Solving Quadratic Equations

Learning Goals

- solve quadratic equations five different ways

- solve real world problems by finding the zeros

Warm - Up - Try on Your Own

- Use Graphing Technology

Janice sells T-shirts. Her profit P is modelled by $P(x)=-x^2 + 22x - 40$, where x represents the number of T-shirts sold.

Find the breakeven point.

What part of the parabola are you looking for?

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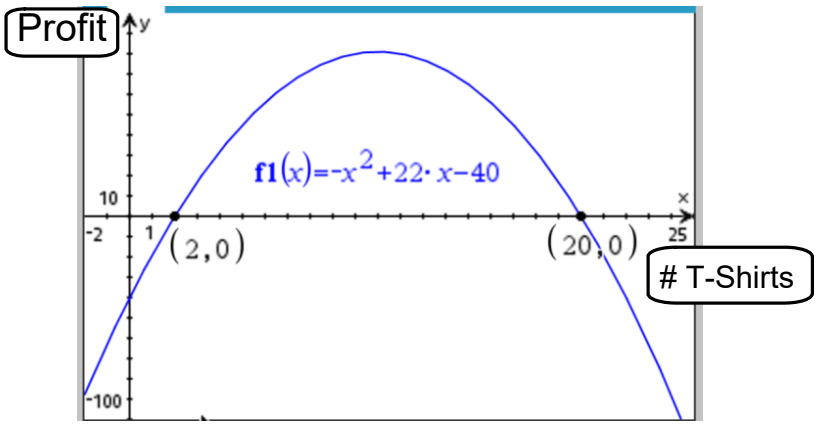
Find the breakeven point.

What part of the parabola are you looking for?

Definition of *breakeven* (Entry 1 of 2)

: the point at which cost and income are equal and there is neither profit nor loss

So Profit = \$0, we need to find zeros.



Video starts here

You should have done the Warm-Up first

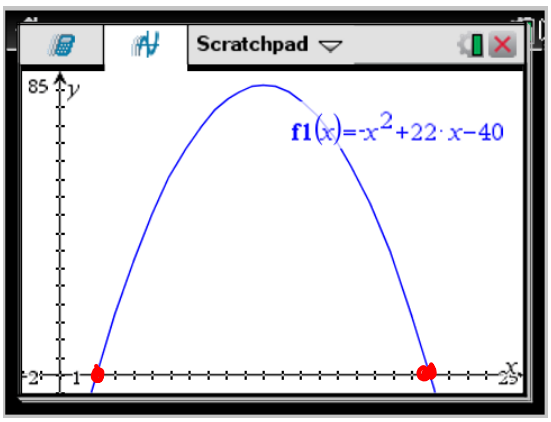
There are 5 ways to find the zeros.

1. Graph - by hand
2. Graphing calculator
3. Factoring
4. Quadratic Formula
5. Vertex Form ⟶ algebra

1. Graph by Hand

- make a table of values
- graph
- read off zeros

2. Graphing calculator - see Warm-Up



- trace

OR

- or menu
  - analyze graph
  - find zeros

3. Factor

$$P(x) = -x^2 + 22x - 40$$

"set P(x) to zero"

factor out negative sign

$0 = -x^2 + 22x - 40$

$0 = -(x^2 - 22x + 40)$

$0 = -(x - 20)(x - 2)$

$x - 20 = 0$ $\quad$ $x - 2 = 0$

$x = 20$ $\quad$ $x = 2$

4. Quadratic Formula - Exact Roots

$M(x) = x^2 - 4x - 6$ "exact roots"

$a=1 \quad b=-4 \quad c=-6$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{4 \pm \sqrt{(-4)^2-4(1)(-6)}}{2(1)}$

$x = \frac{4 \pm \sqrt{16+24}}{2}$

$x = \frac{4 \pm \sqrt{40}}{2}$

$\sqrt{40} = \sqrt{4}\sqrt{10} = 2\sqrt{10}$

$x = \frac{4 \pm 2\sqrt{10}}{2}$

$x = \frac{2(2 \pm \sqrt{10})}{2}$

$x = 2 \pm \sqrt{10}$

$x_1 = 2+\sqrt{10}$ $\quad$ $x_2 = 2-\sqrt{10}$ "EXACT"

$x_1 \doteq 2+3.2$ $\quad$ $x_2 = 2-3.2$ "APPROX"

$\doteq 5.2$ $\quad$ $\doteq -1.2$

when there are decimals, a decimal answer is allowed and use a calculator.

5. Vertex Form

$$x^2 - 49 = 0$$

$x^2 = 49$

$\sqrt{x^2} = \pm\sqrt{49}$

$x = \pm 7$

$x_1 = 7 \qquad x_2 = -7$

$x^2 - 49 = 0$

$(x-7)(x+7) = 0$

$x = 7 \qquad x = -7$

$$(x-1)^2 - 25 = 0$$

$(x-1)^2 - 25 + 25 = 0 + 25$

$(x-1)^2 = 25$

$\sqrt{(x-1)^2} = \pm\sqrt{25}$

$x - 1 = \pm 5$

$x - 1 = 5$

$x - 1 + 1 = 5 + 1$

$x_1 = 6$

$x - 1 = -5$

$x - 1 + 1 = -5 + 1$

$x_2 = -4$

1st two are Grade 10 but sometimes we forget how easy this can be

Third ... !!!

5. Vertex Form - Exact Roots

$$x^2 - 2x - 4 = 0$$

convert to Vertex by completing Square

$(x^2 - 2x + 1 - 1) - 4 = 0$

$(x-1)^2 - 1 - 4 = 0$

$(x-1)^2 - 5 = 0$

$(x-1)^2 = 5$

Then solve by "square-rooting"

$\sqrt{(x-1)^2} = \pm\sqrt{5}$

$(x-1) = \pm\sqrt{5}$

Now create two equations and solve

$x - 1 = \sqrt{5}$ $\quad x - 1 = -\sqrt{5}$

$x - 1 + 1 = \sqrt{5} + 1$ $\quad x - 1 + 1 = -\sqrt{5} + 1$

$x_1 = 1 + \sqrt{5}$ $\quad x_2 = 1 - \sqrt{5}$

These are 'exact roots'

End of the video

do both Try on Your Owns

- full solutions provided

textbook questions are OPTIONAL

Try This on Your Own

1. ***Graphing – Using Nspire***

$$-5t^2+40t+45=0$$

To graph this, what equation are you putting into the Ti-Nspire?

What part of the parabola are you looking for?

What is the solution?

On Your Own #1

1. ***Graphing – Using Nspire***

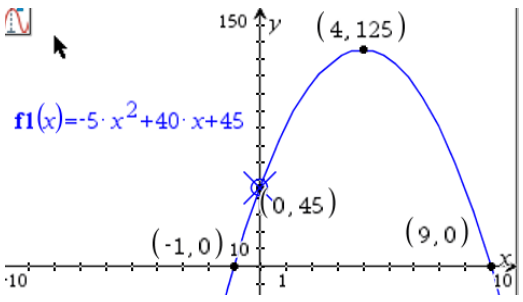
$$-5t^2+40t+45=0$$

To graph this, what equation are you putting into the Ti-Nspire?

What part of the parabola are you looking for?

zeros

What is the solution?



From the graph $x_1 = -1$ $x_2 = 9$

Therefore solution is $t_1=-1$ and $t_2=9$

Now use your graph to answer the following …

The flight of a ball is modelled by the function h(t) = -5t² + 40t + 45 where h is the height in metres and t is the time in seconds.

Determine how long the ball will be above 80m.

1. *What does the 80 represent?* _____

2. *What 'quadratic equation" are you trying to solve.* _____

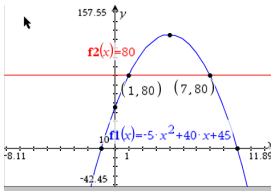
Now use your graph to answer the following …

The flight of a ball is modelled by the function h(t) = -5t² + 40t + 45 where h is the height in metres and t is the time in seconds.

Determine how long the ball will be above 80m.

1. *What does the 80 represent?* Height of the ball

2. *What 'quadratic equation" are you trying to solve.* $-5t^2 + 40t + 45 = 80$



The ball is above 80 m from 1 sec to 7 sec,
so the ball is above 80 m for 6 seconds.

Using Algebra - Method 3 - Factor

$$-5t^2 + 40t + 45 = 80$$

$$-5t^2 + 40t + 45 - 80 = 80 - 80$$

$$-5t^2 + 40t - 35 = 0$$

$$-5(t^2 - 8t - 7) = 0$$

$$-5(t - 7)(t - 1) = 0$$

$$(t - 7) = 0 \qquad (t - 1) = 0$$

$$t = 7 \qquad t = 1$$

On Your Own #2

Example - Solve the following Quadratic Equation using Four Methods.

$$3x^2 - 5x - 30 = x(x-1)$$

1. Graphing
2. Factoring
3. Vertex Form
4. Quadratic Formula

Worked Example - Solve the following Quadratic Equation using Four Methods.

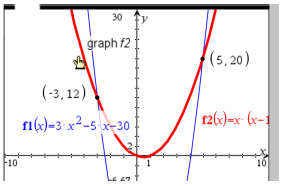
$3x^2 - 5x - 30 = x(x-1)$ → expand and "set to zero"

$3x^2 - 5x - 30 = x^2 - x$

$2x^2 - 4x - 30 = 0$

Graph two functions

1. Graphing



2. Factored Form

$2x^2 - 4x - 30 = 0$

$2(x^2 - 2x - 15) = 0$

$2(x-5)(x+3) = 0$

$x = 5$ $x = -3$

3. Vertex Form

$2x^2 - 4x - 30 = 0$

$2(x^2 - 2x + 1 - 1) - 30 = 0$

$2(x-1)^2 - 30 - 2 = 0$

$2(x-1)^2 - 32 = 0$

$\frac{2(x-1)^2}{2} = \frac{32}{2}$

$\sqrt{(x-1)^2} = \pm\sqrt{16}$

$x - 1 = \pm 4$

$x - 1 = -4$ $x - 1 = 4$

$x = -3$ $x = 5$

4. Quadratic Formula

$a = 2 \quad b = -4 \quad c = -30$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{(4) \pm \sqrt{(-4)^2 - 4(2)(30)}}{2(2)}$

$x = -3 \quad x = 5$

On Your Own #3

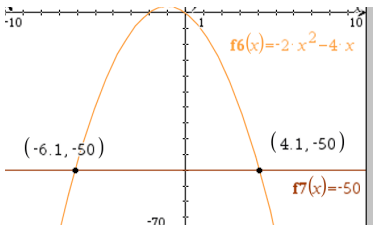
3. Solve the following Quadratic Equation using Four Methods.

$$-2x^2 - 4x = -50$$

Practise - Solve the following Quadratic Equation using Four Methods.

$$-2x^2 - 4x = -50$$

1. Graphing



2. Factored Form

$-2x^2-4x+50 = 0$

$-2(x^2+2x-25) = 0$

doesn't factor

3. Vertex Form

$-2x^2+4x+50 = 0$

$-2(x^2-2x+1-1)+50 = 0$

$-2(x-1)^2+50+2 = 0$

$-2(x-1)^2+52 = 0$

$\frac{-2(x-1)^2}{-2} = \frac{-52}{-2}$

$\sqrt{(x-1)^2} = \pm\sqrt{26}$

$x-1 = \sqrt{26}$ OR $x-1 = -\sqrt{26}$

$x = 1+\sqrt{26}$ $x = 1-\sqrt{26}$

4. Quadratic Formula

$a=-2$, $b=4$, $c=50$

$x = \frac{-4 \pm \sqrt{4^2-4(-2)(50)}}{2(-2)}$

$x = \frac{-4 \pm \sqrt{16+200}}{-4}$

$x = \frac{-4 \pm \sqrt{216}}{-4}$

$x = \frac{-4 \pm 4\sqrt{26}}{-4}$

$x = \frac{-4}{-4} \pm \frac{4\sqrt{26}}{-4}$

$x = 1 \pm \sqrt{26}$

Optional Additional Practise

pg 177 # 1ab, 2ab, 5, 6a, 7, 8, 12, 14

1. Determine the roots of each equation by factoring.

a) $x^2 + 5x + 4 = 0$ **c)** $4x^2 - 9 = 0$

b) $x^2 - 11x + 18 = 0$ **d)** $2x^2 - 7x - 4 = 0$

2. Use the quadratic formula to determine each of the roots to two decimal places.

a) $x^2 - 4x - 9 = 0$ **c)** $-2x^2 + 3x - 6 = 0$

b) $3x^2 + 2x - 8 = 0$ **d)** $0.5x^2 - 2.2x - 4.7 = 0$

5. Locate the x-intercepts of the graph of each function.

a) $f(x) = 3x^2 - 7x - 2$ **b)** $f(x) = -4x^2 + 25x - 21$

6. Determine the break-even quantities for each profit function, where x is the number sold, in thousands.

a) $P(x) = -x^2 + 12x + 28$ **c)** $P(x) = -2x^2 + 22x - 17$

b) $P(x) = -2x^2 + 18x - 40$ **d)** $P(x) = -0.5x^2 + 6x - 5$

7. A The flight of a ball hit from a tee that is 0.6 m tall can be modelled by the function $h(t) = -4.9t^2 + 6t + 0.6$, where $h(t)$ is the height in metres at time t seconds. How long will it take for the ball to hit the ground?

8. The population of a region can be modelled by the function $P(t) = 0.4t^2 + 10t + 50$, where $P(t)$ is the population in thousands and t is the time in years since the year 1995.

a) What was the population in 1995?

b) What will be the population in 2010?

c) In what year will the population be at least 450 000? Explain your answer.

12. T Jackie mows a strip of uniform width around her 25 m by 15 m rectangular lawn and leaves a patch of lawn that is 60% of the original area. What is the width of the strip?

13. A small flare is launched off the deck of a ship. The height of the flare above the water is given by the function $h(t) = -4.9t^2 + 92t + 9$, where $h(t)$ is measured in metres and t is time in seconds.

a) When will the flare's height be 150 m?

b) How long will the flare's height be above 150 m?

Lesson 3.5, pp. 177–178

1. **a)** $x = -1$ or -4 **b)** $x = 2$ or 9 **c)** $x = \pm\frac{3}{2}$ **d)** $x = -\frac{1}{2}$ or 4

2. **a)** $x = 5.61$ or -1.61 **c)** no real roots
b) $x = 1.33$ or -2 **d)** $x = -1.57$ or 5.97

3. **a)** $x = -1$ or -0.25 **b)** $x = 1$ or 4.5

4. **a)** **i)** Solve by factoring, function factors **ii)** $x = 0$ or 10
b) **i)** Quadratic formula, function does not factor
ii) $x = \frac{-3 \pm \sqrt{5}}{4}$
c) **i)** Quadratic formula, function does not factor
ii) $x = -2 \pm \sqrt{7}$
d) **i)** Quadratic formula, function does not factor
ii) $x = -4 \pm \sqrt{7}$
e) **i)** Solve by factoring, function factors
ii) $x = -1$ or 10
f) **i)** Quadratic formula, function does not factor
ii) $x = 2 \pm \sqrt{19}$

5. **a)** $(2.59, 0), (-0.26, 0)$ **b)** $(1, 0), \left(\frac{21}{4}, 0\right)$

6. **a)** 14 000 **b)** 4000 or 5000 **c)** 836 or 10 164 **d)** 901 or 11 099

7. 1.32 s

8. **a)** 50 000 **b)** 290 000 **c)** 2017

11. base = 8 cm, height = 24 cm

12. 2.1 m

13. **a)** after 1.68 s and again at 17.09 s
b) The rocket will be above 150 m for $17.09 - 1.68 = 15.41$ s.