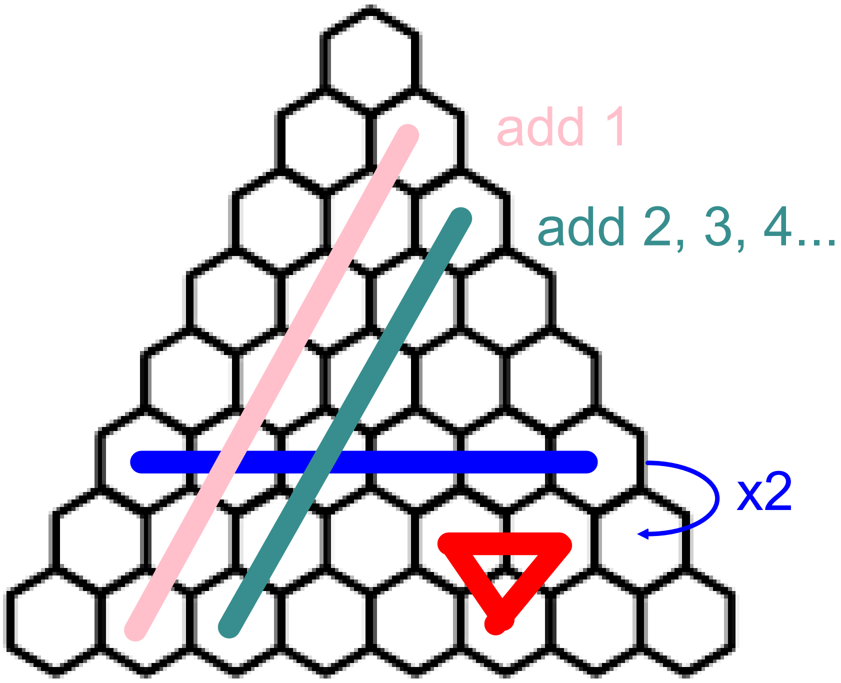
# Pascal's Triangle

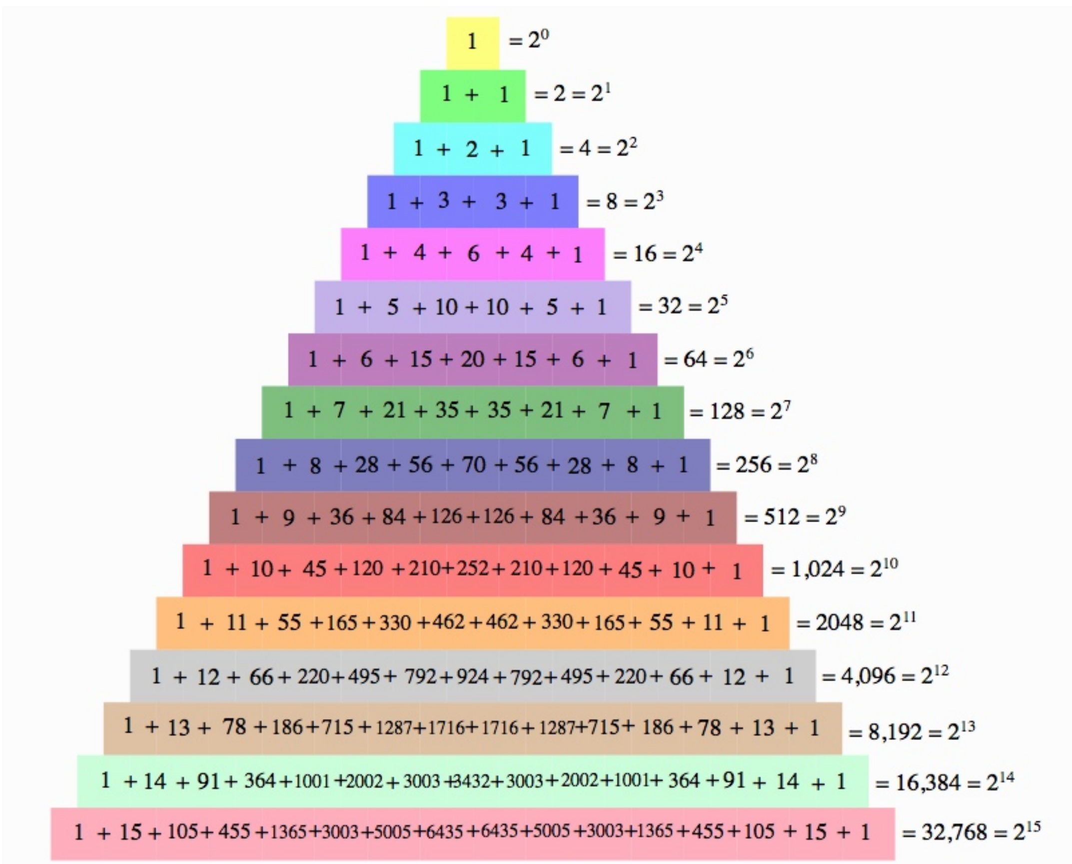
**Learning Goal**
**- able to use patterning to expand a binomial**

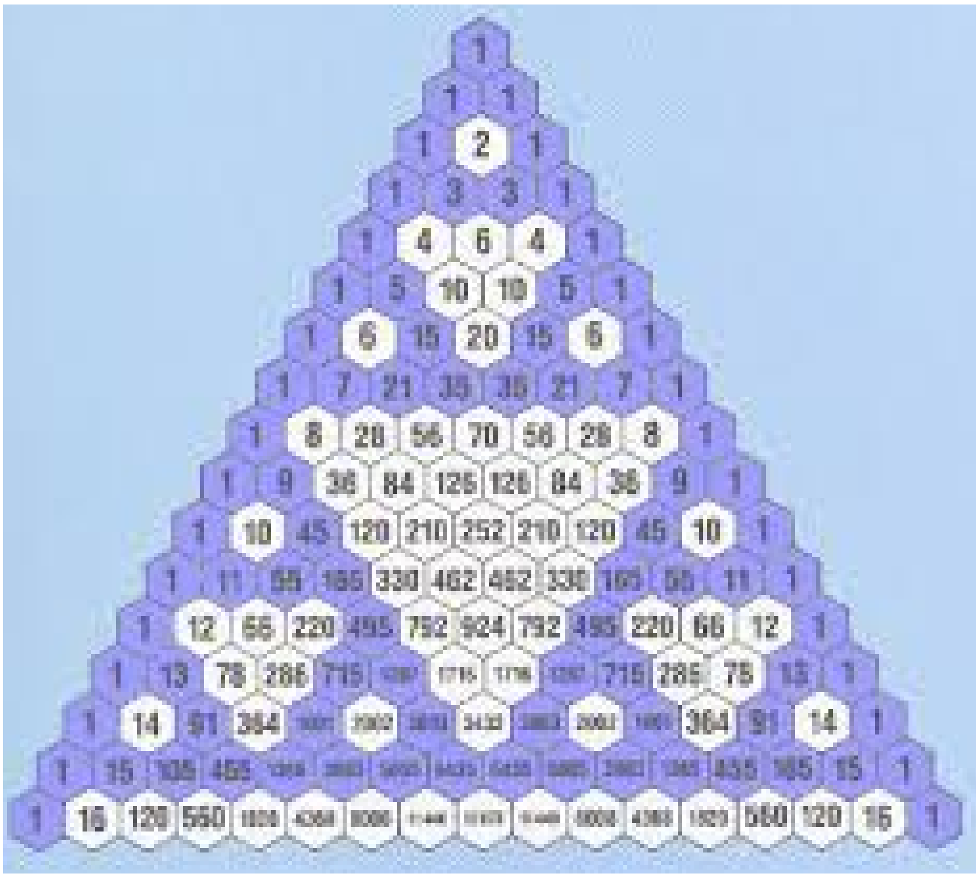
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

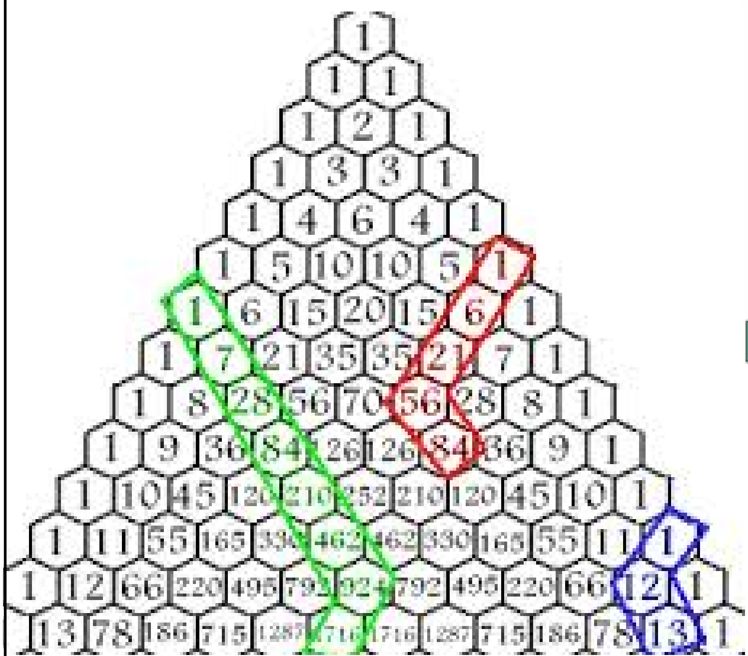
Can you find any other patterns?

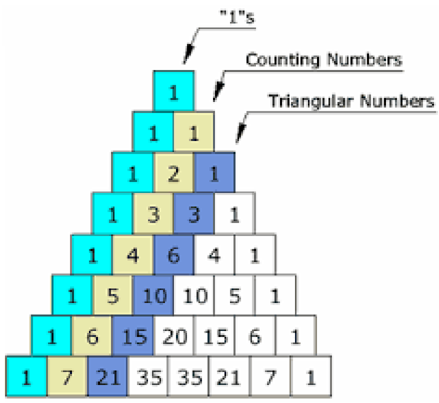
# Pascal's Triangle

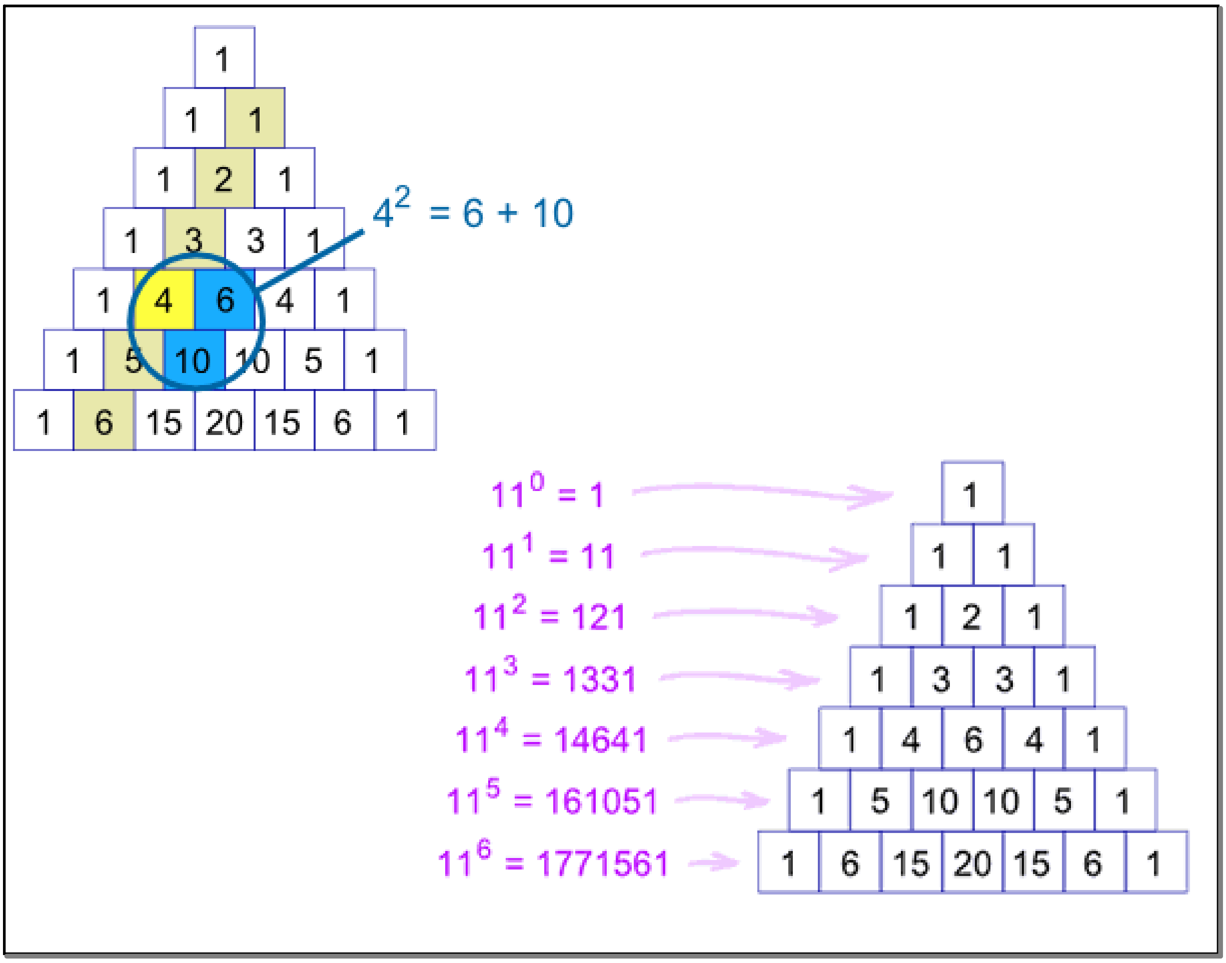







"1"s
Counting Numbers
Triangular Numbers
1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1




$\text{expand}\left((a+b)^0\right)$ $\quad 1$

$\text{expand}\left((a+b)^1\right)$ $\quad a+b$

$\text{expand}\left((a+b)^2\right)$ $\quad a^2+2\cdot a\cdot b+b^2$

$\text{expand}\left((a+b)^3\right)$ $\quad a^3+3\cdot a^2\cdot b+3\cdot a\cdot b^2+b^3$

$\text{expand}\left((a+b)^4\right)$

$a^4+4\cdot a^3\cdot b+6\cdot a^2\cdot b^2+4\cdot a\cdot b^3+b^4$

$\text{expand}\left((a+b)^5\right)$

$a^5+5\cdot a^4\cdot b+10\cdot a^3\cdot b^2+10\cdot a^2\cdot b^3+5\cdot a\cdot b^4$ ▸

Handout

Do you see a relationship to Pascal's Triangle?

$$1$$

$$a+b$$

$(a+b)^2$ → $$1a^2+2ab+1b^2$$

$(a+b)^3$ → $$1a^3b^0+3a^2b^1+3a^1b^2+1a^0b^3$$

$$1a^4b^0+4a^3b^1+6a^2b^2+4a^1b^3+1a^0b^4$$

Circle the coefficients. Do they look familiar?

Yes, they are from Pascal's triangle

What do you notice about the exponents as you move from left to right?

a → down

b → up

What do the exponents in each term add up to?

the original exponent

$(x+2)^3$

$(2x-1)^4$ (with $a = 2x$, $b = -1$)

$1(2x)^4(-1)^0 + 4(2x)^3(-1)^1 + 6(2x)^2(-1)^2 + 4(2x)^1(-1)^3 + 1(2x)^0(-1)^4$

$= 16x^4 - 32x^3 + 24x^2 - 8x + 1$

$(3x+y)^5$

$= 1(3x)^5(y)^0 + 5(3x)^4y^1 + 10(3x)^3y^2 + \ldots$

$= 3^5x^5 + 5(3^4)x^4y + 10(3^3)x^3y^2 + \ldots$

$= 243x^5 + 405x^4y + 270x^3y^2 + \ldots$

$(m^2-2)^3$

$= 1(m^2)^3(-2)^0 + 3(m^2)^2(-2)^1 + 3(m^2)(-2)^2 + 1(-2)^3$

$= m^6 - 6m^4 + 12m^2 - 8$

$(4p^3+p^2)^6$

$= 1(4p^3)^6(p^2)^0 + 6(4p^3)^5(p^2)^1 + \ldots$

$= 4096p^{18} + 6(1024)p^{15}p^2 + \ldots$

$= 4096p^{18} + 6144p^{17} + \ldots$

Find the 3rd term in $(2x-3y)^6$

1, 6, 15
↑

$15(2x)^4(-3y)^2$

$= 15(2^4)(-3)^2x^4y^2$

$= 15(16)(9)x^4y^2$

$= 2160x^4y^2$

# Seatwork

# Pg 466 # 2ac, 4ace, 5f, 10

pg. 466

**2.** Expand and simplify each binomial power.

a) $(x + 2)^5$  b) $(x - 1)^6$  c) $(2x - 3)^3$

**3.** Expand and simplify the first three terms of each binomial power.

a) $(x + 5)^{10}$  b) $(x - 2)^8$  c) $(2x - 7)^9$

## PRACTISING

**4.** Expand and simplify each binomial power.

K a) $(k + 3)^4$  c) $(3q - 4)^4$  e) $(\sqrt{2}x + \sqrt{3})^6$

b) $(y - 5)^6$  d) $(2x + 7y)^3$  f) $(2z^3 - 3y^2)^5$

**5.** Expand and simplify the first three terms of each binomial power.

a) $(x - 2)^{13}$  c) $(z^5 - z^3)^{11}$  e) $\left(3b^2 - \frac{2}{b}\right)^{14}$

b) $(3y + 5)^9$  d) $(\sqrt{a} + \sqrt{5})^{10}$  f) $(5x^3 + 3y^2)^8$

**10.** Expand and simplify $(3x - 5y)^6$.

Answers

**1.** 1, 13, 78, and 286

**2.** a) $(x + 2)^5 = x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$

b) $(x - 1)^6 = x^6 - 6x^5 + 15x^4 - 20x^3 + 15x^2 - 6x + 1$

c) $(2x - 3)^3 = 8x^3 - 36x^2 + 54x - 27$

**3.** a) $(x + 5)^{10} = x^{10} + 50x^9 + 1125x^8 + \ldots$

b) $(x - 2)^8 = x^8 - 16x^7 + 112x^6 - \ldots$

c) $(2x - 7)^9 = 512x^9 - 16\,128x^8 + 225\,792x^7 - \ldots$

**4.** a) $(k + 3)^4 = k^4 + 12k^3 + 54k^2 + 108k + 81$

b) $(y - 5)^6 = y^6 - 30y^5 + 375y^4 - 2500y^3 + 9375y^2 - 18\,750y + 15\,625$

c) $(3q - 4)^4 = 81q^4 - 432q^3 + 864q^2 - 768q + 256$

d) $(2x + 7y)^3 = 8x^3 + 84x^2y + 294xy^2 + 343y^3$

e) $(\sqrt{2}x + \sqrt{3})^6 = 8x^6 + 24\sqrt{6}x^5 + 180x^4 + 120\sqrt{6}x^3 + \sqrt{270}x^2 + 54\sqrt{6}x + 27$

f) $(2z^3 - 3y^2)^5 = 32z^{15} - 240z^{12}y^2 + 720z^9y^4 - 1080z^6y^6 + 810z^3y^8 - 243y^{10}$

**5.** a) $(x - 2)^{13} = x^{13} - 26x^{12} + 312x^{11} - \ldots$

b) $(3y + 5)^9 = 19\,683y^9 + 295\,245y^8 + 1\,968\,300y^7 + \ldots$

c) $(z^5 - z^3)^{11} = z^{55} - 11z^{53} + 55z^{51} - \ldots$

d) $(\sqrt{a} + \sqrt{5})^{10} = a^5 + 10\sqrt{5a}\,a^4 + 225a^4 + \ldots$

e) $\left(3b^2 - \frac{2}{b}\right)^{14} = 4\,782\,969b^{28} - 44\,641\,044b^{25} + 193\,444\,524b^{22} + \ldots$

f) $(5x^3 + 3y^2)^8 = 390\,625x^{24} + 1\,875\,000x^{21}y^2 + 3\,937\,500x^{18}y^4 + \ldots$

**10.** $(3x - 5y)^6 = 729x^6 - 7290x^5y + 30\,375x^4y^2 - 67\,500x^3y^3 + 84\,375x^2y^4 - 56\,250xy^5 + 15\,625y^6$