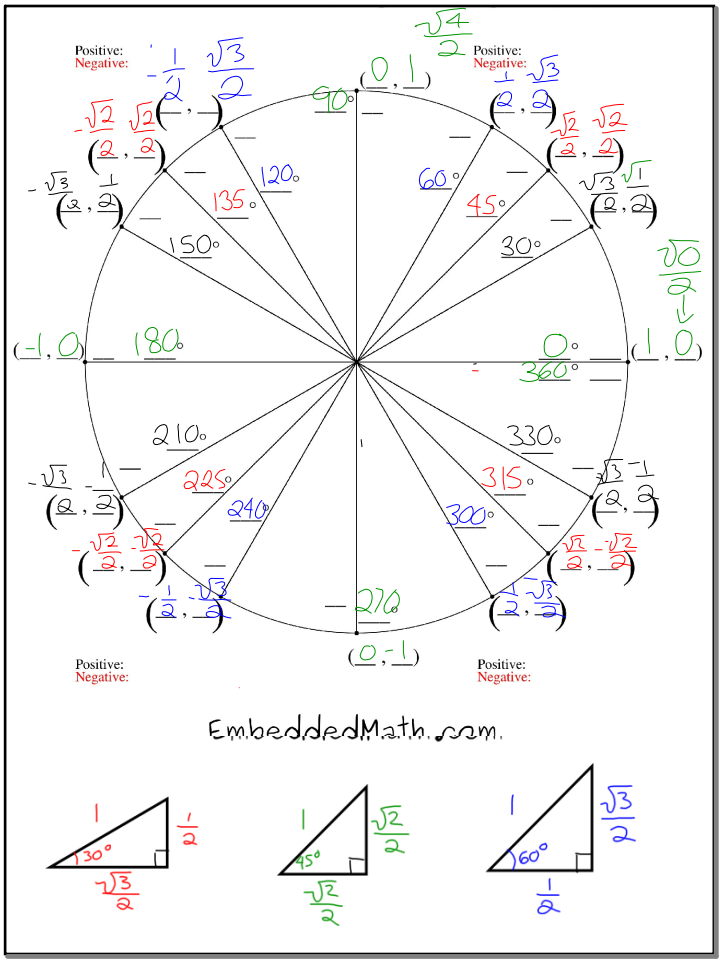
You will need the following for this Lesson ...

- *Two handouts (print them if possible)*
- *Your Unit Circle*
- *A Protractor (look in your Grade School math kit)*

You will NOT need a calculator



Pause the video and Try on your own

Warm up #1

- Use Your Unit Circle to determine the Exact Trig Ratios and Then Evaluate

$1 - \dfrac{\sin 45^\circ}{\cos 45^\circ}$

c) $\tan^2 30^\circ - \cos^2 45^\circ$

Warm up #1

$1 - \dfrac{\sin 45^\circ}{\cos 45^\circ}$

$= 1 - \dfrac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}}$

$= 1 - 1$

$= 0$

c) $\tan^2 30^\circ - \cos^2 45^\circ$

$\left(\dfrac{\sin 30^\circ}{\cos 30^\circ}\right)^2 - \cos^2 45^\circ$

$= \left(\dfrac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right)^2 - \left(\dfrac{\sqrt{2}}{2}\right)^2$

$= \left(\dfrac{1}{2} \times \dfrac{2}{\sqrt{3}}\right)^2 - \dfrac{2}{4}$

$= \left(\dfrac{1}{\sqrt{3}}\right)^2 - \dfrac{1}{2}$

$= \dfrac{1}{3} - \dfrac{1}{2}$

$= \dfrac{2 - 3}{6}$

$= \dfrac{-1}{6}$

Pause the video and Try on your own

Warm up #2

Determine θ, 0° < θ < 360°

Hint - *Isolate the Trig Ratio,*

- *Use the Unit Circle* -- *there will be two answers*

$2\cos\theta = \sqrt{3}$ $\sqrt{3}\tan\theta = 1$

Warm up #2

$2\cos\theta = \sqrt{3}$

$\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = 30°, 330°$

$\sqrt{3}\tan\theta = 1$

$\tan\theta = \frac{1}{\sqrt{3}}$

$\tan\theta = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}$ ← $\frac{\sin\theta}{\cos\theta}$

$\theta = 30°, 210°$

$= \frac{1}{3} - \frac{1}{2}$

$= \frac{2-3}{6}$

$= \frac{-1}{6}$

Trigonometric Ratios for all Angles

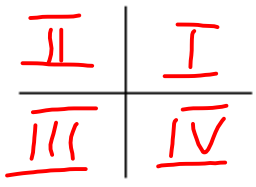
Learning Goals
- identify all the "special terms" related to angles in a Cartesian Plane
- be able to determine the Related Acute Angle of any angle
- be able to determine the exact trigonometric ratio of any angle between 0° and 360°

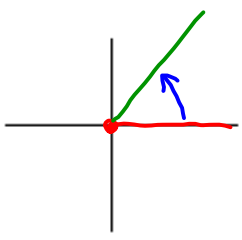
If you have a printer - Print the Note

If you don't have a printer you MUST copy this as a note.

Understanding Angles

- angles can be located anywhere in the x-y plane
- the x and y-axis divide the x-y plane into 4 quadrants



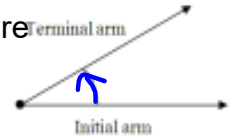
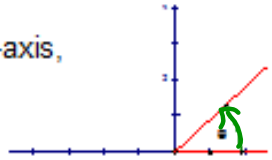


Vertex – at the origin

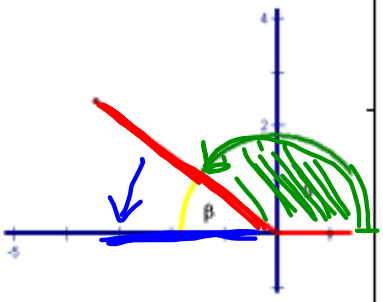
Initial arm – start of the angle

Terminal arm - end of the angle

Angle - between initial and terminal arms

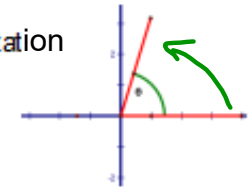
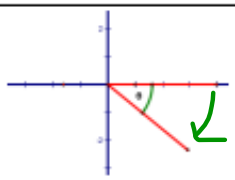
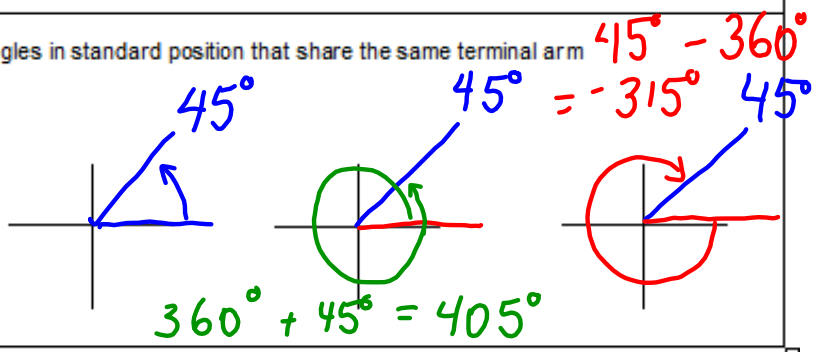
Angle	Formed by two rays that share a common endpoint.  $\alpha, \beta, \gamma, \theta$
Measure of an Angle	The size of an angle. The rotation between the initial arm and terminal arm of the angle. Angles are often measured in degrees.
Standard Position	An angle drawn with its initial arm on the x-axis, and endpoint the origin. 

Principal Angle	The counterclockwise angle between the initial arm and the terminal arm of an angle in standard position. θ is the principal angle. Its value lies between $0° \le \theta \le 360°$
Related Acute Angle RAA	An angle formed between the terminal arm and the x-axis. β is the related acute angle. -always positive -always between 0 and 90°



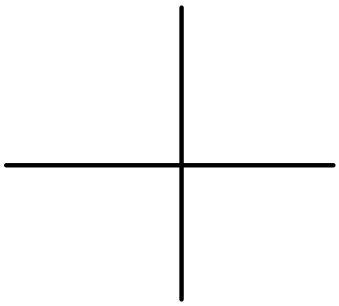
This is VERY important

Greek Letter		Name	Equivalent
Α	α	Alpha	A
Β	β	Beta	B
Γ	γ	Gamma	G
Δ	δ	Delta	D
Ε	ε	Epsilon	E
Ζ	ζ	Zeta	Z
Η	η	Eta	E
Θ	θ	Theta	Th
Ι	ι	Iota	I
Κ	κ	Kappa	K
Λ	λ	Lambda	L
Μ	μ	Mu	M
Ν	ν	Nu	N
Ξ	ξ	Xi	X
Ο	ο	Omicron	O
Π	π	Pi	P
Ρ	ρ	Rho	R
Σ	σ	Sigma	S
Τ	τ	Tau	T
Υ	υ	Upsilon	U
Φ	φ	Phi	Ph
Χ	χ	Chi	Ch
Ψ	ψ	Psi	Ps
Ω	ω	Omega	O

Positive Angle	Angle formed by counter-clockwise rotation 
Negative Angle	Angle formed by clockwise rotation. 
Coterminal Angle	Angles in standard position that share the same terminal arm  415° - 360° = -315°

Pause the video and Try on your own

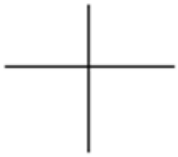
Example 1: Given $\theta = -150°$, determine the principal angle and the related acute angle.



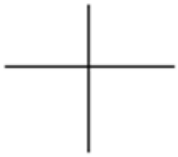
Example 2: Given the following angles, sketch the angle, the next coterminal angle, the first negative angle and determine the related acute angle.

a) $\theta = 35°$

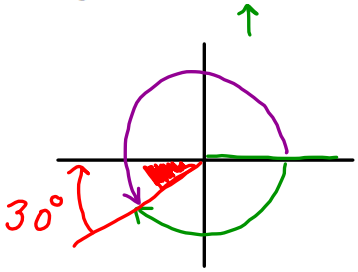
b) $\theta = 140°$



c) $\theta = 240°$



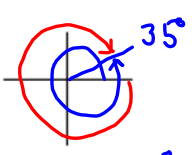
Example 1: Given $\theta = -150°$, determine the principal angle and the related acute angle.



$180° + 30° = 210°$

Example 2: Given the following angles, sketch the angle, the next coterminal angle, the first negative angle and determine the related acute angle.

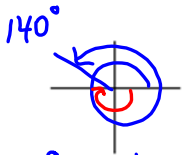
a) $\theta = 35°$



$35° + 360° = 395°$

$35° - 360° = -325°$

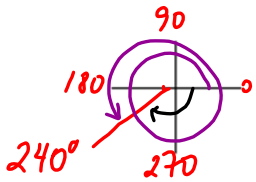
b) $\theta = 140°$



$140° + 360° = 500°$

$140° - 360° = -220$

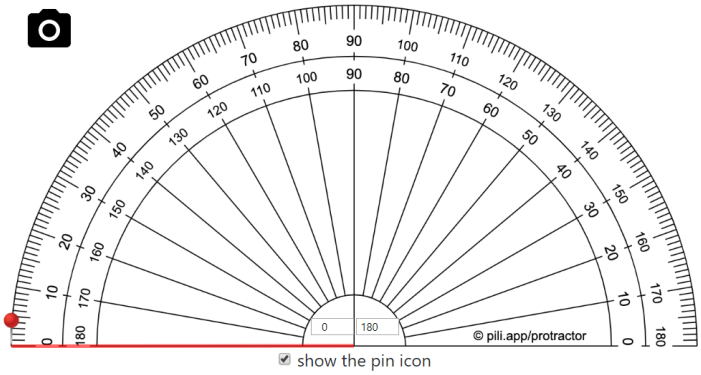
c) $\theta = 240°$



$240° + 360° = 600°$

$240° - 360° = -120°$

You will need a protractor



Link to online protractor

https://www.piliapp.com/protractor/

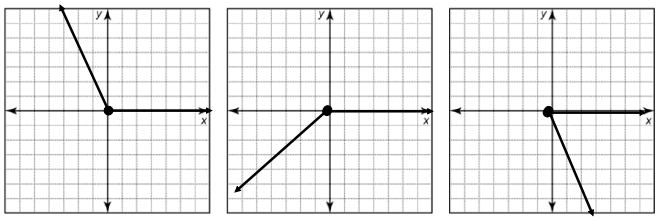
If you have a printer - Print the pdf

If you don't have a printer you MUST copy this as a note.

Pause the video and Try on your own

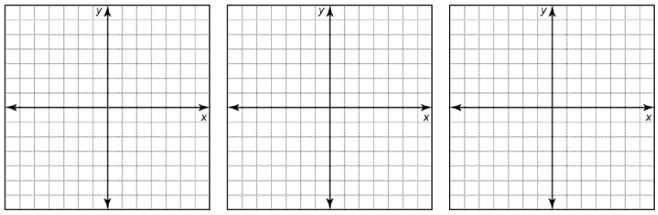
5.3 Related Acute Angles

1. Label the Principal Angle θ.
2. Use a protractor to measure Angle θ.
3. Identify the Related Acute Angle β and determine its' measure.



4. Determine the Related Acute Angle β of the following angles.

θ = 140° θ = 260° θ = 350°



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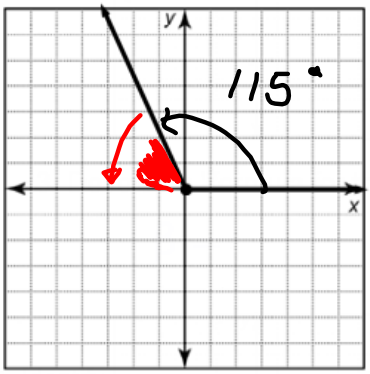
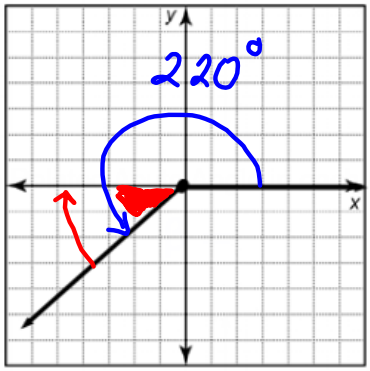
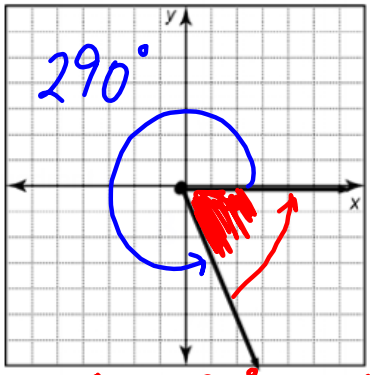


$180° - 115° = 65°$



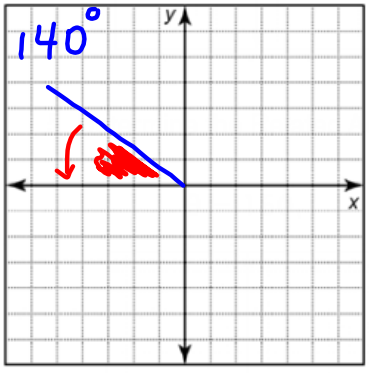
$220° - 180° = 40°$



$360° - 290° = 70°$

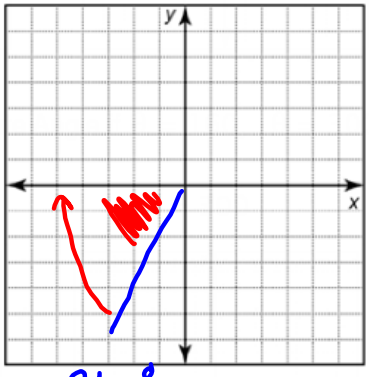
4. Determine the Related Acute Angle β of the following angles.

θ = 140°



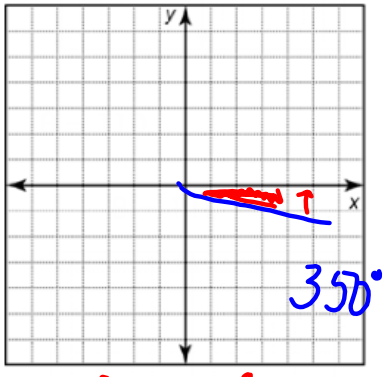
$180° - 140° = 40°$

θ = 260°



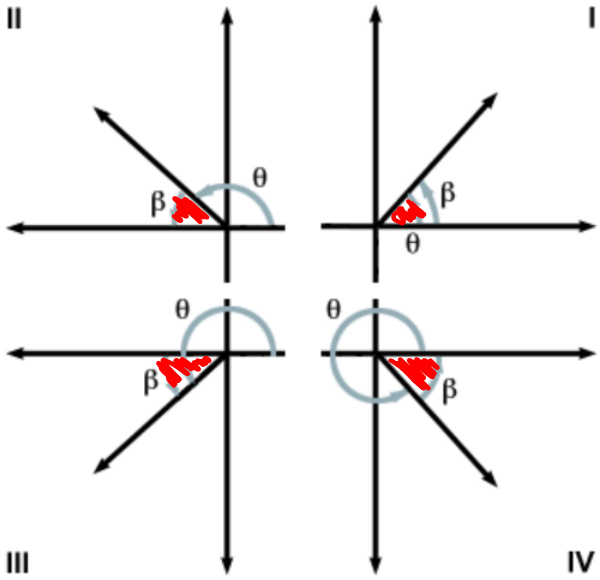
$260° - 180° = 80°$

θ = 350°



$360° - 350° = 10°$

Summary



quadrant	β (reference angle)
I	$\beta = \theta$
II	$\beta = 180 - \theta$
III	$\beta = \theta - 180$
IV	$\beta = 360 - \theta$

Try on Your Own

1. For each trigonometric ratio, use a sketch to determine in which quadrant the terminal arm of the principal angle lies, the value of the related acute angle β,

a) $\sin 315°$ **b)** $\tan 110°$ **c)** $\cos 285°$ **d)** $\tan 225°$

2. Determine all the angles between $0°$ and $360°$ with the following Related Acute Angles.

a) $\beta = 10^0$ b) $\beta = 45^0$ c) $\beta = 71^0$

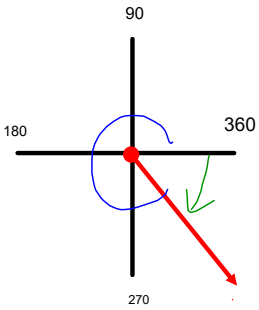
3. Determine two coterminal angles for each of the following Principal Angles.

a) $\theta = 170^0$ b) $\theta = 207^0$ c) $\theta = -53^0$

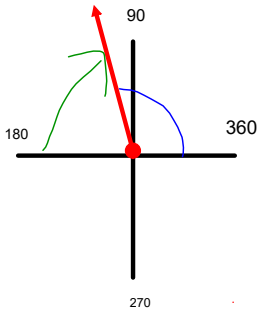
Try on Your Own - Solutions

1. For each trigonometric ratio, use a sketch to determine in which quadrant the terminal arm of the principal angle lies, the value of the related acute angle β,

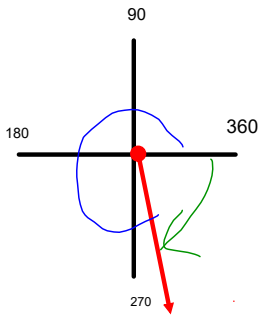
a) $\sin 315°$ **b)** $\tan 110°$ **c)** $\cos 285°$ **d)** $\tan 225°$



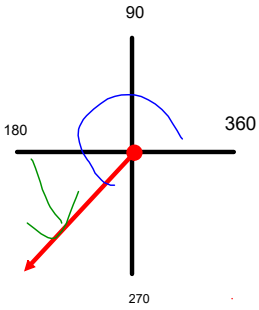
RAA = 360 - 315
= 45



RAA = 180 - 110
= 70



RAA = 360 - 285
= 75

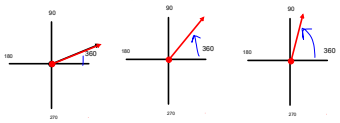


RAA = 225 - 180
= 45

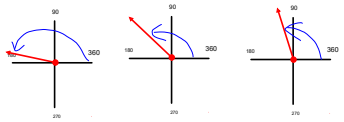
Try on Your Own - Solutions

2. Determine all the angles between 0° and 360° with the following Related Acute Angles.

a) $\beta = 10^0$ b) $\beta = 45^0$ c) $\beta = 71^0$



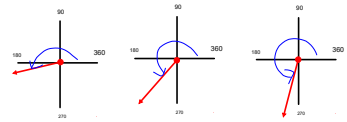
$\theta = 10°$ $\theta = 45°$ $\theta = 71°$



$\theta = 180° - 10°$
$= 170°$

$\theta = 180° - 45°$
$= 135°$

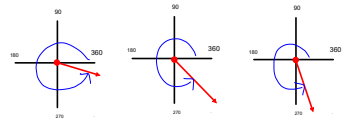
$\theta = 180° - 71°$
$= 109°$



$\theta = 180° + 10°$
$= 190°$

$\theta = 180° + 45°$
$= 225°$

$\theta = 180° + 71°$
$= 251°$



$\theta = 360° - 10°$
$= 350°$

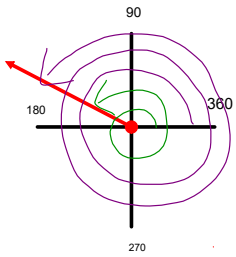
$\theta = 360° - 45°$
$= 315°$

$\theta = 360° - 71°$
$= 289°$

Try on Your Own - Solutions

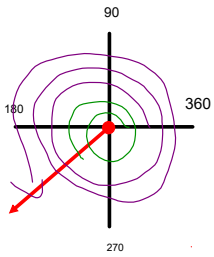
3. Determine two coterminal angles for each of the following Principal Angles.

a) $\theta = 170^0$ b) $\theta = 207^0$ c) $\theta = -53^0$



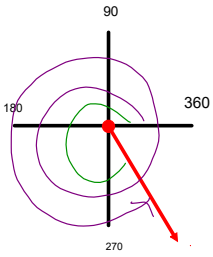
170 + 360 = 530

170 + 720 = 890



207 + 360 = 567

207 + 720 = 927



-53 + 360 = 307

-53 + 720 = 667