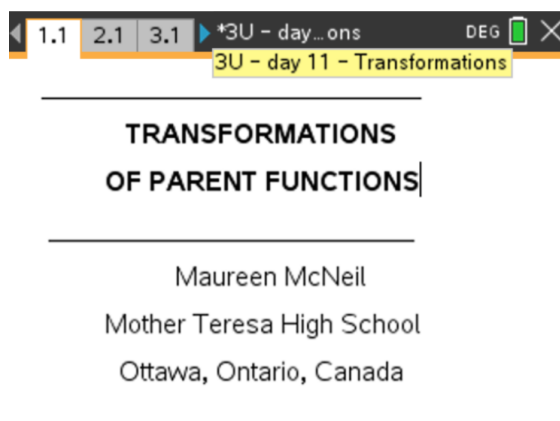


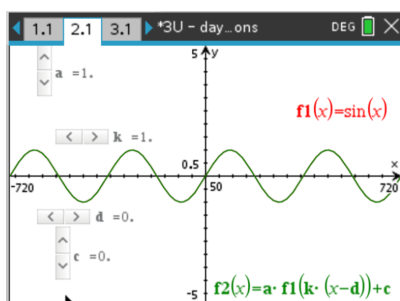
## Warm-Up

Open the Nspire file

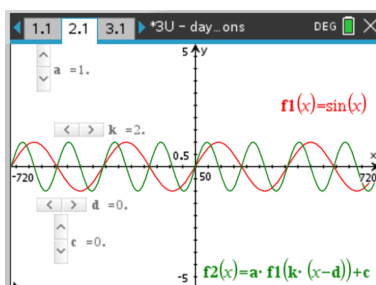
3U - C3 - day 11 - Transformations.tns



Spend about ten minutes convincing yourself that you understand all 4 Transformations on all 5 Parent Functions.



If you change  $k = 2$  What happens?



Period = 180

Now change other transformations on the sinusoidal function.

Now Continue with all Parent Functions

*Turn on the Video*

## Transformations using Points

### Learning Goals

- Review and understand the four transformations we can apply to functions
- Transform functions using a Table of Points

MCR3U: Understanding Transformations with Piecewise Functions

$$g(x) = -3f[2(x-10)] - 2$$

The graphs below illustrate the function  $f(x)$ .

a) Sketch each transformation in order from left to right  
 b) Fill in the co-ordinate points for each of the functions

Handwritten notes: (1) x, -3y; (2) x/2, y; (3) x+10, y; (4) x, y-2

$f(x)$	$-3f(x)$	$-3f(2x)$	$-3f[2(x-10)]$	$-3f[2(x-10)]-2$
A (-8, 1)	(-8, -3)	(-4, -3)	(6, -3)	(6, -5)
B (-4, 2)	(-4, -6)	(-2, -6)	(8, -6)	(8, -8)
C (0, 0)	(0, 0)	(0, 0)	(10, 0)	(10, -2)
D (6, 2)	(6, -6)	(3, -6)	(13, -6)	(13, -8)

MCR3U: Understanding Transformations with Piecewise Functions

$$g(x) = -3f[2(x-10)] - 2$$

The graphs below illustrate the function  $f(x)$ .

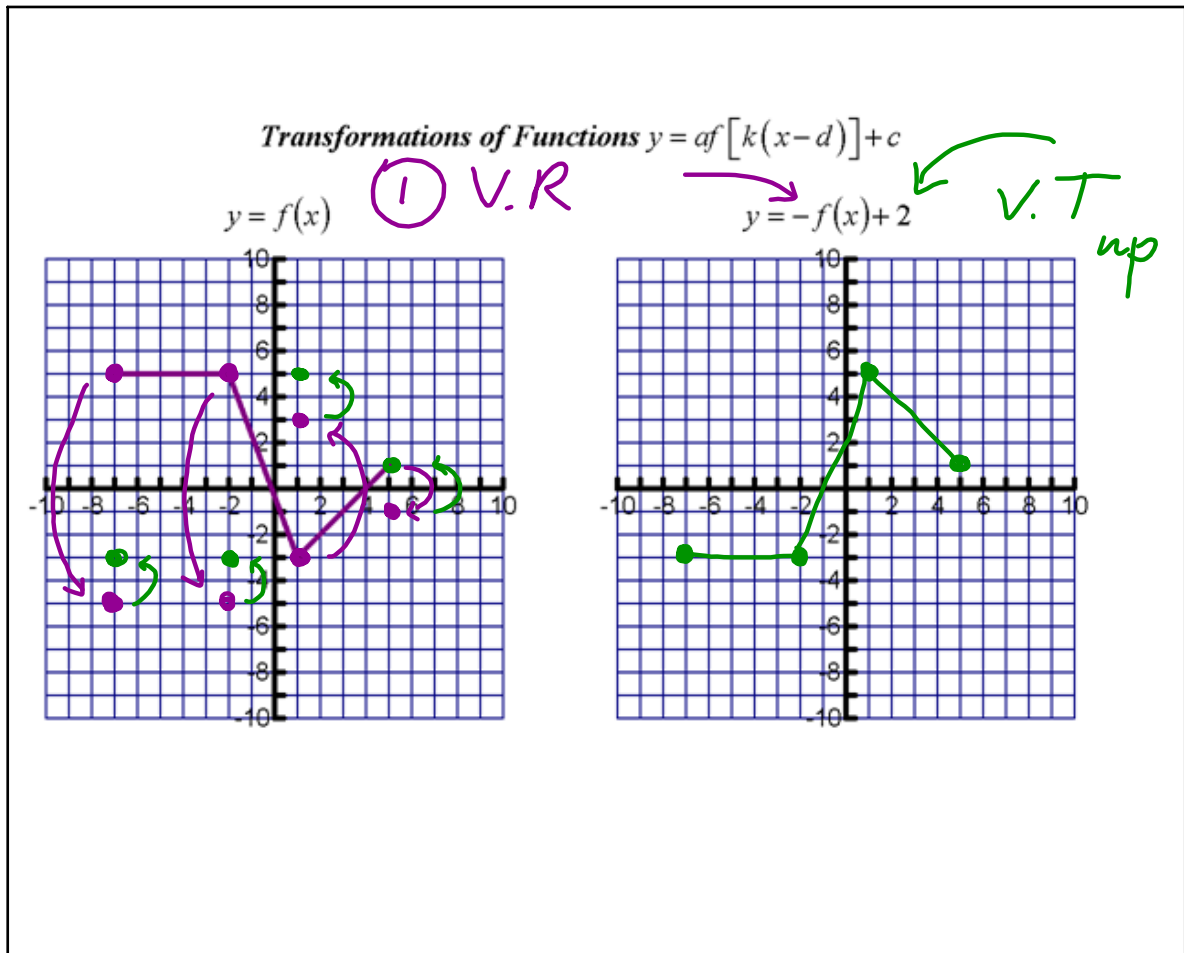
a) Sketch each transformation in order from left to right  
 b) Fill in the co-ordinate points for each of the functions

Handwritten notes: V.R V.S by 3 H.C by 1/2 H.T right 10 V.T down 2

$f(x)$	$-3f(x)$	$-3f(2x)$	$-3f[2(x-10)]$	$-3f[2(x-10)]-2$
A (-8, 1)	(-8, -3)	(-4, -3)	(6, -3)	(6, -5)
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D (6, 2)	(6, -6)	(3, -6)	(13, -6)	(13, -8)

In Summary ...

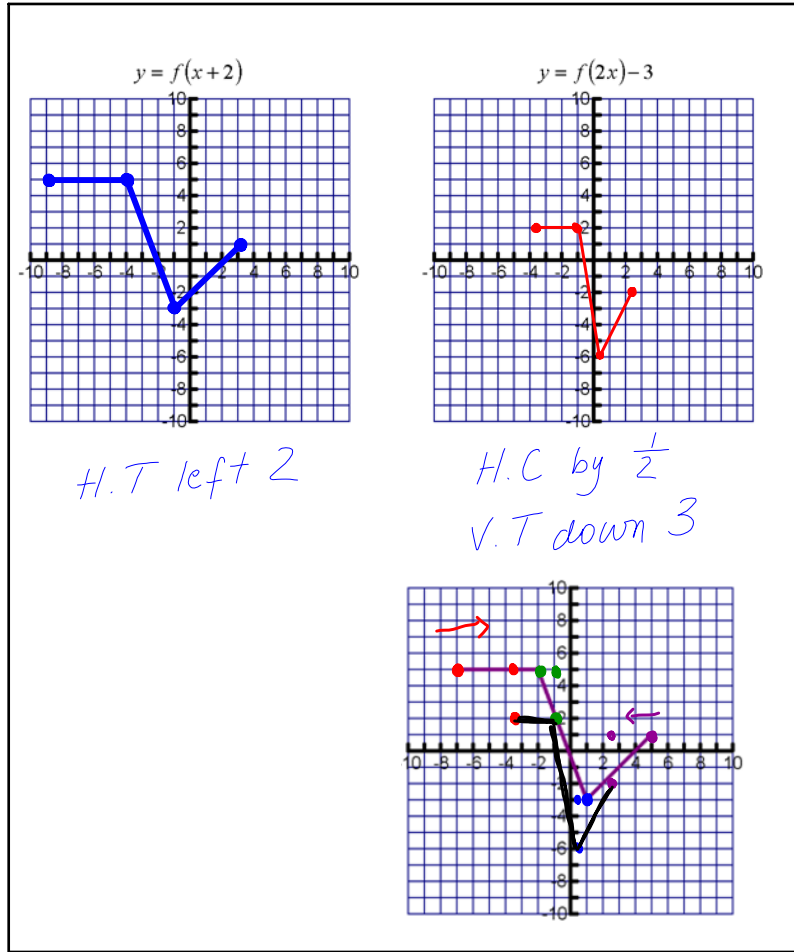
$$(x, y) \rightarrow \left(\frac{x}{k}, ay\right) \rightarrow \left(\frac{x}{k} + d, ay + c\right)$$



$y = f(x+2)$

$y = f(2x) - 3$

1. Name transformations
2. Decide the order of transformations
3. Do the transformations one at a time



$y = -3f(x-3)+1$

$y = 2f[-(x+5)]$

$f(x)$ 
1
2
3
4

A				
B				
C				
D				

$$y = -3f(x-3)+1$$

$$y = 2f[-(x+5)]$$

I always do these one with a table -  
because it gets too confusing for some.....  
included from old notebook

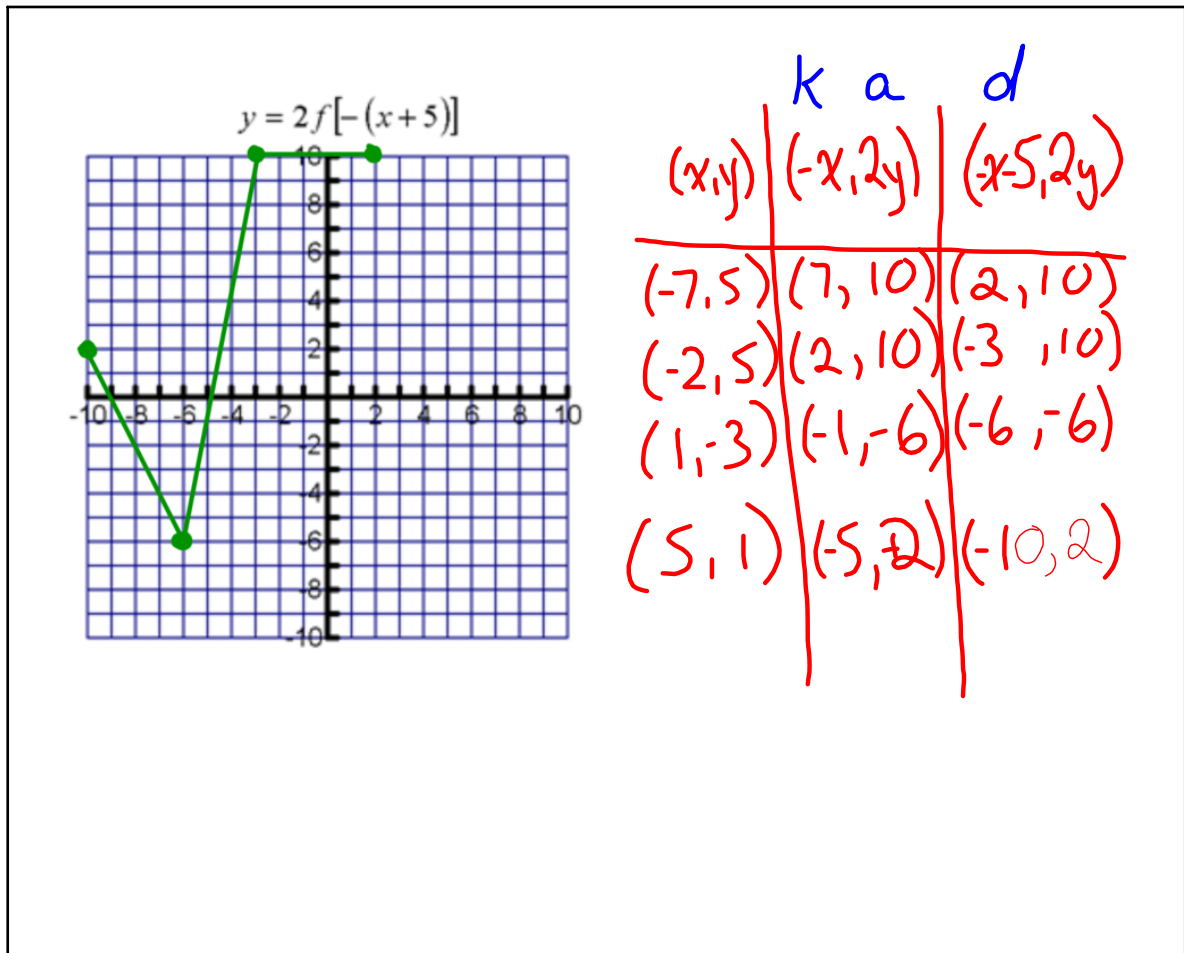
$$y = -3f(x-3)+1$$

a = -3  
k = 1  
d = 3  
c = 1

$$g(x) = a + f(k(x-d)) + c$$

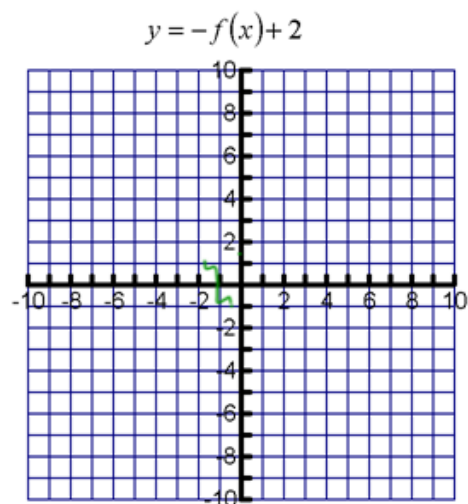
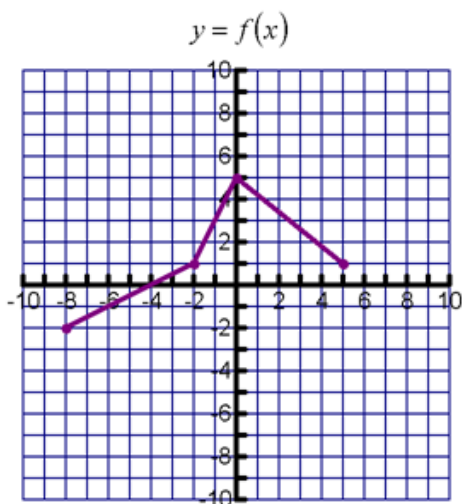
$$(x, y) \rightarrow \left(\frac{x}{k}, ay\right) \rightarrow \left(\frac{x}{k} + d, ay + c\right)$$

$(x, y)$	$k=1 \quad a=-3$ $(x, -3y)$	$d=3 \quad c=1$ $(x+3, -3y+1)$
$(-7, 5)$	$(-7, -15)$	$(-4, -14)$
$(-2, 5)$	$(-2, -15)$	$(1, -14)$
$(1, -3)$	$(1, +9)$	$(4, 10)$
$(5, 1)$	$(5, -3)$	$(8, -2)$



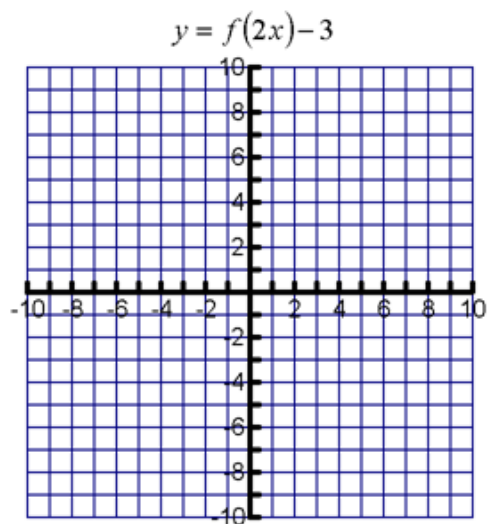
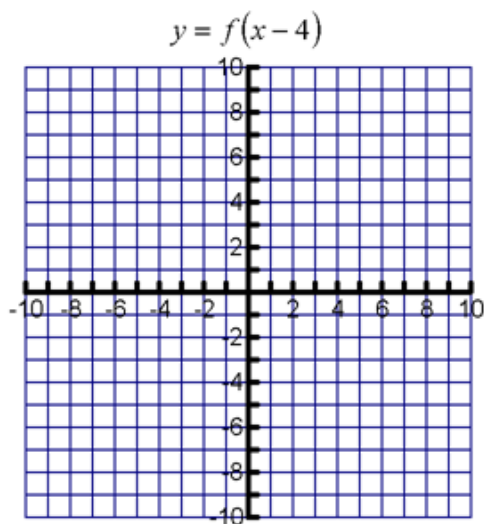
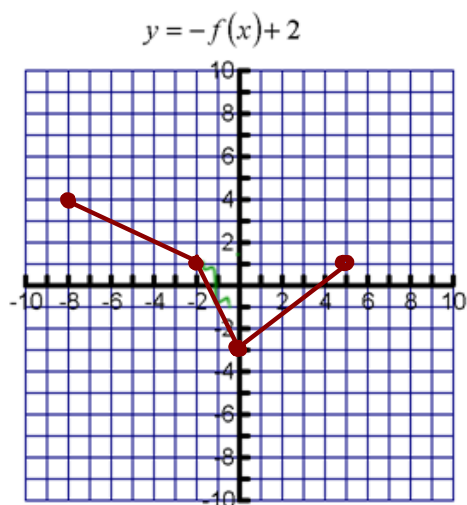
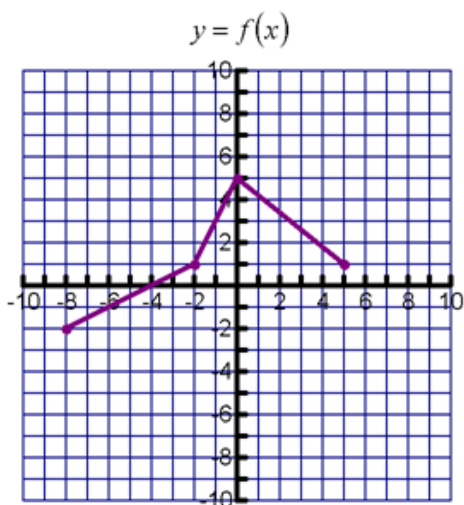
Try On Your Own #1 ...

Transformations of Functions  $y = af[k(x-d)] + c$



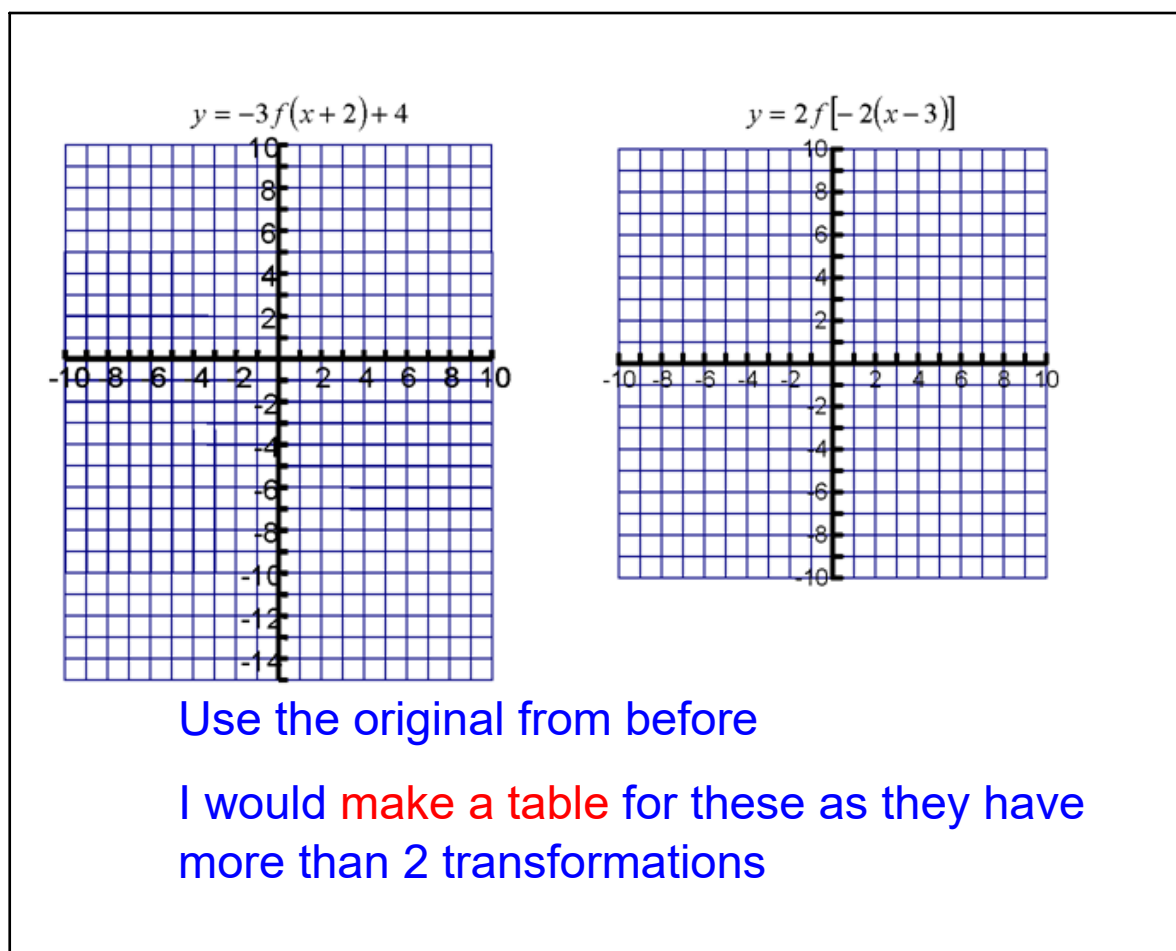
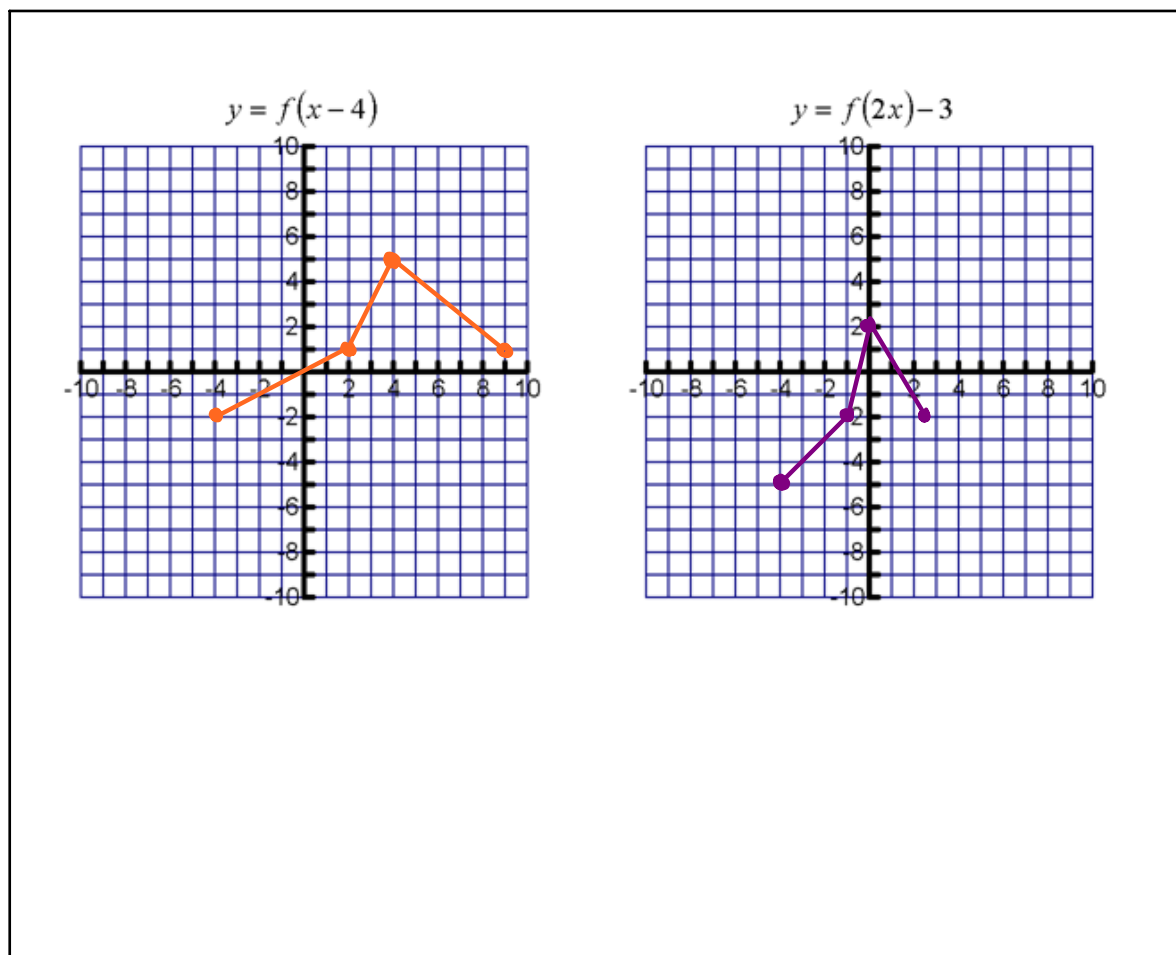
## Try On Your Own #1 ...

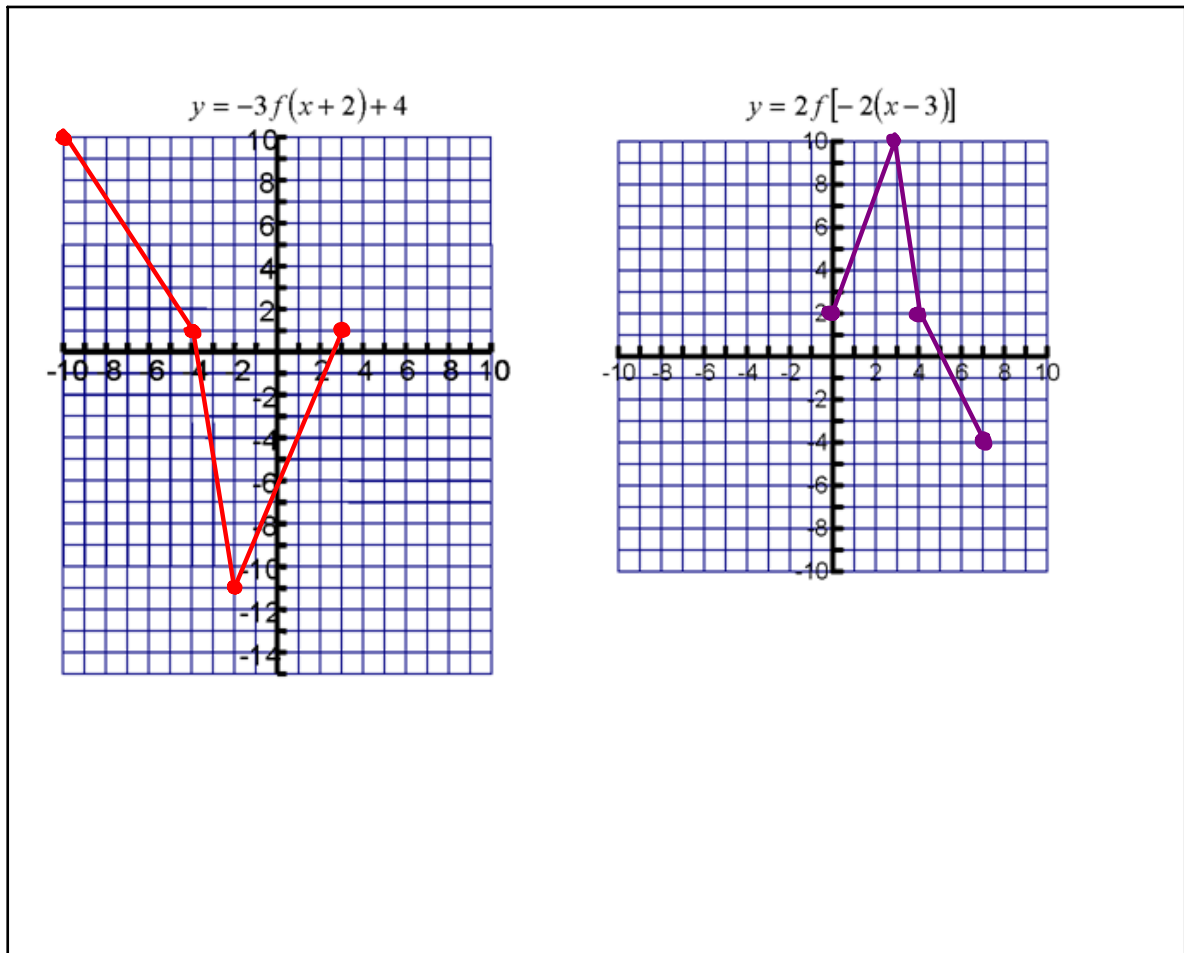
Transformations of Functions  $y = af[k(x-d)] + c$



Use the original from the previous page







$y = f(x)$

$y = -3f(x+2)+4$

$y = 2f[-2(x-3)]$

$(x, y)$	$(x, -3y)$	$(x-2, -3y+4)$	$(\frac{x}{2}, 2y)$	$(\frac{x}{2}+3, 2y)$
$(-8, -2)$	$(-8, 6)$	$(-10, 10)$	$(-8, -4)$	$(-7, -4)$
$(-2, 1)$	$(-2, -3)$	$(-4, 1)$	$(-2, 2)$	$(-1, 2)$
$(0, 5)$	$(0, -15)$	$(-2, -11)$	$(0, 10)$	$(3, 10)$
$(5, 1)$	$(5, -3)$	$(3, 1)$	$(5, 2)$	$(0.5, 2)$

# Homework:

Pg 58 #3,

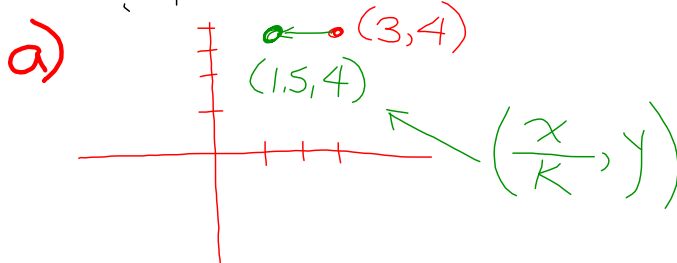
Pg. 70 # 4, 11, 12, 19, 20

3. The point  $(3, 4)$  is on the graph of  $y = f(x)$ . State the coordinates of the image of this point on each graph.

a)  $y = f(2x)$       b)  $y = f(0.5x)$       c)  $y = f\left(\frac{1}{3}x\right)$       d)  $y = f(-4x)$

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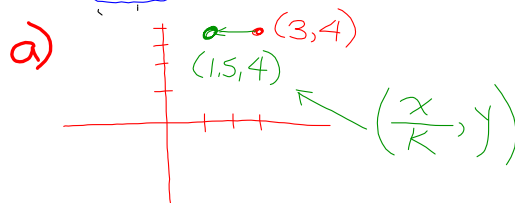
- a)  $y = f(2x)$     b)  $y = f(0.5x)$     c)  $y = f\left(\frac{1}{3}x\right)$     d)  $y = f(-4x)$



Now try b) c) and d)

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b)  $(x, y) \rightarrow (2x, y)$   
 $(3, 4) \rightarrow (6, 4)$

c)  $(x, y) \rightarrow (3x, y)$   
 $(3, 4) \rightarrow (9, 4)$

d)  $(x, y) \rightarrow \left(-\frac{x}{4}, y\right)$   
 $(3, 4) \rightarrow \left(-\frac{3}{4}, 4\right)$

## Correct Terminology is Important

4. Explain what transformations you would need to apply to the graph of

**K**  $y = f(x)$  to graph each function.

a)  $y = 3f(x) - 1$       c)  $y = f(2x) - 5$       e)  $y = \frac{2}{3}f(x + 3) + 1$

b)  $y = f(x - 2) + 3$       d)  $y = -f\left(\frac{1}{2}x\right) - 2$       f)  $y = 4f(-x) - 4$

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b)  $y = f(x - 2) + 3$       d)  $y = -f\left(\frac{1}{2}x\right) - 2$       f)  $y = 4f(-x) - 4$

4. a) Vertical stretch, factor 3, then translation 1 unit down  
 b) Translation 2 units right and 3 units up  
 c) Horizontal compression, factor  $\frac{1}{2}$ , then translation 5 units down  
 d) Reflection in  $x$ -axis, horizontal stretch with factor 2, and then translation 2 units down  
 e) Vertical compression, factor  $\frac{2}{3}$ , then translation 3 units left and 1 unit up  
 f) Vertical stretch with factor 4, reflection in  $y$ -axis, and then translation 4 units down

12. For  $f(x) = \sqrt{x}$ , sketch the graph of  $h(x) = f(-3x - 12)$ .

11. For  $f(x) = x^2$ , sketch the graph of  $g(x) = f(2x + 6)$ .

Hint ...

factor out the k

$$g(x) = a f(k(x-d)) + c$$

12. For  $f(x) = \sqrt{x}$ , sketch the graph of  $h(x) = f(-3x - 12)$ .

$$h(x) = f(-3(x+4))$$

11. For  $f(x) = x^2$ , sketch the graph of  $g(x) = f(2x + 6)$ .

This question uses the correct words  
you need to know them

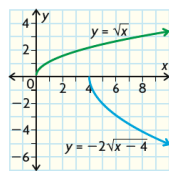
19. The function  $y = f(x)$  has been transformed to  $y = af[k(x - d)] + c$ . Determine  $a$ ,  $k$ ,  $c$ , and  $d$ ; sketch the graph; and state the domain and range for each transformation.
- A vertical stretch by the factor 2, a reflection in the  $x$ -axis, and a translation 4 units right are applied to  $y = \sqrt{x}$ .
  - A vertical compression by the factor  $\frac{1}{2}$ , a reflection in the  $y$ -axis, a translation 3 units left, and a translation 4 units down are applied to  $f(x) = \frac{1}{x}$ .

This question uses the correct words  
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19. The function  $y = f(x)$  has been transformed to  $y = af[k(x - d)] + c$ . Determine  $a$ ,  $k$ ,  $c$ , and  $d$ ; sketch the graph; and state the domain and range for each transformation.
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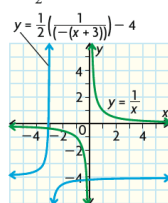
Answer

19. a)  $a = -2, k = 1, c = 0, d = 4$



domain =  $\{x \in \mathbb{R} \mid x \geq 4\}$ , range =  $\{y \in \mathbb{R} \mid y \leq 0\}$

- b)  $a = \frac{1}{2}, k = -1, c = -3, d = -4$



domain =  $\{x \in \mathbb{R} \mid x \neq -3\}$ , range =  $\{y \in \mathbb{R} \mid y \neq -4\}$

20. If  $f(x) = (x - 2)(x + 5)$ , determine the  $x$ -intercepts for each function.

a)  $y = f(x)$

c)  $y = f\left(-\frac{1}{3}x\right)$

b)  $y = -4f(x)$

d)  $y = f(-(x + 2))$

20. If  $f(x) = (x - 2)(x + 5)$ , determine the  $x$ -intercepts for each function.

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b)  $y = -4f(x)$

d)  $y = f(-(x + 2))$

a)  $f(x) = (x - 2)(x + 5)$   
 $x$  intercepts  $\rightarrow (2, 0) \quad (-5, 0)$

Remember transformations.....

$$(x, y) \rightarrow \left(\frac{x}{k}, ay\right) \rightarrow \left(\frac{x}{k} + d, ay + c\right)$$

b)  $y = -4f(x)$   
 $\downarrow$   
 $a = -4$

$$\therefore (x, y) \rightarrow (x, -4y)$$

$$(2, 0) \rightarrow (2, 0)$$

$$(-5, 0) \rightarrow (-5, 0)$$

Now try c) and d)



20. If  $f(x) = (x - 2)(x + 5)$ , determine the x-intercepts for each function.

- a)  $y = f(x)$                       c)  $y = f\left(-\frac{1}{3}x\right)$   
 b)  $y = -4f(x)$                     d)  $y = f(-(x + 2))$

a)  $f(x) = (x - 2)(x + 5)$   
 x intercepts  $\rightarrow (2, 0)$   $(-5, 0)$

Remember transformations...

$$(x, y) \rightarrow \left(\frac{x}{k}, ay\right) \rightarrow \left(\frac{x}{k} + d, ay + c\right)$$

b) $y = -4f(x)$ $\downarrow$ $a = -4$ $\therefore (x, y) \rightarrow (x, -4y)$ $(2, 0) \rightarrow (2, 0)$ $(-5, 0) \rightarrow (-5, 0)$	c) $y = \left(-\frac{1}{3}x\right)$ $\downarrow$ $k = -\frac{1}{3}$ $\therefore (x, y) \rightarrow \left(\frac{x}{-\frac{1}{3}}, y\right)$ $(-3x, y)$ $(2, 0) \rightarrow (-6, 0)$ $(-5, 0) \rightarrow (15, 0)$
---	--

d)  $f(-(x + 2))$   
 $\uparrow \quad \uparrow$   
 $k = -1 \quad d = -2$   
 $(2, 0) \rightarrow (-2, 0) \rightarrow (4, 0)$   
 $(-5, 0) \rightarrow (5, 0) \rightarrow (3, 0)$   
 "k" "d"