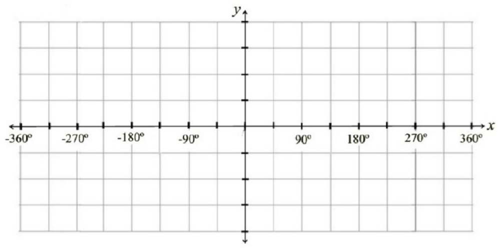
Warm - up - Print the pdf

Review of Transformations - "a" and "c"

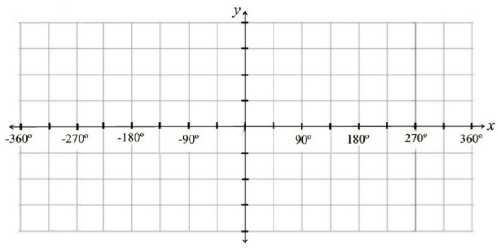
Warm-Up

Accurately Graph the following functions using a different colour for each, $-360 \le x \le 360$

$f(x) = \sin x$ $\quad f(x) = 2\sin x$ $\quad f(x) = \sin x - 3$ $\quad f(x) = 2\sin x + 2$



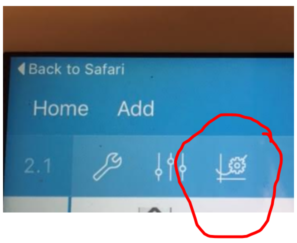
$f(x) = \cos x$ $\quad f(x) = -\cos x$ $\quad f(x) = -3\cos x$ $\quad f(x) = -3\cos x - 1$

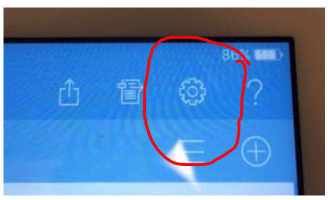


Check your answer on your Nspire

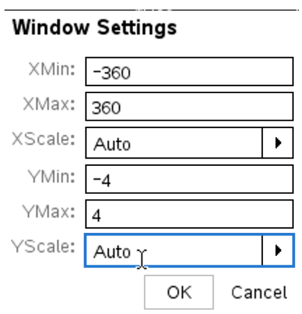
Check your answer on your TI-Nspire

Make sure your Nspire is set to degrees (there are two settings)

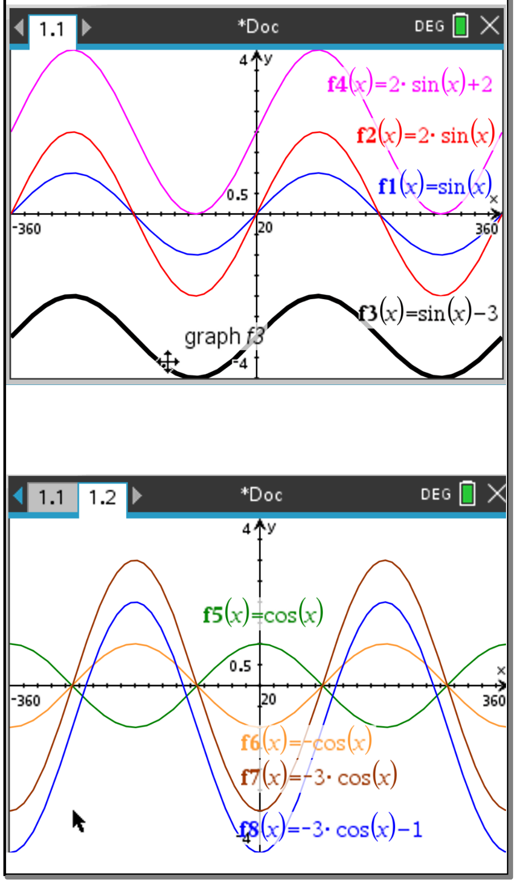




and set window to match the graph



Do not move on ... until you have graphed these with Technology



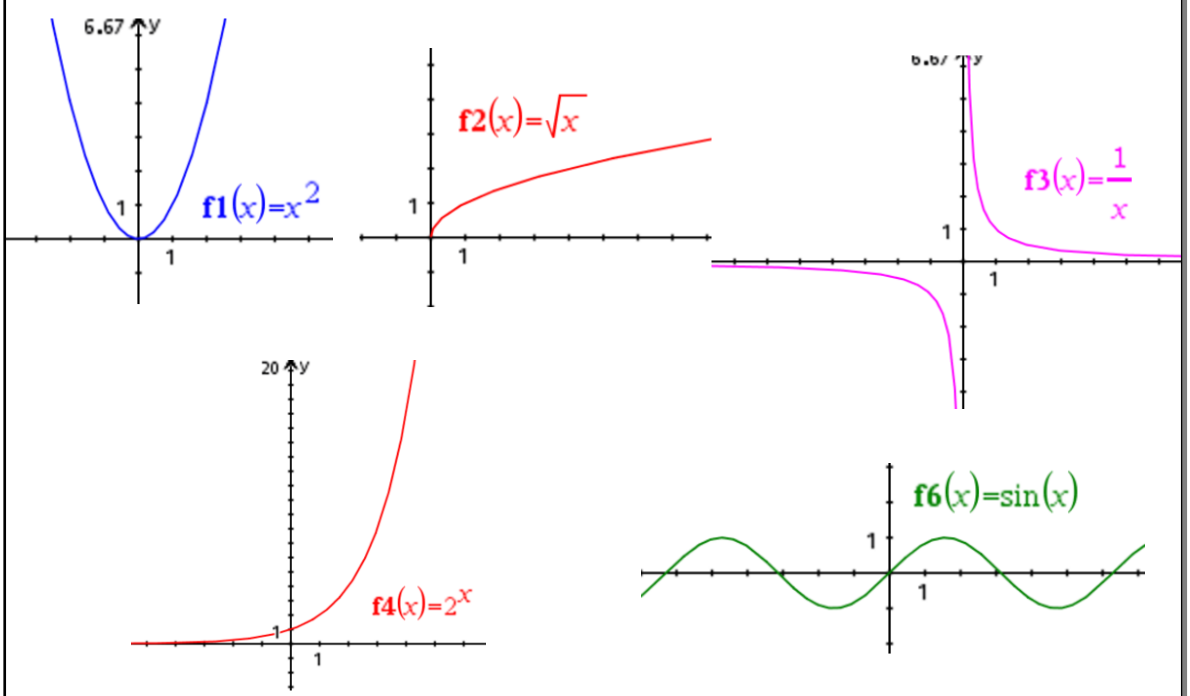
Horizontal Transformations

"k" and "d"

Learning goal:

- explore horizontal transformations of parent functions

What parent functions have we learned about?



Your window settings should be

$-15 \leq x \leq 15$

$-15 \leq y \leq 15$

For the Sine Function $y = a \sin(k(x-d))+c$

Make sure your calculator is set to degrees.

Suggested window settings Xmin = -180°
Xmax = 540°
Xscl = 90°
Ymin = - 4
Ymax = 4

The role of "k" $y=a\sqrt{k(x-d)}+c$

1. Use the TI-Nspire to graph each of the functions.
2. In Your Notes sketch and label on a single set of axis.

$y=\sqrt{x}$

$y=\sqrt{2x}$ $y=\sqrt{0.5x}$

$y=\sqrt{5x}$ $y=\sqrt{-2x}$

Predict

$y=\sqrt{4x}+3$

3. Describe the effect "k" has on the graph of $y=\sqrt{x}$

The role of "k" $y=\dfrac{a}{k(x-d)}+c$

1. Use the TI-Nspire to graph each of the functions.
2. In Your Notes.sketch and label on a single set of axis.

$y=\dfrac{1}{x}$ $y=\dfrac{1}{0.5x}$ $y=\dfrac{1}{3x}$

Predict

$y=\dfrac{1}{3x}+5$

3. Does the change in "k" affect which quadrant the graph is in?
4. Describe the effect "k" has on the shape of the graph.
5. Does the change in "k" affect the location of the asymptotes?

$y=a\sin(k(x-d))+c$

$y=\sin x$

$y=\sin 2x$ $y=\sin 0.5x$

$y=\sin 3x$ $y=\sin(-x)$

Predict

$y=3\sin 0.5x$

$y=\sin(-x)+3$

1. Sketch on the same axes.
2. Describe the effect "k" has on the graph of y=sin x

$g(x)=ab^{k(x-d)}+c$

$y=2^{x}$

$y=2^{2x}$ $y=2^{0.3x}$ $y=2^{-x}$

$y=2^{3x}$ $y=2^{0.5x}$

1. Sketch on the same axes.
2. Describe the effect "k" has on the graph of $y=2^x$

The role of "d" $y=a(x-d)^2+c$

1. Use the TI-Nspire to graph each of the functions.
2. In Your Notes.sketch and label on a single set of axis.

$y=x^2$

$y=(x+3)^2$ $y=(x-2)^2$

$y=(x+5)^2$ $y=(x-7)^2$

Predict

$y=0.5(x+3)^2$

$y=2(x-5)^2+7$

3. Describe the effect "d" has on the graph of $y=x^2$

The role of "d" $y=a\sqrt{k(x-d)}+c$

1. Use the TI-Nspire to graph each of the functions.
2. In Your Notes.sketch and label on a single set of axis.

$y=\sqrt{x}$

$y=\sqrt{(x+3)}$ $y=\sqrt{(x-2)}$

$y=\sqrt{(x+5)}$ $y=\sqrt{(x-7)}$

Predict

$y=0.5\sqrt{(x+3)}$

$y=2\sqrt{(x-5)}+7$

3. Describe the effect "d" has on the graph of $y=\sqrt{x}$

The role of "d"

$$y = \frac{a}{k(x-d)} + c$$

1. Use the TI-Nspire to graph each of the functions.
2. In Your Notes sketch and label on a single set of axis.

$y = \frac{1}{x}$ $y = \frac{1}{x-7}$ $y = \frac{1}{x+3}$

Predict

$y = \frac{1}{x-4} + 3$

3. Does the change in "d" affect which quadrant the graph is in?
4. Describe the effect "d" has on the shape of the graph.
5. Does the change in "d" affect the location of the asymptotes?

$y = a\sin(k(x-d)) + c$

$y = \sin x$

$y = \sin(x - 90°)$

$y = \sin(x + 45°)$

Predict

$y = 3\sin(x + 45°)$

1. Sketch on the same axes.
2. Describe the effect "d" has on the graph of y=sin x

$$g(x) = ab^{k(x-d)} + c$$

$y = 2^x$

$y = 2^{(x-5)}$

$y = 2^{(x+3)}$

$y = 2^{(x+10)}$

Predict

$y = 2^{3x} - 4$

1. Sketch on the same axes.
2. Describe the effect of "d" has on the graph of $y=2^x$

Turn on the video

$k > 1$ H. Compression → ←

$0 < k < 1$ H. Stretch ← →

$k < 0$ H. Reflection

$d > 0$ H. Translation right $(x - d)$ $(x-5)$

$d < 0$ H. Translation left $(x - {-d})$ $(x+5)$

Summary $y=a \sin (k(x-d))+c$

$k > 1$ H. Compression

$0 < k < 1$ H. Stretch

$k < 0$ H. Reflection

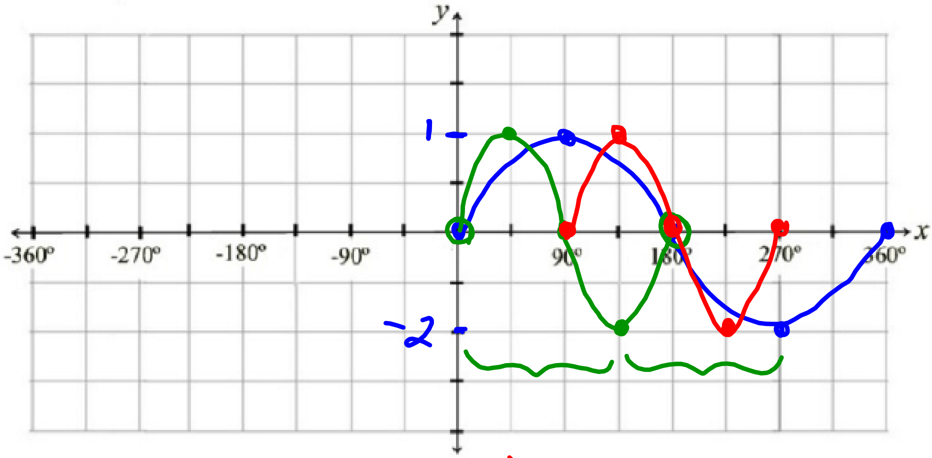
How many cycles / 360°?

$$\text{period} = \frac{360^\circ}{k}$$

$d > 0$ Phase shift right

$d < 0$ Phase shift left

$y = \sin(2x - 180)$

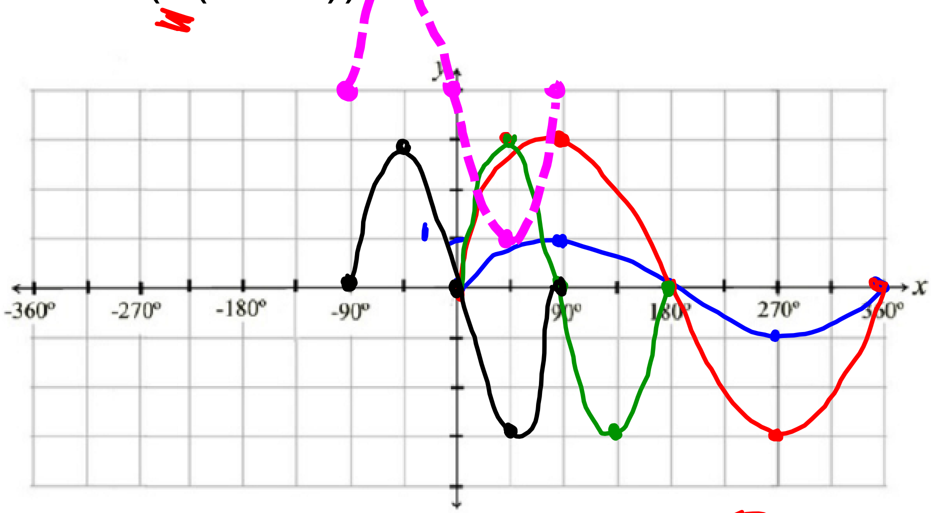


$y = 1\sin(2(x - 90))$

H. Compression

Phase shift right 90°

$y = 3\sin(2(x+90)) + 4$



$a = 3$ V. Stretch by 3 (1)

$k = 2$ H. Compression $\frac{1}{2}$ (2)

$d = -90$ Phase shift 90 left (3)

$c = 4$ V. Translation up 4 (4)

Try on Your Own #1

These transformations work the same on ALL the Parent Functions BUT are most interesting on the sinusoidal because they change the characteristics of the functions.

Investigate how the "k" changes the Period

5. Using a graphing calculator in DEGREE mode, graph each function. Use the WINDOW settings shown. After you have the graph, state the period, the equation of the axis, and the amplitude for each function.

```
WINDOW
 Xmin=0
 Xmax=1440
 Xscl=180
 Ymin=-2
 Ymax=6
 Yscl=1
 Xres=1
```

a) $f(x) = 2\sin x + 3$
b) $f(x) = 3\sin x + 1$
c) $f(x) = \sin(0.5x) + 2$
d) $f(x) = \sin(2x) - 1$
e) $f(x) = 2\sin(0.25x)$
f) $f(x) = 3\sin(0.5x) + 2$

Equation	Period	Equation of Axis	Amplitude
f(x) = 2 sinx + 3			
f(x) = 3 sinx +1			
f(x) = sin(0.5x) + 3			
f(x) = sin(2x) + 3			
f(x) = 2 sin(0.25x) + 3			
f(x) = 3 sin(0.5x) + 2			

Try on Your Own #1 Solution

These transformations work the same on ALL the Parent Functions BUT are most interesting on the sinusoidal because they change the characteristics of the functions.

Investigate how the "k" changes the Period

5. Using a graphing calculator in DEGREE mode, graph each function. Use the WINDOW settings shown. After you have the graph, state the period, the equation of the axis, and the amplitude for each function.

$X_{min} = -360$
$X_{max} = 360$
$Y_{min} = -6$
$Y_{max} = 6$

a) $f(x) = 2\sin x + 3$
b) $f(x) = 3\sin x + 1$
c) $f(x) = \sin(0.5x) + 2$
d) $f(x) = \sin(2x) - 1$
e) $f(x) = 2\sin(0.25x)$
f) $f(x) = 3\sin(0.5x) + 2$

Equation	Period	Equation of Axis	Amplitude
f(x) = 2 sinx + 3	360	y=3	2
f(x) = 3 sinx +1	360	y=1	3
f(x) = sin(0.5x) + 3	720	y=3	1
f(x) = sin(2x) + 3	180	y=3	1
f(x) = 2 sin(0.25x) + 3	1440	y=3	2
f(x) = 3 sin(0.5x) + 2	720	y=2	3

How is 'k' related to the period?

$$\text{Period} = \frac{360}{K}$$

$$k = \frac{360}{\text{Period}}$$

Try On Your Own #2 P. 383

1. State the transformations, in the order you would apply them, for each sinusoidal function.

a) $f(x) = \sin(4x) + 2$

b) $y = 0.25\cos(x - 20°)$

c) $g(x) = -\sin(0.5x)$

d) $y = 12\cos(18x) + 3$

e) $f(x) = -20\sin\left[\frac{1}{3}(x - 40°)\right]$

2. If the function $f(x) = 4\cos 3x + 6$ starts at $x = 0$ and completes two full cycles, determine the period, amplitude, equation of the axis, domain, and range.

3. Use transformations to predict what the graph of $g(x) = 5\sin(2(x - 30°)) + 4$ will look like. Verify with a graphing calculator.

Answers

1. a) horizontal compression: $\frac{1}{4}$, vertical translation: 2

b) horizontal translation: 20, vertical compression: $\frac{1}{4}$

c) horizontal stretch: 2; reflection in x-axis

d) horizontal compression: $\frac{1}{18}$; vertical stretch: 12; vertical translation: 3

e) horizontal stretch: 3; horizontal translation: 40; vertical stretch: 20; reflection in x-axis

2. period: 120°; amplitude: 4; axis: $y = 6$; domain: $\{x \in \mathbf{R} \mid 0° \leq x \leq 240°\}$; range: $\{y \in \mathbf{R} \mid 2 \leq y \leq 10\}$

3.