

Factoring Polynomials

Learning Goals

- review factoring from grade 10
- factoring by grouping

Difference of Squares

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^2 - 9$$

$$= (a - 3)(a + 3)$$

$$16x^2 - 25$$

$$= (4x - 5)(4x + 5)$$

Perfect Squares

$$\underline{a^2x^2} + \underline{2abx} + \underline{b^2} = (ax + b)^2$$

$$x^2 + 10x + 25$$

$$= (x + 5)^2$$

$$1x^2 - 12x + 36$$

$\begin{matrix} \nearrow & & \nwarrow \\ 1^2 & & 6^2 \\ & 2ab & \\ & = 2(1)(6) & \\ & = (1x - 6)^2 & \end{matrix}$

$$4x^2 + 12x + 9$$

$\begin{matrix} \nearrow & & \nwarrow \\ a=2 & & b=3 \\ & 2(2)(3) & \\ & = (2x + 3)^2 & \end{matrix}$

Factoring by Grouping - group terms together and factor

$$n^3 + 3n^2 + 2n + 6$$

$$= \underline{n^2(n + 3)} + 2(n + 3)$$

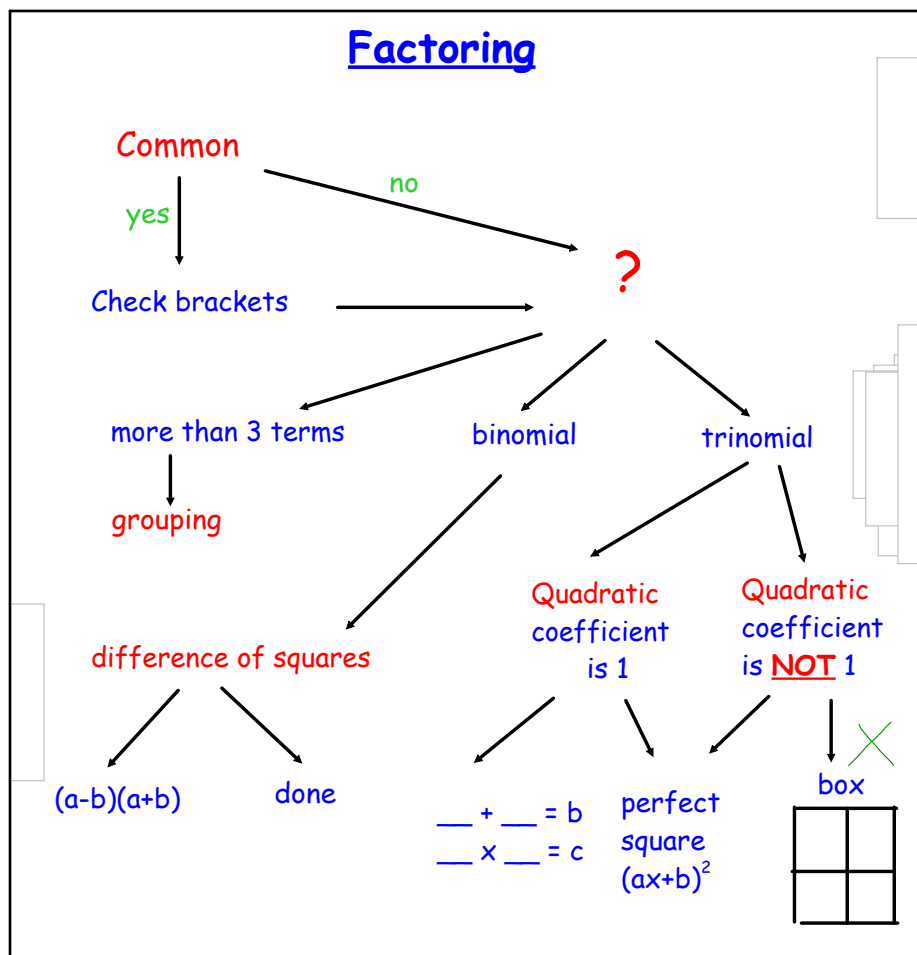
$$= (n + 3)(n^2 + 2)$$

$$x^3 + x^2 + x + 1$$

$$= x(x^2 + x) + x + 1$$

$$= x^2(x + 1) + 1(x + 1)$$

$$= (x + 1)(x^2 + 1)$$



On the Boards...

$$\begin{aligned}
 &9a^2b^2 - 16b^6 \\
 &= b^2(9a^2 - 16b^4) \\
 &= b^2(3a - 4b^2)(3a + 4b^2)
 \end{aligned}$$

$$\begin{aligned}
 &x^2 + 18x + 81 \\
 &= (x + 9)^2
 \end{aligned}$$

$$\begin{aligned}
 &x^2 - 6x + 9 - 4y^2 \\
 &= (x - 3)^2 - 4y^2 \\
 &= [x - 3 + 2y][x - 3 - 2y]
 \end{aligned}$$

$$\begin{aligned}
 &16a^8 - 81 \\
 &= (8a^4 - 9)(8a^4 + 9)
 \end{aligned}$$

$$\begin{aligned}
 &4x^2 + 12x + 9 \\
 &\begin{array}{c}
 \nearrow a=2 \quad \uparrow \quad \nwarrow b=3 \\
 2(2)(3)
 \end{array} \\
 &= (2x + 3)^2
 \end{aligned}$$

Seatwork

- pg 102 # 6 - 9

- Handout

Handout

Factor each completely.

1) $12a^3 - 9a^2 + 4a - 3$

$$(3a^2 + 1)(4a - 3)$$

2) $2p^3 + 5p^2 + 6p + 15$

$$(p^2 + 3)(2p + 5)$$

3) $3n^3 - 4n^2 + 9n - 12$

$$(n^2 + 3)(3n - 4)$$

4) $12n^3 + 4n^2 + 3n + 1$

$$(4n^2 + 1)(3n + 1)$$

5) $m^3 - m^2 + 2m - 2$

$$(m^2 + 2)(m - 1)$$

6) $5n^3 - 10n^2 + 3n - 6$

$$(5n^2 + 3)(n - 2)$$

$$7) 35xy - 5x - 56y + 8$$
$$(5x - 8)(7y - 1)$$

$$8) 224az + 56ac - 84yz - 21yc$$
$$7(8a - 3y)(4z + c)$$

$$9) mz - 5mh^2 - 5nz + 25nh^2$$
$$(m - 5n)(z - 5h^2)$$

$$10) 12xy - 28x - 15y + 35$$
$$(4x - 5)(3y - 7)$$

$$11) 40xy + 30x - 100y - 75$$
$$5(2x - 5)(4y + 3)$$

$$12) 75a^2c - 45a^2d - 30bc + 18bd$$
$$3(5a^2 - 2b)(5c - 3d)$$

$$13) 192x^2y + 72x^3 - 24rxy - 9rx^2$$
$$3x(8x - r)(8y + 3x)$$

$$14) 90au - 36av - 150yu + 60yv$$
$$6(3a - 5y)(5u - 2v)$$

$$15) 140ab - 60a^2 + 168b - 72a$$
$$4(5a + 6)(7b - 3a)$$

$$16) 105ab - 90a - 21b + 18$$
$$3(5a - 1)(7b - 6)$$

17) $16x^2c + 8xyd - 16x^2d - 8xyc$

$8x(2x - y)(c - d)$

18) $150m^2nz + 20mn^2c - 120m^2nc - 25mn^2z$

$5mn(6m - n)(5z - 4c)$

19) $105xuv + 60xv - 70xu - 90xv^2$

$5x(7u - 6v)(3v - 2)$

20) $112xy - 16x + 128x^2 - 14y$

$2(8x - 1)(7y + 8x)$

pg. 102 # 6-9

6. Factor.

a) $x^2 - 9$

b) $4n^2 - 49$

c) $x^8 - 1$

d) $9(y - 1)^2 - 25$

e) $3x^2 - 27(2 - x)^2$

f) $-p^2q^2 + 81$

7. Factor.

a) $ax + ay + bx + by$

b) $2ab + 2a - 3b - 3$

c) $x^3 + x^2 - x - 1$

d) $1 - x^2 + 6x - 9$

e) $a^2 - b^2 + 25 + 10a$

f) $2m^2 + 10m + 10n - 2n^2$

8. Andrij claims that the following statement is true:

K $x^3 - y^3 = (x - y)(x^2 + y^2)$

Is Andrij correct? Justify your decision.

9. Factor.

a) $2x(x - 3) + 7(3 - x)$

b) $xy + 6x + 5y + 30$

c) $x^3 - x^2 - 4x + 4$

d) $y^2 - 49 + 14x - x^2$

e) $6x^2 - 21x - 12x + 42$

f) $12m^3 - 14m^2 - 30m + 35$

Answers

6. a) $(x - 3)(x + 3)$ d) $(3y - 8)(3y + 2)$
b) $(2n - 7)(2n + 7)$ e) $-12(2x - 3)(x - 3)$
c) $(x^4 + 1)(x^2 + 1)(x + 1)(x - 1)$ f) $-(pq + 9)(pq - 9)$
7. a) $(x + y)(a + b)$ d) $(4 - x)(x - 2)$
b) $(b + 1)(2a - 3)$ e) $(a - b + 5)(a + b + 5)$
c) $(x + 1)^2(x - 1)$ f) $2(m + n)(m - n + 5)$
8. no; $(x - y)(x^2 + y^2) = x^3 - x^2y + xy^2 - y^3$
9. a) $(x - 3)(2x - 7)$ d) $(y - x + 7)(y + x - 7)$
b) $(x + 5)(y + 6)$ e) $3(2x - 7)(x - 2)$
c) $(x - 1)(x - 2)(x + 2)$ f) $(2m^2 - 5)(6m - 7)$