Determining Maximum and Minimum Values of a Quadratic Function

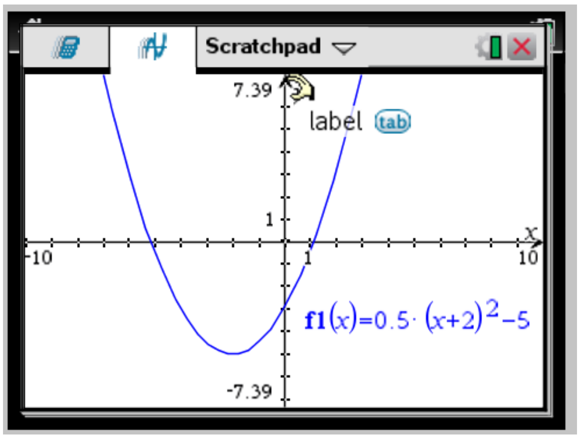
Learning Goals

- **recall how to convert a quadratic function to vertex form by completing the square**
- **determine four ways to find the max /min of a quadratic function**
- **investigate "Max / Min " word problems**

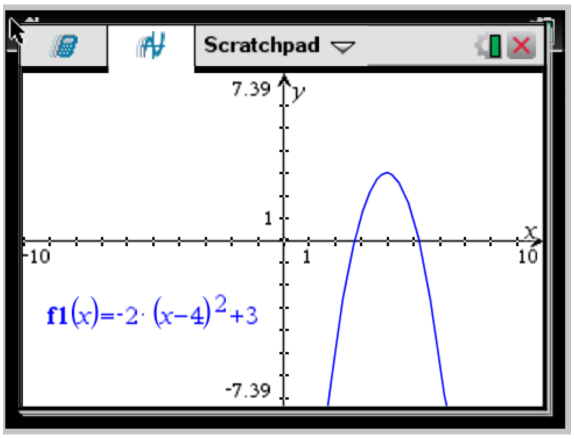
Maximum / Minimum Value (Optimum Value)

- refers to the y- coordinate of the vertex
- we have a maximum if $a<0$
- we have a minimum if $a>0$

$a = 0.5$



$a = -2$



Finding vertex from standard form: $f(x) = ax^2 + bx + c$

1. **completing the square** - read off vertex form

2. **factoring**
   - find zeros
   - find average of zeros → x-coordinate of the vertex

3. **partial factoring**
   - find two points equal distance from the axis of symmetry
   - averaging the x-coordinates → x-coordinate of the vertex

4. **formula** $-\dfrac{b}{2a}$ to find the x-coordinate of the vertex

5. using a **TI-nspire calculator**

MCR3U *3.2 Maximum and Minimum of Quadratic Functions*

The vertex is the optimal value of a quadratic function. The x-coordinate is the input value required to achieve the optimal value, and the y-coordinate is the optimal value of the function.

If the parabola opens up the y-coordinate is a min value

If the parabola opens down, the y-coordinate is a max value

Completing the Square

Completing the square allows us to convert from Standard Form to Vertex Form

The essential steps:	Example: $f(x) = 2x^2 + 12x - 7$
1. Make sure your equation is in standard form. $y = ax^2 + bx + c$	$f(x) = \underbrace{2x^2 + 12x}\, - 7$
2. Factor out "*a*", from the first two terms	$f(x) = 2(x^2 + 6x) - 7$
3. Create a perfect square in the bracket by adding and subtracting $\left(\frac{b}{2}\right)^2$	$f(x) = 2(x^2 + 6x + 9 - 9) - 7$
4. Group the perfect square and remove the extra number out. Remember to multiply by "*a*" in front.	$f(x) = 2(x^2 + 6x + 9) - 18 - 7$
5. Factor the perfect square you created in the bracket and collect the like terms outside the bracket. (Make it pretty)	$f(x) = 2(x+3)^2 - 25$ $x^2 + \overset{+}{6}x + \overset{\times}{9} = (x+3)(x+3)$

Do 1. in class, the rest is homework

Practise:

1. $f(x) = -3x^2 + 6x + 1$ 2. $f(x) = x^2 - 10x + 1$ 3. $f(x) = 0.5x^2 + 5x + 7$

Answers:

$f(x) = -3(x-1)^2 + 4$ $f(x) = (x-5)^2 - 24$ $f(x) = 0.5(x+5)^2 - 5.5$

Factoring and Using the Roots

This allows us to convert from Standard Form to Factored Form and then to Vertex Form

The essential steps:	Example: $f(x) = 4x^2 - 12x - 40$
1. Factor to determine the roots. $x-5=0$ $x=5$ The Factored Form of the function is … zeros	$4(x^2-3x-10)$ $= 4(x-5)(x+2)$ 5 -2
2. Determine the Axis of Symmetry (the x co-ordinate of the Vertex)	$\frac{5+(-2)}{2} = \frac{3}{2}$ $x = \frac{3}{2}$
3. Determine the y co-ordinate of the Vertex	$f(\frac{3}{2}) = -49$
4. State the function in Vertex Form	$f(x) = 4(x-\frac{3}{2})^2 - 49$

Practise: Do 1. in class, 2. is homework

1. $fx) = x^2 – 8x + 12$

2. $fx) = -3x^2 - 12x + 15$

Answers:

$f(x)=(x-4)^2-4$

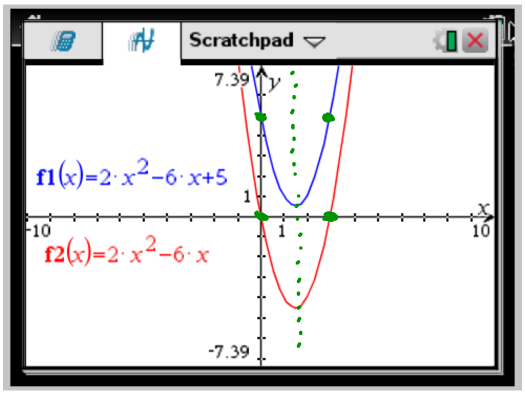
$f(x)=-3(x+2)^2+27$

Partial Factoring

Graph $y=2x^2-6x\boxed{+5}$

$y=2x^2-6x$

a same



Blue graph has no zeros. We can still find the axis of symmetry by finding two points that are the same distance away from it. If we move the graph down, now we can find zeros of the new graph. We can use these points to find the axis of symmetry.

What characteristic do these quadratic functions share?

Partial Factoring

$y=\underbrace{2x^2-6x}_{\text{Red}}+5$ blue

$y=(2x)(x-3)$

$2x=0$ $\quad$ $x-3=0$

$x=0$ $\quad$ $x=3$

AOS $\quad \frac{0+3}{2}=\frac{3}{2}$

$f\left(\frac{3}{2}\right)=2\left(\frac{3}{2}\right)^2-6\left(\frac{3}{2}\right)+5$

$=\frac{1}{2}$

$\therefore$ vertex is $\left(\frac{3}{2},\frac{1}{2}\right)$

Find the vertex using partial factoring.

$f(x)=x^2-10x+1$

$= x(x-10)$

zeros 0 10

AOS $\frac{0+10}{2} = 5$

$f(5) = 5^2-10(5)+1$

$=25-50+1$

$=-24$

$\therefore$ vertex $(5,-24)$

Using the formula

$\left(-\frac{b}{2a},\ f\left(-\frac{b}{2a}\right)\right)$

Find the vertex using the formula

$g(x)=\overset{a}{x^2}+\overset{b}{6}x+\overset{c}{1}$

x-coordinate

$\frac{-b}{2a} = \frac{-6}{2(1)} = -\frac{6}{2} = -3$

y-coordinate

$g(-3) = (-3)^2+6(-3)+1$

$= -8$

$\therefore$ vertex is $(-3,-8)$

Using the TI-Nspire

1. Graph

2. Menu

Analyse Graph

Max/Min

3. Trace

11. The height of a rocket above the ground is modelled by the quadratic function $h(t) = -4t^2 + 32t$, where $h(t)$ is the height in metres t seconds after the rocket was launched.

What is the maximum height that the rocket will reach?

What part of the parabola are we looking for?

y-coordinate of vertex

How are we going to find it?

complete the square

$h = -4t^2 + 32t$

$= -4(t^2 - 8t)$

$= -4(t^2 - 8t + 16 - 16)$

$= -4(t^2 - 8t + 16) + 64$

$= -4(t - 4)^2 + 64$

$\therefore$ height is 64

factor

$h = -4t^2 + 32t$

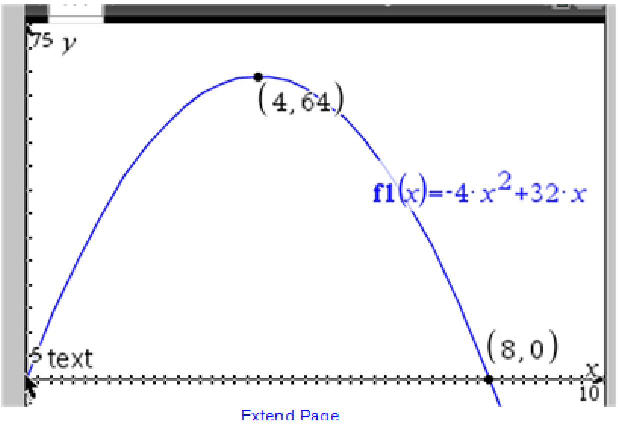
$= -4t(t - 8)$

zeros 0 8

AOS $\frac{0+8}{2} = 4$

$h(4) = -4(4)^2 + 32(4)$

$= 64$



Seatwork

pg 153 # 3bd, 4ab, 6, 8, 10

3. Determine the maximum or minimum value for each.
   a) $y = -4(x + 1)^2 + 6$
   b) $f(x) = (x - 5)^2$
   c) $f(x) = -2x(x - 4)$
   d) $g(x) = 2x^2 - 7$

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4. Determine the maximum or minimum value. Use at least two different methods.
   a) $y = x^2 - 4x - 1$
   b) $f(x) = x^2 - 8x + 12$
   c) $y = 2x^2 + 12x$
   d) $y = -3x^2 - 12x + 15$
   e) $y = 3x(x - 2) + 5$
   f) $g(x) = -2(x + 1)^2 - 5$

6. Use a graphing calculator to determine the maximum or minimum value. Round to two decimal places where necessary.
   a) $f(x) = 2x^2 - 6.5x + 3.2$
   b) $f(x) = -3.6x^2 + 4.8x$

8. The height of a ball thrown vertically upward from a rooftop is modelled by $h(t) = -5t^2 + 20t + 50$, where $h(t)$ is the ball's height above the ground, in metres, at time t seconds after the throw.
   a) Determine the maximum height of the ball.
   b) How long does it take for the ball to reach its maximum height?
   c) How high is the rooftop?

10. Show that the value of $3x^2 - 6x + 5$ cannot be less than 1.

Answers

3. a) maximum: 6 c) maximum: 8
   b) minimum: 0 d) minimum: -7
4. a) complete the square; minimum: -5
   b) factor or complete the square; minimum: -4
   c) factor or complete the square; minimum: -18
   d) factor or complete the square; maximum: 27
   e) use partial factoring; minimum: 2
   f) use vertex form; maximum: -5

6. a) minimum: -2.08 b) maximum: 1.6

8. a) 70 m b) 2 s c) 50 m
9. $562 500
10. Minimum value is 2, therefore $3x^2 - 6x + 5$ cannot be less than 1.