Warm up #1 - Try On Your Own

Find the zeros.

$(x-5)(2x-1) = 0$

$x^2 - 25 = 0$

$x^2 - 2x = 12$

Warm up #1 - On Your Own

Find the zeros.

$(x-5)(2x-1) = 0$

$x-5=0$

$x_1 = 5$

$2x-1=0$

$\frac{2x}{2} = \frac{1}{2}$

$x_2 = \frac{1}{2}$

$x^2 - 25 = 0$

$x^2 = 25$

$\sqrt{x^2} = \pm\sqrt{25}$

$x = \pm 5$

$x_1 = 5 \quad x_2 = -5$

$x^2 - 2x = 12$

$x^2 - 2x - 12 = 0$

-12, -2

Doesn't factor

QF $a=1 \; b=-2 \; c=-12$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \frac{2 \pm \sqrt{(-2)^2 - 4(1)(-12)}}{2(1)}$

$x = \frac{2 \pm \sqrt{52}}{2}$

$x = \frac{2 \pm 2\sqrt{13}}{2}$

$x = 1 \pm \sqrt{13}$

Warm up #2 - Try On Your Own

The demand function for a new automotive part is $p(x)=-0.5x+7.8$, where p is the price in dollars and x is the quantity sold in thousands. The new part can be manufactured by three different processes, A, B, or C. The cost function for each process is given by:

A $C(x)=4.6x+5.12$

B $C(x)=3.8x+5.12$

C $C(x)=5.3x+3.8$

Use the TI-Nspire to investigate the break-even quantities.

Which process would you recommend to the company?

BIG HINT provided on the next page.

BIG Hint ...

Revenue = (Price)(# sold)

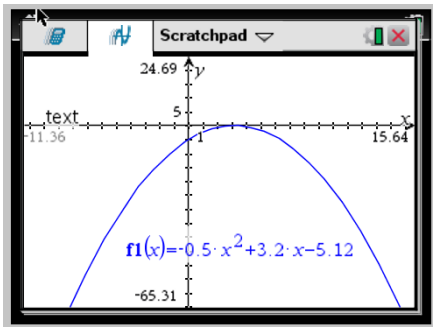
Profit = Revenue - Cost

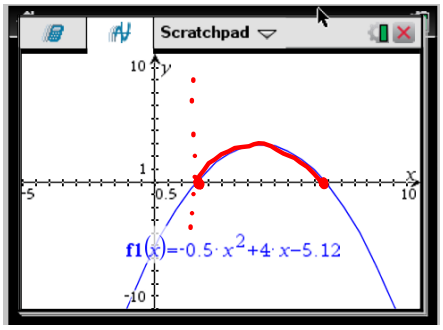
A) $p(x)=-0.5x+7.8$, $C(x)=4.6x+5.12$

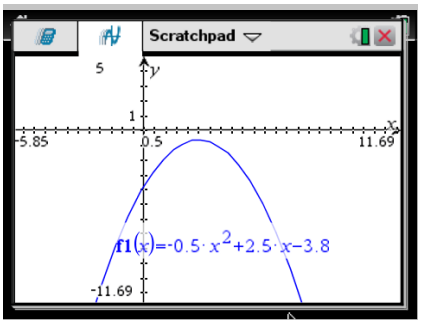
$$P(x) = (-0.5x+7.8)(x) - (4.6x+5.12)$$

$$= -0.5x^2+7.8x-4.6x-5.12$$

$$= -0.5x^2+3.2x-5.12$$







Recommendation

The company will make a profit after selling 1600 items.

Turn on the Video

The Number of Zeros in a Quadratic Function

Learning goal
- determine the number of zeros of a quadratic function

We can't always graph everything.

Algebraic Methods:

1. Factored Form
2. Vertex Form
3. Standard form - Discriminant

If you have a printer a Note has been provided on the website.

Factored Form $f(x) = a(x-r)(x-s)$

If the quadratic is expressed in factored form then there are

$f(x) = 3(x-5)(x+1)$

2 different → 2 zeros

$f(x) = 3(x-5)(x-5)$

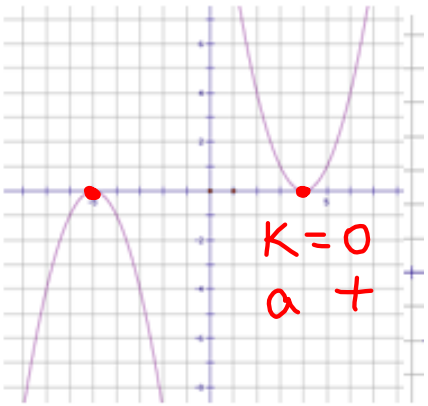
same → 1 zero

$f(x) = 3(x-5)^2 = 3(x-5)(x-5)$

Note: If there are no zeros, we can't use factored form.

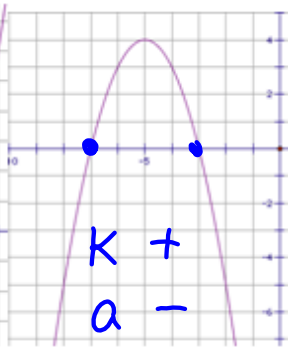
Vertex Form $f(x) = a(x-h)^2 + k$

Consider the following graphs: What are the values of a? k?



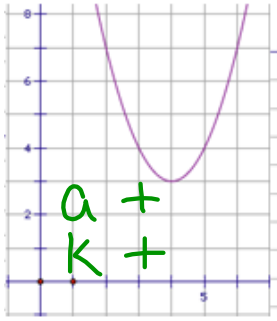
k = 0
a +

a +
k −



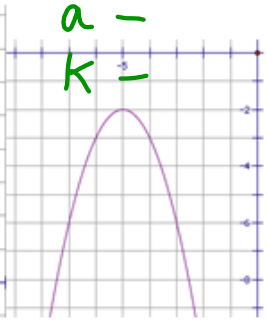
k +
a −

a −
k = 0



a +
k +

a −
k −



Summary

$a \neq 0$ and $k = 0$	One root	
$a > 0$ and $k > 0$	No real roots	Hint: a and k are same sign
$a < 0$ and $k < 0$		
$a > 0$ and $k < 0$	Two roots	Hint: a and k are opposite signs
$a < 0$ and $k > 0$		

Standard Form: $f(x) = ax^2 + bx + c$

The **discriminant** is $b^2 - 4ac$

Value of $b^2 - 4ac$	Number of Zeros
$b^2 - 4ac > 0$	2
$b^2 - 4ac = 0$	1
$b^2 - 4ac < 0$	0

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$\frac{2 \pm \sqrt{9}}{2}$ $\overset{\sqrt{9}=3}{=}$ $\frac{2 \pm 3}{2}$ → 2 zeros

$\frac{2 \pm \sqrt{0}}{2} = \frac{2 \pm 0}{2}$ → 1 zero

$\frac{2 \pm \sqrt{-16}}{2}$ = not a real result → NO zeros

This is the end of the video

Do the Try on your own...

Textbook questions are always Optional

On Your Own #1

Example:
Predict how many zeros (i.e. f(x) = 0) each of the following functions have.

$$f(x) = (x-1)(x-10)$$
$$f(x) = 0.5x(x+4)$$
$$f(x) = 2(x+3)^2$$
$$f(x) = -3(x-2)(x+7)$$

On Your Own #1

Example:
Predict how many zeros (i.e. f(x) = 0) each of the following functions have.

$f(x) = (x-1)(x-10)$ — 2

$f(x) = 0.5x(x+4)$ — 2

$f(x) = 2(x+3)^2$ — 1

$f(x) = -3(x-2)(x+7)$ — 2

On Your Own #2

Example:
Predict how many zeros (i.e. f(x) = 0) each of the following functions have.

$f(x) = 2(x+3)^2 - 7$

$f(x) = 2(x+3)^2 + 7$

$f(x) = 2(x+3)^2$

$f(x) = -\frac{1}{2}(x-5)^2 + 10$

On Your Own #2

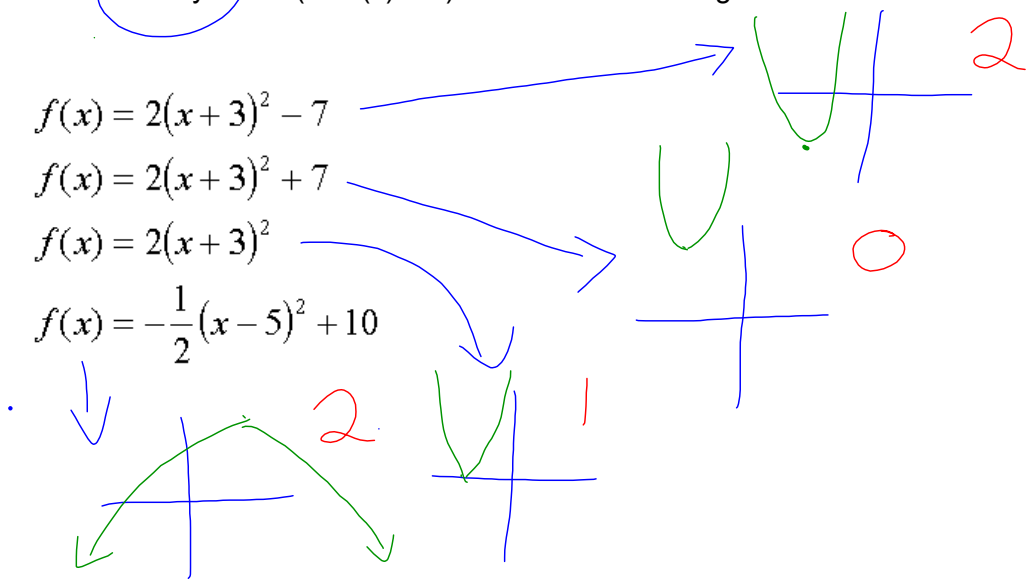
Example:
Predict how many zeros (i.e. f(x) = 0) each of the following functions have.

$$f(x) = 2(x+3)^2 - 7$$

$$f(x) = 2(x+3)^2 + 7$$

$$f(x) = 2(x+3)^2$$

$$f(x) = -\frac{1}{2}(x-5)^2 + 10$$



On Your Own #3

Example:
Use the discriminant to determine the number of zeroes.

$$f(x) = -2x^2 + 12x - 18$$

$$g(x) = 2x^2 + 6x - 8$$

$$h(x) = x^2 - 4x + 7$$

On Your Own #3

Example:
Use the discriminant to determine the number of zeroes.

$f(x) = -2x^2 + 12x - 18$ → $D = b^2 - 4ac$
$= (12)^2 - 4(-2)(-18)$
$= 0$ one

$g(x) = 2x^2 + 6x - 8$ → $D = (6)^2 - 4(2)(-8)$
$= 100$
Two

$h(x) = x^2 - 4x + 7$ → $D = (-4)^2 - 4(1)(7)$
$= 16 - 28$
$=$ Negative
None

On Your Own #4

What is the value of k that ensures that $f(x) = kx^2 - 4x + 6$ has NO zeros?

and verify by graphing with technology

On Your Own #4 - HINT

What is the value of k that ensures that $f(x) = kx^2 - 4x + 6$ has NO zeros?

discriminant must be negative

On Your Own #4

What is the value of k that ensures that $f(x) = kx^2 - 4x + 6$ has NO zeros?

discriminant must be negative

$a = k \quad b = -4 \quad c = 6$

$\therefore \quad b^2 - 4ac < 0$

$(-4)^2 - 4(k)(6) < 0$

$16 - 24k < 0$

$16 - 16 - 24k < 0 - 16$

$-24k < -16$

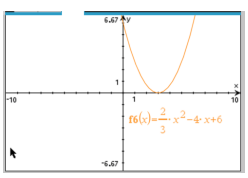
$\frac{-24k}{-24} > \frac{-16}{-24}$

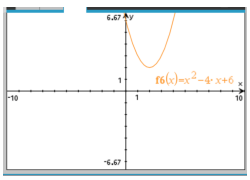
$k > \frac{2}{3}$

we can solve inequalities like equations EXCEPT if we multiply or divide by a negative we need to change the direction of the sign.

since we are dividing by -24 the less than becomes a greater than

and verify by graphing with technology





if k = 2/3 only one zero

if k > 2/3, I chose k= 1, NO zeros

On Your Own #5

Example:
Determine the value of k so that f(x) has only one zero.

$$f(x) = x^2 - kx + 3$$

and verify by graphing with technology

On Your Own #5 - HINT

Example:
Determine the value of k so that f(x) has only one zero.

$$f(x) = x^2 - kx + 3$$

discriminant must be zero

On Your Own #5

Example:
Determine the value of k so that f(x) has only one zero.

$f(x) = x^2 - kx + 3$

discriminant must be zero

$b^2 - 4ac = 0$

$a = 1$
$b = -k$
$c = 3$

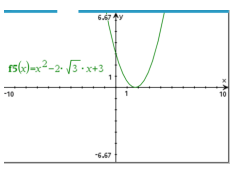
$(-k)^2 - 4(1)(3) = 0$
$k^2 - 12 = 0$
$\sqrt{k^2} = \pm\sqrt{12}$
$k = \pm\sqrt{12}$
$k = \pm 2\sqrt{3}$
$k = 2\sqrt{3}$ or $k = -2\sqrt{3}$

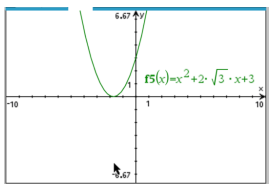
Solve to find two exact values of "K"

$f(x) = x^2 - 2\sqrt{3}x + 3$

$f(x) = x^2 - (-2\sqrt{3})x + 3$
$= x^2 + 2\sqrt{3}x + 3$

Now verify by graphing with technology





On Your Own #6

A market researcher predicted that the profit for the first year of a business would be, $P(x) = -0.3x^2 + 3x - 15$

Will it be possible for the business to break even in its first year?

What part of the parabola are you looking for?

On Your Own #6

A market researcher predicted that the profit for the first year of a business would be, $P(x) = -0.3x^2 + 3x - 15$

Will it be possible for the business to break even in its first year?

zeros

discriminant

$b^2 - 4ac$

$= 3^2 - 4(-0.3)(-15)$

$= -9$

$-9 < 0$

∴ no zeros

∴ it is always loosing money

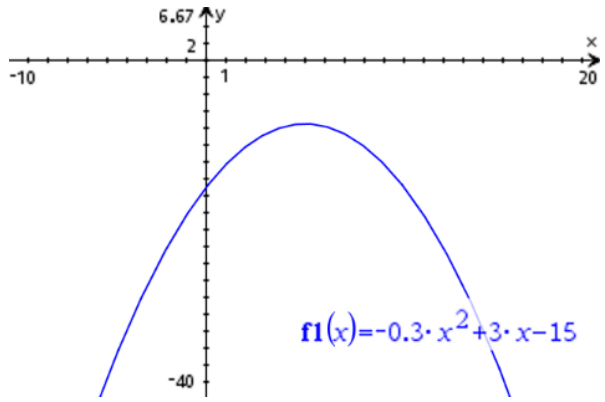
∴ it is not possible to break even

On Your Own #6

A market researcher predicted that the profit for the first year of a business would be, $P(x) = -0.3x^2 + 3x - 15$

Will it be possible for the business to break even in its first year?

You can graph to check as well



no zeros

Optional Extra Practise

pg 185 # 4ab, 5ab, 6, 7, 8, 9

4. Determine the number of zeros. Do not use the same method for all four parts. K

a) $f(x) = -3(x-2)^2 + 4$

b) $f(x) = 5(x-3)(x+4)$

c) $f(x) = 4x^2 - 2x$

d) $f(x) = 3x^2 - x + 5$

5. For each profit function, determine whether the company can break even. If the company can break even, determine in how many ways it can do so. A

a) $P(x) = -2.1x^2 + 9.06x - 5.4$

b) $P(x) = -0.3x^2 + 2x - 7.8$

c) $P(x) = -2x^2 + 6.4x - 5.12$

d) $P(x) = -2.4x^2 + x - 1.2$

6. For what value(s) of k will the function $f(x) = 3x^2 - 4x + k$ have one x-intercept?

7. For what value(s) of k will the function $f(x) = kx^2 - 4x + k$ have no zeros?

8. For what values of k will the function $f(x) = 3x^2 + 4x + k = 0$ have no zeros? one zero? two zeros?

9. The graph of the function $f(x) = x^2 - kx + k + 8$ touches the x-axis at one point. What are the possible values of k?

3. a) 2 zeros b) no zeros c) 1 zero d) 1 zero
4. a) 2 zeros b) 2 zeros c) 2 zeros d) no zeros
5. a) 2 break-even points c) 1 break-even point
   b) Cannot break even d) Cannot break even
6. $k = \frac{4}{3}$
7. $k < -2$ or $k > 2$
8. $k > \frac{4}{3}, k = \frac{4}{3}, k < \frac{4}{3}$
9. $k = -4$ or 8
10. No, resulting quadratic has no solutions.

Lesson 3.6, pp. 185–186

1. a) vertex $(0, -5)$, up, 2 zeros d) vertex $(-2, 0)$, up, 1 zero
   b) vertex$(0, 7)$, down, 2 zeros e) vertex $(-3, -5)$, down, no zeros
   c) vertex $(0, 3)$, up, no zeros f) vertex $(4, -2)$, up, 2 zeros
2. a) 2 zeros b) 2 zeros c) 2 zeros d) 1 zero
3. a) 2 zeros b) no zeros c) 1 zero d) 1 zero
4. a) 2 zeros b) 2 zeros c) 2 zeros d) no zeros
5. a) 2 break-even points c) 1 break-even point
   b) Cannot break even d) Cannot break even
6. $k = \frac{4}{3}$
7. $k < -2$ or $k > 2$
8. $k > \frac{4}{3}, k = \frac{4}{3}, k < \frac{4}{3}$
9. $k = -4$ or 8