Warm-Up

odd

Determine the next three terms. Explain your reasoning.

4, 9, 19, 39, 79,...

1, 1, 2, 3, 5, 8, 13, 21, ...

6, 13, 27, 55, ...

Warm-Up

even

Determine the next three terms. Explain your reasoning.

100, 99, 97, 94, 90, ...

1, -8, 27, -64, 125, ...

3, 5, 10, 12, 24, 26, 52, ...

Sequence - ordered list of numbers

Term - position of a number in a sequence

- subscripts are used to identify positions

t_1 is the first term

t_2 is the second term

t_n is the n^{th} term $\longrightarrow$ general term

$n \in N \longrightarrow \{1, 2, 3, ...\}$

Finite Sequence - has a set number of terms

Infinite Sequence - doesn't have a set number of terms

- no last term

Create a Formula

$n \in I$

+6, +6, +6, +6

2,8,14,..... 20, 26, 32 ...... t_n

t_1 t_2 t_3 t_4 t_5 t_6

"term"

$t_6 = t_5 + 6$

$$\boxed{t_n = t_{n-1} + 6,\ t_1 = 2,\ n \in I,\ n > 1}$$

so if $n = 10$, then

$t_{10} = t_{10-1} + 6$

$t_{10} = t_9 + 6$!!

Starting value?

A formula relating the general term of a sequence to the previous term is called the RECURSIVE FORMULA

Find the Recursive Formula:

−4, −4

15,11,7,........

Starting value?

$t_n = t_{n-1} - 4$, $t_1 = 15$

$n \in I$, $n > 1$

Find t_{10}?

$t_{10} = t_9 - 4$, ∴ need t_9!

$t_9 = t_8 - 4$, ∴ Need t_8

"compute" backwards and $t_{10} = -21$

Let's consider this as a Function

n (x, 0)	t (y, 19)
1	15
2	11
3	7

−4, −4

$y = mx + b$

when, $x=10 \rightarrow$ $y = -4x + 19$

$-4(10) + 19$

$-40 + 19$

$y = -21$

Find the Recursive Formula:

$\times 2$

6, 12, 24, 48... . . . . . .

t_1 t_2 t_3 t_4 t_n

Starting value?

$t_n = 2 \cdot t_{n-1}$, $t_1 = 6$

$n \in I, n > 1$

What is the pattern?

What are the next four numbers ?

1, 1, 2, 3, 5, 8, 13,21

t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_8 t_9

Create a formula ...

Starting value?

$t_9 = 21 + 13$
$= 34$

$t_9 = t_8 + t_7$

$t_n = t_{n-1} + t_{n-2}$

$t_1 = 1$ $t_2 = 1$

$n \in I, n \geq 3$

At the boards ...
Create the Recursive formula

2,5,8,11, ...

$t_n = t_{n-1} + 3, \; t_1 = 2$
$n \in I, \; n > 1$

Starting value?

2,4,8,16, ...

$t_n = 2 \cdot t_{n-1}, \; t_1 = 2$
$n \in I, \; n > 1$

3, 6, 9, 15, 24, 39, ...

$t_n = t_{n-1} + t_{n-2}, \; t_1 = 3, \; t_2 = 6$
$n \in I, \; n \geq 3$

t_1 t_2 $\underline{\underline{t_3}}$

Homework
1. Handout

2. Pg. 440 # 7
what is the pattern
can you create a formula?

MCR 3U

Patterning, The Recursive Formula, The General Term

Example 1: Given the sequence 1, 1, 2, 3, 5, 8. …
a) Describe the pattern
b) Determine the next three terms
c) Determine a **Recursive Formula**

Example 2: Given the sequence 5, 8, 11, 14, ...
a) Describe the pattern
b) Determine the next three terms
c) Determine a **Recursive Formula**
d) Determine the **General Term**

MCR 3U

Example 3: Given the sequence 5, 14, 41, 122, 365, 1094, 3281, ...
a) Describe the pattern
b) Determine the next three terms
c) Determine a **Recursive Formula**
d) Determine the **General Term**

Example 4: Given the sequence $\frac{3}{4}, \frac{5}{9}, \frac{7}{16}, \frac{9}{25}, \frac{11}{36}, \frac{13}{49}, \frac{15}{64} \ldots$

a) Describe the pattern for the numerator and the denominator
b) Determine the next three terms
c) Is the pattern **Recursive?**
d) Determine the **General Term**

MCR 3U

Patterning, The Recursive Formula, The General Term

Example 1: Given the sequence 1, 1, 2, 3, 5, 8. ...
a) Describe the pattern → add two previous terms
b) Determine the next three terms 13, 21, 34
c) Determine a **Recursive Formula**

$t_n = t_{n-1} + t_{n-2}\ ,\ t_1 = 1,\ t_1 = 1,\ n \in I,\ n \geq 3$

Example 2: Given the sequence 5, 8, 11, 14, ...
a) Describe the pattern — add 3
b) Determine the next three terms 17, 20, 23
c) Determine a **Recursive Formula**
~~d) Determine the **General Term**~~

$t_n = t_{n-1} + 3,\ t_1 = 5,\ n \in I,\ n \geq 2$

MCR 3U

Example 3: Given the sequence 5, 14, 41, 122, 365, 1094, 3281, ...
a) Describe the pattern — x 3 minus 1
b) Determine the next three terms — 9842, 29525, 88574
c) Determine a **Recursive Formula**
~~d) Determine the **General Term**~~

$t_n = 3t_{n-1} - 1,\ t_1 = 5,\ n \in I,\ n \geq 2$

Example 4: Given the sequence $\frac{3}{4}, \frac{5}{9}, \frac{7}{16}, \frac{9}{25}, \frac{11}{36}, \frac{13}{49}, \frac{15}{64} \ldots$

go to the boards ...

a) Describe the pattern for the numerator and the denominator
b) Determine the next three terms
c) Is the pattern **Recursive?**
~~d) Determine the **General Term**~~

numerator

a) 3, 5, 7, 9, 11, 13, 15, 17, 19, 21

b) add 2

c) $t_n = t_{n-1} + 2,\ t_1 = 3\ ,\ n \geq 2$

denominator

a) 4, 9, 16, 25, 36, 49, 64, 81, 100, 121
$t_1 \quad t_2 \quad t_3$

$t_n = (n+1)^2\ ,\ n \in I,\ n > 0$ ← Not Recursive! Depends on "n"

Can these be combined?

$t_n = \dfrac{t_{n-1} + 2}{(n+1)^2}\ ,\ t_1 = 3,\ n \geq 2$?

Homework - Question #7

7 a) +5 +10 +20 +40 +80 +160 +320

4, 9, 19, 39, 79, 159, 319, 639

"The Pattern" → Recursive

→ Add to Previous Term

→ +5, +10, +20, +40, ...

$t_2 = t_1 + 5 \rightarrow t_2 = t_1 + 1(5)$ $\quad t_2 = t_1 + 2^0(5)$

$t_3 = t_2 + 10 \quad t_3 = t_2 + 2(5)$ $\quad t_3 = t_2 + 2^1(5)$

$t_4 = t_3 + 20 \quad t_4 = t_3 + 4(5) \rightarrow t_4 = t_3 + 2^2(5)$

$t_5 = t_4 + 40 \quad t_5 = t_4 + 8(5) \rightarrow t_5 = t_4 + 2^3(5)$

$t_n = t_{n-1} + 2^{n-2}(5)$

$t_1 = 4, n \in I, n > 1$

-1 -2 -3 -4 -5 -6 -7

7 b) 100, 99, 97, 94, 90, 85, 79, 72

$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5$

$t_2 = t_1 - 1$

$t_3 = t_2 - 2$

$t_4 = t_3 - 3$

$t_n = t_{n-1} - (n-1), t_1 = 100, n > 1$

$t_1 \ t_2 \ t_3 \ t_4 \ t_5 \ t_6$

c) 1, 1, 2, 3, 5, 8, 13, $\underline{21}$, $\underline{34}$, $\underline{55}$.

Fibonacci !! add 2 previous terms.

$t_n = t_{n-1} + t_{n-2}, \ t_1 = 1, t_2 = 1, \ n \geq 3$

d) 3, 5, 10, 12, 24, 26, 52, $\underline{54}$, $\underline{108}$, $\underline{110}$

+2 +2 +2 +2 +2

×2 ×2 ×2 ×2

$t_1 \ t_2 \ t_3 \ t_4$

$t_2 = t_1 + 2$ $\quad t_3 = 2 \cdot t_2$

$t_4 = t_3 + 2$ $\quad t_5 = 2 \cdot t_4$

so.... n is even

$t_n = t_{n-1} + 2$

$t_1 = 3, \ n \geq 2$

n is odd

$t_n = 2 \cdot t_{n-1}$

$t_2 = 5, \ n \geq 3$

e)

1, -8, 27, -64, 125, ____, ____, ____

$1^3 \quad (-2)^3 \quad 3^3 \quad (-4)^3 \quad 5^3 \quad (-6)^3 \quad 7^3 \quad (-8)^3$

$t_1 \quad t_2 \quad t_3 \quad t_4 \quad t_5 \quad \ldots$

$t_1 = (1)^3 \longrightarrow t_1 = +(1)^3$

$t_2 = (-2)^3 \longrightarrow t_2 = -(2)^3$

$t_3 = (3)^3 \longrightarrow t_3 = +(3)^3$

$t_4 = (-4)^3 \longrightarrow t_4 = -(4)^3$

Not Recursive $\quad t_n = (-1)^{n-1}(n)^3, \; n \in I, \; n > 0$

f)

6, 13, 27, 55, 111, 223, 347

$\times 2 + 1$

$t_n = 2 \cdot t_{n-1} + 1, \; t_1 = 6, \; n \in I, \; n \geq 2$