

Welcome to Cycle 3 - day 2

**Please turn on the Video
when you get to slide 3**

**Simplifying
Algebraic Expressions
with
Integer & Rational
Exponents**

Recall

$$a^n \times a^m = a^{n+m}$$

$$a^n \div a^m = a^{n-m}$$

$$(a^n)^m = a^{n \cdot m}$$

$$a^0 = 1$$

$$a^{-n} = \frac{1}{a^n}$$

$$a^n \times b^n = (ab)^n$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$(a+b)^n \neq a^n + b^n$$

$$\text{even}\sqrt{-a} \Rightarrow \text{imaginary}$$

$$\text{odd}\sqrt{-a} \Rightarrow \text{real}$$

$$a^{\frac{n}{m}} = \sqrt[m]{a^n}$$

The Lesson - Some Worked Examples

Simplify

(leave answer with a positive exponent)

$$1. \quad \frac{2x^5}{8x^6}$$

$$= \frac{1}{4} x^{-1}$$

$$= \frac{1}{4x^1}$$

$$2. \quad \frac{(27x^3)^2}{(8x^{-2})^5}$$

$$= \frac{27^2 x^6}{8^5 x^{-10}}$$

$$= \frac{27^2}{8^5} x^{16}$$

Pause the video and **Try on your own...**

$$3a. \frac{(3x^2y^{-4})^3}{(4x^{-5}y^4)^2}$$

$$3b. \left(\frac{6x^4y^3}{3x^2y^5}\right)^2$$

Remember BEDMAS

$$3a. \frac{(3x^2y^{-4})^3}{(4x^{-5}y^4)^2}$$

$$= \frac{3^3 x^6 y^{-12}}{4^2 x^{-10} y^8}$$

$$= \frac{27}{16} x^{16} y^{-20}$$

can't leave it as a negative exponent, so it has to be on the bottom of the fraction

$$= \frac{27 x^{16}}{16 y^{20}}$$

According to BEDMAS, you should divide inside the bracket first

$$3b. \left(\frac{6x^4y^3}{3x^2y^5}\right)^2$$

$$= (2x^2y^{-2})^2$$

$$= 2^2 x^4 y^{-4}$$

$$= \frac{4x^4}{y^4}$$

4. $\left(\frac{\sqrt[3]{x^4}}{\sqrt[5]{x^3}}\right)^2$

BIG HINT:
CHANGE RADICALS
TO RATIONAL
EXPONENTS

$$= \left(\frac{x^{\frac{4}{3}}}{x^{\frac{3}{5}}}\right)^2$$

$$= \left(x^{\frac{4}{3} - \frac{3}{5}}\right)^2$$

$$= \left(x^{\frac{20-9}{15}}\right)^2$$

$$= \left(x^{\frac{11}{15}}\right)^2$$

$$= x^{\frac{22}{15}} = \sqrt[15]{x^{22}}$$

Pause the video and **Try on your own...**

5. $\sqrt[3]{8x^6y^{-12}}$

$$\begin{aligned}
 5. \quad \sqrt[3]{8x^6y^{-12}} &= \sqrt[3]{8} \cdot \sqrt[3]{x^6y^{-12}} \\
 &= 2 \cdot (x^6y^{-12})^{\frac{1}{3}} \\
 &= 2 \cdot x^2y^{-4} \\
 &= \frac{2x^2}{y^4}
 \end{aligned}$$

6. Simplify and Evaluate

$$\begin{aligned}
 x &= 2 \\
 n &= 3
 \end{aligned}$$

$$\begin{aligned}
 &\frac{(x^{2n+1})(x^{3n-4})}{x^{n+4}} \\
 &= \frac{x^{2n+1+(3n-4)}}{x^{n+4}} \\
 &= \frac{x^{5n-3}}{x^{n+4}} \\
 &= x^{5n-3-(n+4)} \\
 &= x^{5n-3-n-4} \\
 &= x^{4n-7} \\
 &= 2^{4(3)-7} \\
 &= 2^{12-7} \\
 &= 2^5 \\
 &= 32
 \end{aligned}$$

Note:
Simplify first
and then
Evaluate

Try On Your Own - Worksheet provided as pdf

MCR3U 4.3 Simplifying Expressions with Rational Exponents

- Follow the exponent laws (multiplying, dividing, power of a power, negative) and order of operations (BEDMAS)
- Express all answers using positive exponents
- Express answers in radical form

Example: Simplify. Express in rational form with positive exponents

a. $\frac{(2x^{-3}y^2)^3}{(x^3y^{-1})^2}$

b. $\frac{(x^{2n+1})(x^{3n-1})}{x^{2n-5}}$

c. $\frac{\sqrt[3]{27a^{-3}b^{12}}}{\sqrt{16a^{-8}b^{12}}}$

d. $\left(\frac{\sqrt[5]{x^8}}{\sqrt{x^3}}\right)^3$

e. $\frac{\sqrt[3]{x^4}\sqrt{x^{-5}}}{\sqrt[4]{x^{-5}}}$

Try On Your Own - Full Solutions follow

MCR3U 4.3 Simplifying Expressions with Rational Exponents

- Follow the exponent laws (multiplying, dividing, power of a power, negative) and order of operations (BEDMAS)
- Express all answers using positive exponents
- Express answers in radical form

Example: Simplify. Express in rational form with positive exponents

a. $\frac{(2x^{-3}y^2)^3}{(x^3y^{-1})^2}$

b. $\frac{(x^{2n+1})(x^{3n-1})}{x^{2n-5}}$

x^{3n+5}

$\frac{2^3 y^{14}}{x^{15}}$

c. $\frac{\sqrt[3]{27a^{-3}b^{12}}}{\sqrt{16a^{-8}b^{12}}}$

d. $\left(\frac{\sqrt[5]{x^8}}{\sqrt{x^3}}\right)^3$

e. $\frac{\sqrt[3]{x^4}\sqrt{x^{-5}}}{\sqrt[4]{x^{-5}}}$

$\frac{3a^{-3}}{4b^2}$

$x^{\frac{3}{10}}$

$x^{\frac{1}{12}}$

$$a. \frac{(2x^{-3}y^2)^3}{(x^3y^4)^2}$$

$$= \frac{2^3 x^{-9} y^6}{x^6 y^{-8}}$$

$$= 2^3 x^{-15} y^{14}$$

$$= \frac{2^3 y^{14}}{x^{15}}$$

$$b. \frac{(x^{2n+1})(x^{3n-1})}{x^{2n-5}}$$

$$= \frac{x^{2n+1+3n-1}}{x^{2n-5}}$$

$$= \frac{x^{5n}}{x^{2n-5}}$$

$$= x^{5n-1(2n-5)}$$

$$= x^{5n-2n+5}$$

$$= x^{3n+5}$$

or all in one step ...

$$x^{(2n+1+3n-1)-(2n-5)}$$

$$x^{3n+5}$$

c)

$$\frac{\sqrt[3]{27a^{-3}b^{12}}}{\sqrt{16a^{-8}b^{12}}}$$

$$= \frac{\sqrt[3]{27} \cdot \sqrt[3]{a^{-3}b^{12}}}{\sqrt{16} \cdot \sqrt{a^{-8}b^{12}}}$$

$$= \frac{3 \cdot (a^{-3}b^{12})^{\frac{1}{3}}}{4 (a^{-8}b^{12})^{\frac{1}{2}}}$$

$$= \frac{3 \cdot a^{-1}b^4}{4 \cdot a^{-4}b^6}$$

$$= \frac{3 \cdot a^3b^{-2}}{4}$$

$$= \frac{3a^3}{4b^2}$$

d.

$$\left(\frac{\sqrt[5]{x^8}}{\sqrt{x^3}} \right)^3 = x^{\frac{3}{10}}$$

$$= \left(\frac{(x^8)^{\frac{1}{5}}}{(x^3)^{\frac{1}{2}}} \right)^3$$

$$= \left(\frac{x^{\frac{8}{5}}}{x^{\frac{3}{2}}} \right)^3$$

$$= \left(x^{\frac{8}{5} - \frac{3}{2}} \right)^3$$

$$= \left(x^{\frac{16}{10} - \frac{15}{10}} \right)^3$$

$$= \left(x^{\frac{1}{10}} \right)^3$$

$$= x^{\frac{3}{10}}$$

BIG HINT:
CHANGE RADICALS
TO RATIONAL
EXPONENTS

$$\begin{aligned}
 \text{e. } & \frac{\sqrt[3]{x^4} \sqrt{x^{-5}}}{\sqrt[4]{x^{-5}}} \\
 & = \frac{(x^4)^{\frac{1}{3}} (x^{-5})^{\frac{1}{2}}}{(x^{-5})^{\frac{1}{4}}} \\
 & = \frac{x^{\frac{4}{3}} \cdot x^{-\frac{5}{2}}}{x^{-\frac{5}{4}}} \\
 & = x^{\frac{4}{3} + (-\frac{5}{2}) - (-\frac{5}{4})} \\
 & = x^{\frac{16}{12} - \frac{30}{12} + \frac{15}{12}} \\
 & = x^{\frac{1}{12}}
 \end{aligned}$$

Try On Your Own #2

Practice on your own ...

MCR 3U Lesson 4.4 Simplifying Algebraic Expressions

Ex. Simplify each of the following.

1. $x^2 + x^{\frac{4}{3}}$

2. $(27y^9)^{\frac{-2}{3}}$

3. $(-a^2b)^4 \left(\frac{16a^4}{b}\right)^{\frac{1}{2}}$

4. $\sqrt[3]{x^{12}}$

5. $\left(\frac{3}{y^4}\right)^4 + (25y^8)^{\frac{-1}{2}}$

6. $\frac{(3x)^{-2}}{2(x^{-3})^{-1}}$

Practice on your own ...

MCR 3U Lesson 4.4 Simplifying Algebraic Expressions

Ex. Simplify each of the following.

1. $x^2 \cdot x^{\frac{4}{3}}$ 2. $(27y^9)^{\frac{2}{3}}$ 3. $(-a^2b)^4 \left(\frac{16a^4}{b}\right)^{\frac{1}{2}}$

$x^{\frac{2}{3}}$ $\frac{1}{9y^6}$ $4a^{10}b^{\frac{7}{2}}$

Examples

4. $\sqrt[3]{x^{12}}$ 5. $\left(\frac{2}{y^4}\right)^4 + (25y^8)^{\frac{-1}{2}}$ 6. $\frac{(3x)^{-2}}{2(x^{-3})^{-1}}$

x^4 $5y^7$ $\frac{1}{18x^5}$

$$\begin{aligned}
 & 3. \left(-a^2b\right)^4 \left(\frac{16a^4}{b}\right)^{\frac{1}{2}} \\
 & = \left((-1)a^2b\right)^4 \left(\frac{16a^4}{b}\right)^{\frac{1}{2}} \\
 & = (-1)^4 (a^2)^4 b^4 \cdot \frac{16^{\frac{1}{2}} (a^4)^{\frac{1}{2}}}{b^{\frac{1}{2}}} \\
 & = 1a^8 b^4 \cdot \frac{\sqrt{16} \cdot a^2}{b^{\frac{1}{2}}} \\
 & = a^8 b^4 \cdot \frac{4a^2}{b^{\frac{1}{2}}} \\
 & = 4a^{10} b^{\frac{7}{2}}
 \end{aligned}$$

$$\begin{aligned}
 5. \left(y^{\frac{3}{4}}\right)^4 + (25y^8)^{\frac{-1}{2}} \\
 = y^3 \div \frac{1}{(25y^8)^{\frac{1}{2}}} \\
 = y^3 \times (25y^8)^{\frac{1}{2}} \\
 = y^3 \times 25^{\frac{1}{2}} \times y^4 \\
 = 5y^7
 \end{aligned}$$

Additional Practise (Optional)

Pg 236 # 2, 4-9 ad,

2. Simplify. Express each answer with positive exponents.

- | | | |
|------------------------------|--|--------------------------------------|
| a) $y^{10}(y^4)^{-3}$ | c) $\frac{(n^{-4})^3}{(n^{-3})^{-4}}$ | e) $\frac{(x^{-1})^4x}{x^{-3}}$ |
| b) $(x^{-3})^{-3}(x^{-1})^5$ | d) $\frac{w^4(w^{-3})}{(w^{-2})^{-1}}$ | f) $\frac{(b^{-7})^2}{b(b^{-5})b^9}$ |

4. Simplify. Express answers with positive exponents.

- | | | |
|--------------------------------------|-----------------------------------|--|
| a) $(pq^2)^{-1}(p^3q^3)$ | c) $\frac{(ab)^{-2}}{b^5}$ | e) $\frac{(w^2x)^2}{(x^{-1})^2w^3}$ |
| b) $\left(\frac{x^3}{y}\right)^{-2}$ | d) $\frac{m^2n^2}{(m^3n^{-2})^2}$ | f) $\left(\frac{(ab)^{-1}}{a^2b^{-3}}\right)^{-2}$ |

5. Simplify. Express answers with positive exponents.

- | | | |
|-----------------------------|--|---|
| a) $(3xy^4)^2(2x^2y)^3$ | c) $\frac{(10x)^{-1}y^3}{15x^3y^{-3}}$ | e) $\frac{p^{-5}(r^3)^2}{(p^2r)^2(p^{-1})^2}$ |
| b) $\frac{(2a^3)^2}{4ab^2}$ | d) $\frac{(3m^4n^2)^2}{12m^{-2}n^6}$ | f) $\left(\frac{(x^3y)^{-1}(x^4y^3)}{(x^2y^{-3})^{-2}}\right)^{-1}$ |

6. Simplify. Express answers with positive exponents.

- | | | |
|--|---|--|
| a) $(x^4)^{\frac{1}{2}}(x^6)^{-\frac{1}{3}}$ | c) $\frac{\sqrt{25m^{-12}}}{\sqrt{36m^{10}}}$ | e) $\left(\frac{(32x^5)^{-2}}{(x^{-1})^{10}}\right)^{0.2}$ |
| b) $\frac{9(c^8)^{0.5}}{(16c^{12})^{0.25}}$ | d) $\sqrt[3]{\frac{(10x^3)^2}{(10x^6)^{-1}}}$ | f) $\frac{\sqrt[10]{1024x^{20}}}{\sqrt[9]{512x^{27}}}$ |

7. Evaluate each expression. Express answers in rational form with positive exponents.

a) $(16x^6y^4)^{\frac{1}{2}}$ for $x = 2, y = 1$

b) $\frac{(9p^{-2})^{\frac{1}{2}}}{6p^2}$ for $p = 3$

c) $\frac{(81x^4y^6)^{\frac{1}{2}}}{8(x^9y^3)^{\frac{1}{3}}}$ for $x = 10, y = 5$

d) $\left(\frac{(25a^4)^{-1}}{(7a^{-2}b)^2}\right)^{\frac{1}{2}}$ for $a = 11, b = 10$

8. Evaluate. Express answers in rational form with positive exponents.

a) $(\sqrt{10\,000x})^{\frac{3}{2}}$ for $x = 16$

b) $\left(\frac{(4x^3)^4}{(x^3)^6}\right)^{-0.5}$ for $x = 5$

c) $(-2a^2b)^{-3}\sqrt{25a^4b^6}$ for $a = 1, b = 2$

d) $\sqrt{\frac{(18m^{-5}n^2)(32m^2n)}{4mn^{-3}}}$ for $m = 10, n = 1$

9. Simplify. Express answers in rational form with positive exponents.

a) $(36m^4n^6)^{0.5}(81m^{12}n^8)^{0.25}$

b) $\left(\frac{(6x^3)^2(6y^3)}{(9xy)^6}\right)^{-\frac{1}{3}}$

c) $\left(\frac{\sqrt{64a^{12}}}{(a^{1.5})^{-6}}\right)^{\frac{2}{3}}$

d) $\left(\frac{(x^{18})^{-\frac{1}{6}}}{\sqrt[5]{243x^{10}}}\right)^{0.5}$

Answers

2. a) $\frac{1}{y^2}$ c) $\frac{1}{n^{24}}$ e) 1

b) x^4 d) $\frac{1}{w}$ f) $\frac{1}{b^{19}}$

3. a) 36 b) $x^2y^2 - 36$

c) Usually it is faster to substitute numbers into the simplified form.

4. a) p^2q c) $\frac{1}{a^2b^7}$ e) ux^4

b) $\frac{y^2}{x^6}$ d) $\frac{n^6}{m^4}$ f) $\frac{a^6}{b^4}$

5. a) $72x^8y^{11}$ c) $\frac{y^6}{150x^4}$ e) $\frac{r^4}{p^7}$

b) $\frac{a^2}{b^2}$ d) $\frac{3m^{10}}{4n^2}$ f) $\frac{y^4}{x^5}$

6. a) 1 c) $\frac{5}{6m^{11}}$ e) $\frac{1}{4}$

b) $\frac{9c}{2}$ d) $10x^4$ f) $\frac{1}{x}$

7. a) $4x^3y^2 = 32$ c) $\frac{9y^2}{8x} = \frac{45}{16}$

b) $\frac{1}{2p^3} = \frac{1}{54}$ d) $\frac{1}{35b} = \frac{1}{350}$

8. a) $1000x^{\frac{3}{4}} = 8000$ c) $\frac{-5}{8a^4} = \frac{-5}{8}$

b) $\frac{x^3}{16} = \frac{125}{16}$ d) $\frac{12n^3}{m^2} = \frac{3}{25}$

9. a) $18m^5n^5$ b) $\frac{27y}{2}$ c) $4a^{10}$ d) $\frac{1}{\sqrt{3x^{\frac{5}{2}}}}$

x⁴

