \theta = -0.7325$

1. Determine Related Acute Angle β

1. Determine Related Acute Angle β

2. Use CAST to sketch β and θ 's



3. Determine θ_1 and θ_2

2. Use CAST to sketch β and θ 's



3. Determine θ_1 and θ_2

c) $\tan \theta = -1.5$

d) $\cos \theta = 0.7777$

1. Calculate Related Acute Angle β

1. Calculate Related Acute Angle β

2. Use CAST to sketch β and θ 's



3. Determine θ_1 and θ_2

2. Use CAST to sketch β and θ 's



3. Determine θ_1 and θ_2

No Calculators - only trig table

MCR 3U - Mrs McNeil

REMEMBER THE RAA RATIO IS ALWAYS POSITIVE

Example #3 Use each trigonometric ratio to determine the Related Acute Angle. Then use this to determine all values of θ , to the nearest degree if $0^\circ < \theta < 360^\circ$.

a) $\sin \theta = -0.3256$

Determine
1. Calculate Related Acute Angle β
 $\sin \beta = .3256$
 $\beta = 19^\circ$

2. Use CAST to sketch β and θ s

3. Determine θ_1 and θ_2
 $\theta_1 = 180 + 19 = 199^\circ$
 $\theta_2 = 360 - 19 = 341^\circ$

b) $\cos \theta = -0.7325$

Determine
1. Calculate Related Acute Angle β
 $\cos \beta = .7325$
 $\beta = 43$

2. Use CAST to sketch β and θ s

3. Determine θ_1 and θ_2
 $\theta_1 = 180 - 43 = 137^\circ$
 $\theta_2 = 180 + 43 = 223^\circ$

c) $\tan \theta = -1.5$

Determine
1. Calculate Related Acute Angle β
 $\tan \beta = 1.5$
 $\beta = 56$

2. Use CAST to sketch β and θ s

3. Determine θ_1 and θ_2
 $\theta_1 = 180 - 56 = 124^\circ$
 $\theta_2 = 360 - 56 = 304^\circ$

d) $\cos \theta = 0.7777$

Determine
1. Calculate Related Acute Angle β
 $\cos \beta = .7777$
 $\beta = 39$

2. Use CAST to sketch β and θ s

3. Determine θ_1 and θ_2
 $\theta_1 = 39^\circ$
 $\theta_2 = 360 - 39 = 321^\circ$

Try On Your Own #2 - No Calculators - only trig table

MCR 3U

Solve for θ where $0 \leq \theta \leq 360$ by first calculating the Related Acute Angle β and then determine the value(s) of θ .

a) $\cos \theta = -0.8667$

b) $\sin \theta = -0.7234$

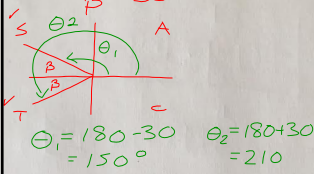
c) $2 \sin \theta = -1$

d) $-5 \cos \theta + 3 = 2$

Solving Linear Trigonometric Equations using the Related Acute Angle

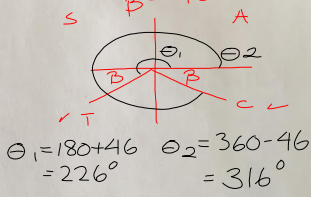
Solve for θ where $0 \leq \theta \leq 360$ by first calculating the Related Acute Angle β and then determine the value(s) of θ .

a) $\cos \theta = -0.8667$
 $\cos \beta = .8667$
 $\beta = 30^\circ$



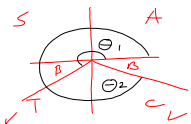
$\theta_1 = 180 - 30 = 150^\circ$
 $\theta_2 = 180 + 30 = 210^\circ$

b) $\sin \theta = -0.7234$
 $\sin \beta = .7234$
 $\beta = 46^\circ$



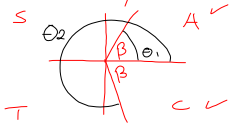
$\theta_1 = 180 + 46 = 226^\circ$
 $\theta_2 = 360 - 46 = 316^\circ$

c) $2 \sin \theta = -1$
 $\sin \theta = -\frac{1}{2}$
 $\sin \beta = \frac{1}{2}$
 $\beta = 30^\circ$



$\theta_1 = 180 + 30 = 210^\circ$
 $\theta_2 = 360 - 30 = 330^\circ$

d) $-5 \cos \theta + 3 = 2$
 $-5 \cos \theta = -1$
 $\cos \theta = \frac{1}{5}$
 $\cos \theta = 0.200$
 $\cos \beta = 0.2000$
 $\beta = 78^\circ$



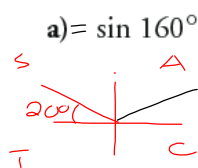
$\theta_1 = 78^\circ$
 $\theta_2 = 360 - 78 = 282^\circ$

Try On Your Own #3

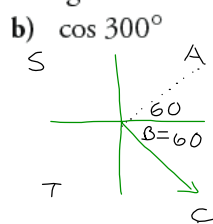
4. Use the related acute angle to state an equivalent expression.

- a) $\sin 160^\circ$ b) $\cos 300^\circ$ c) $\tan 110^\circ$ d) $\sin 350^\circ$

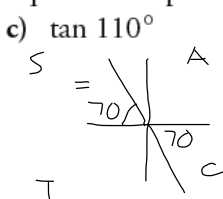
4. Use the related acute angle to state an equivalent expression.



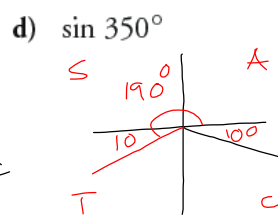
$$\sin 20$$



$$= \cos 60^\circ$$



$$= \tan 70$$



$$= \sin 190^\circ$$

P. 299 #4 - In textbook