

Trigonometric Ratios for Special Angles and the Unit Circle

Learning Goals

- describe the unit circle
- describe and use special angles

Watch the Kahn Academy Video

Unit Circle Introduction

Sal Kahn connects the Unit Circle we created in the last lesson to SOHCAHTOA and Sine and Cosine values.

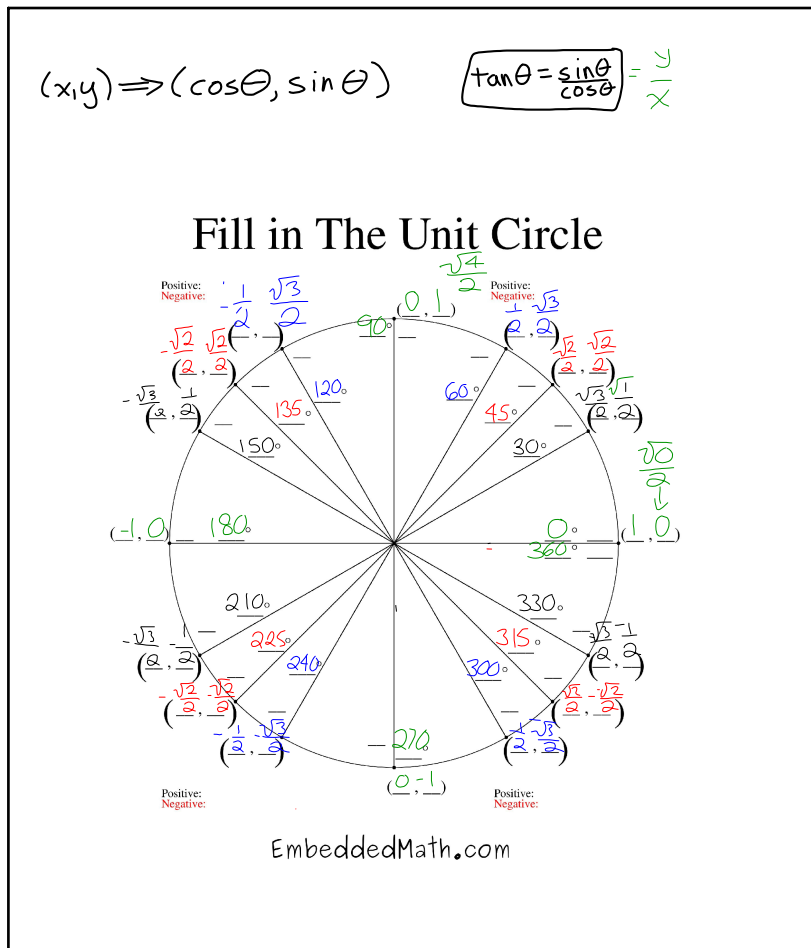
<https://youtu.be/1m9p9iubMLU>

Sal Kahn's Unit Circle is based on SOHCAHTOA and his BIG conclusion is that anywhere on the Unit Circle these things are true.

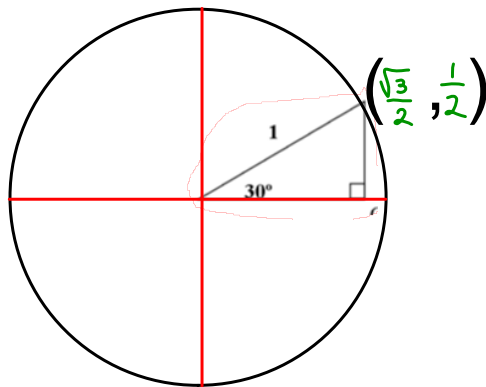
$$(x,y) \Rightarrow (\cos\theta, \sin\theta)$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{y}{x}$$

This is a BIG deal and you should copy this to the top of the sheet of your Unit Circle.



Check this out .. $(x,y) \Rightarrow (\cos\theta, \sin\theta)$



This must mean that $\sin 30^\circ = \frac{1}{2}$

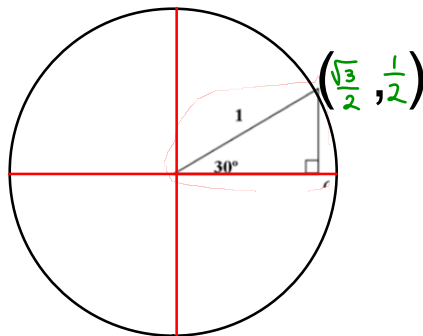
This must mean that $\cos 30^\circ = \frac{\sqrt{3}}{2}$

Verify on TI Nspire ...

Yes do this on your own !!

Calculator settings must be set to degrees.

Check this out .. $(x,y) \Rightarrow (\cos\theta, \sin\theta)$



This must mean that $\sin 30^\circ = \frac{1}{2}$

This must mean that $\cos 30^\circ = \frac{\sqrt{3}}{2}$

Verify on TI Nspire ...

$\sin(30)$	$\frac{1}{2}$
$\cos(30)$	$\frac{\sqrt{3}}{2}$

Try on your own

Use only your Unit Circle to determine the values of the following ...

$$\sin 45^\circ =$$

$$\cos 45^\circ =$$

$$\tan 45^\circ =$$

Verify on TI Nspire ...

Try on your own - Big Hint

Use only your Unit Circle to determine the values of the following ...

$$(x, y) \Rightarrow (\cos \theta, \sin \theta)$$

$$\boxed{\tan \theta = \frac{\sin \theta}{\cos \theta}} = \frac{y}{x}$$

The co-ordinate point for 45° $(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2})$.

$$\sin 45^\circ =$$

$$\cos 45^\circ =$$

$$\tan 45^\circ =$$

Verify on TI Nspire ...

Try on your own - The Answer

Use only your Unit Circle to determine the values of the following ...

$$(x, y) \Rightarrow (\cos \theta, \sin \theta) \quad \boxed{\tan \theta = \frac{\sin \theta}{\cos \theta}} = \frac{y}{x}$$

$$\sin 45^\circ = \frac{\sqrt{2}}{2}$$

$$\cos 45^\circ = \frac{\sqrt{2}}{2}$$

$$\tan 45^\circ = \frac{\sin \theta}{\cos \theta} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$

remember ...
anything divided by itself is one.

Verify on TI Nspire ...

$\sin(45)$	$\frac{\sqrt{2}}{2}$
$\cos(45)$	$\frac{\sqrt{2}}{2}$
$\tan(45)$	1

Try on your own

Use only your Unit Circle to determine the values of the following ...

$$\sin 60^\circ =$$

$$\cos 60^\circ =$$

$$\tan 60^\circ =$$

Verify on TI Nspire ...

Try on your own - Big Hint

Use only your Unit Circle to determine the values of the following ...

$$(x, y) \Rightarrow (\cos \theta, \sin \theta)$$

$$\boxed{\tan \theta = \frac{\sin \theta}{\cos \theta}} = \frac{y}{x}$$

The co-ordinate point for 60° $(\frac{1}{2}, \frac{\sqrt{3}}{2})$.

$$\sin 60^\circ =$$

$$\cos 60^\circ =$$

$$\tan 60^\circ =$$

Verify on TI Nspire ...

Try on your own - Big Hint

Use only your Unit Circle to determine the values of the following ...

$$(x, y) \Rightarrow (\cos \theta, \sin \theta)$$

$$\boxed{\tan \theta = \frac{\sin \theta}{\cos \theta}} = \frac{y}{x}$$

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$\tan 60^\circ = \frac{\sin \theta}{\cos \theta} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$$

Verify on TI Nspire ...

$\sin(60)$	$\frac{\sqrt{3}}{2}$
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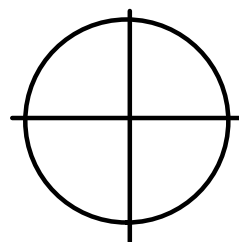
$\cos(60)$	$\frac{1}{2}$
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$\tan(60)$	$\sqrt{3}$
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Try on your own

Given $\sin \theta = \frac{-\sqrt{3}}{2}$

Find θ



Verify on TI Nspire ...

Try on your own - a hint

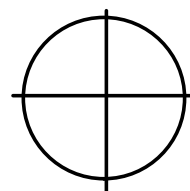
$$(x, y) \Rightarrow (\cos \theta, \sin \theta)$$

Use your Unit Circle find when $y = \frac{-\sqrt{3}}{2}$

There will be **two** answers

Given $\sin \theta = \frac{-\sqrt{3}}{2}$

Find θ



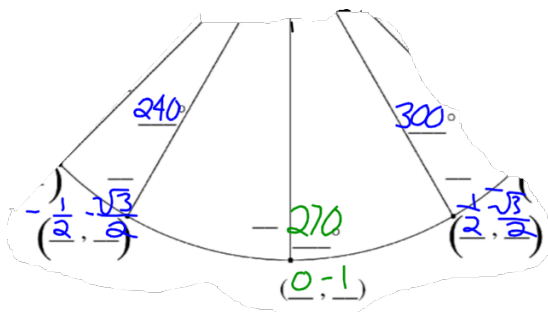
Verify on TI Nspire ...

Try on your own - a Big Hint

$$(x,y) \Rightarrow (\cos\theta, \sin\theta)$$

Use your Unit Circle find when $y = \frac{-\sqrt{3}}{2}$

There will be **two** answers



Given $\sin\theta = \frac{-\sqrt{3}}{2}$

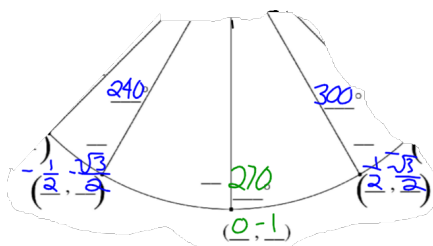
Find θ

Verify on TI Nspire ...

Try on your own - The Answer

$$(x,y) \Rightarrow (\cos\theta, \sin\theta)$$

Use your Unit Circle find when $y = \frac{-\sqrt{3}}{2}$



Given $\sin\theta = \frac{-\sqrt{3}}{2}$

Find θ

$$\theta = 240^\circ \text{ and } 300^\circ$$

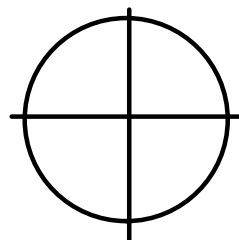
Verify on TI Nspire ...

$\sin(240)$	$\frac{-\sqrt{3}}{2}$
$\sin(300)$	$\frac{-\sqrt{3}}{2}$

Try on your own

Given $\cos \theta = \frac{-\sqrt{2}}{2}$

Find θ

**Try on your own - a hint**

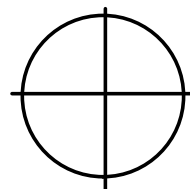
$$(x, y) \Rightarrow (\cos \theta, \sin \theta)$$

Using your Unit Circle find when $x = \frac{-\sqrt{2}}{2}$

There will be **two** answers

Given $\cos \theta = \frac{-\sqrt{2}}{2}$

Find θ

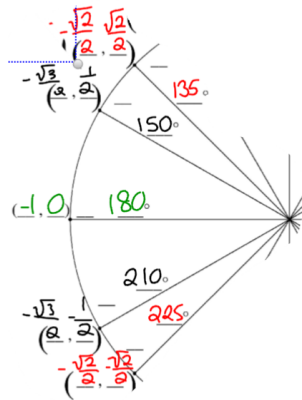


Verify on TI Nspire ...

Try on your own - a Big Hint

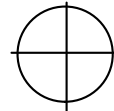
$$(x,y) \Rightarrow (\cos\theta, \sin\theta)$$

Using your Unit Circle find when $x = \frac{-\sqrt{2}}{2}$
 There will be **two** answers



Given $\cos\theta = \frac{-\sqrt{2}}{2}$

Find θ

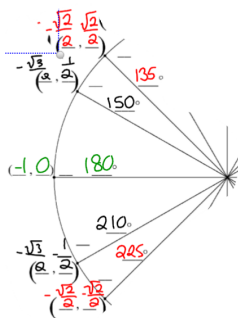


Verify on TI Nspire ...

Try on your own - the Answer

$$(x,y) \Rightarrow (\cos\theta, \sin\theta)$$

Using your Unit Circle find when $x = \frac{-\sqrt{2}}{2}$
 There will be **two** answers



Given $\cos\theta = \frac{-\sqrt{2}}{2}$

Find θ



$$\theta = 135^\circ \text{ AND } 225^\circ$$

Verify on TI Nspire ...

$\cos(135)$	$\frac{-\sqrt{2}}{2}$
$\cos(225)$	$\frac{-\sqrt{2}}{2}$

Print the pdf or copy the questions and complete, no technology - just your Unit Circle

Ex. Evaluate each of the following. Answers must be exact. No calculators permitted.

- $\sin 120^\circ$ _____ $\cos 180^\circ$ _____ $\tan 30^\circ$ _____ $\sin 315^\circ$ _____
 $\cos 210^\circ$ _____ $\tan 300^\circ$ _____ $\sin 270^\circ$ _____ $\cos 60^\circ$ _____
 $\sin 240^\circ$ _____ $\tan 225^\circ$ _____ $\tan 0^\circ$ _____ $\cos 135^\circ$ _____
 $\sin 150^\circ$ _____ $\sin 300^\circ$ _____ $\cos 330^\circ$ _____ $\tan 150^\circ$ _____

Ex. Solve each of the following, where $0 \leq \theta < 360^\circ$.

- a) $\sin \theta = \frac{\sqrt{3}}{2}$ $\theta =$ _____ or $\theta =$ _____ .
 b) $\cos \theta = \frac{\sqrt{2}}{2}$ $\theta =$ _____ or $\theta =$ _____ .
 c) $\sin \theta = -\frac{1}{2}$ $\theta =$ _____ or $\theta =$ _____ .
 d) $\tan \theta = \sqrt{3}$ $\theta =$ _____ or $\theta =$ _____ .
 e) $\cos \theta = 0$ $\theta =$ _____ or $\theta =$ _____ .
 f) $\cos \theta = -\frac{\sqrt{3}}{2}$ $\theta =$ _____ or $\theta =$ _____ .
 g) $\tan \theta = -1$ $\theta =$ _____ or $\theta =$ _____ .
 h) $\sin \theta = -1$ $\theta =$ _____ or $\theta =$ _____ .

Ex. Evaluate each of the following. Answers must be exact. No calculators permitted.

- $\sin 120^\circ$ $\frac{\sqrt{3}}{2}$ $\cos 180^\circ$ -1 $\tan 30^\circ$ $\frac{\sqrt{3}}{3}$ $\sin 315^\circ$ $-\frac{\sqrt{2}}{2}$
 $\cos 210^\circ$ $-\frac{\sqrt{3}}{2}$ $\tan 300^\circ$ $-\sqrt{3}$ $\sin 270^\circ$ -1 $\cos 60^\circ$ $\frac{1}{2}$
 $\sin 240^\circ$ $-\frac{\sqrt{3}}{2}$ $\tan 225^\circ$ 1 $\tan 0^\circ$ 0 $\cos 135^\circ$ $-\frac{\sqrt{2}}{2}$
 $\sin 150^\circ$ $\frac{1}{2}$ $\sin 300^\circ$ $-\frac{\sqrt{3}}{2}$ $\cos 330^\circ$ $\frac{\sqrt{3}}{2}$ $\tan 150^\circ$ $-\frac{\sqrt{3}}{3}$

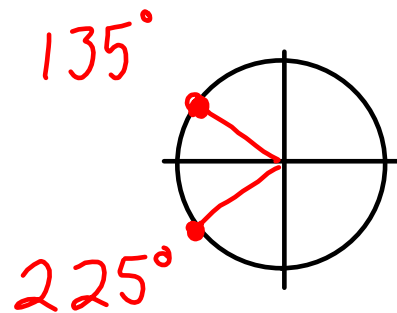
Ex. Solve each of the following, where $0 \leq \theta < 360^\circ$.

- a) $\sin \theta = \frac{\sqrt{3}}{2}$ $\theta = 60^\circ$ or $\theta = 120^\circ$.
 b) $\cos \theta = \frac{\sqrt{2}}{2}$ $\theta = 45$ or $\theta = 315$.
 c) $\sin \theta = -\frac{1}{2}$ $\theta = 210$ or $\theta = 330$.
 d) $\tan \theta = \sqrt{3}$ $\theta = 60$ or $\theta = 240$.
 e) $\cos \theta = 0$ $\theta = 90$ or $\theta = 270$.
 f) $\cos \theta = -\frac{\sqrt{3}}{2}$ $\theta = 210$ or $\theta = 150$.
 g) $\tan \theta = -1$ $\theta = 135$ or $\theta = 315$.
 h) $\sin \theta = -1$ $\theta = 270$ or $\theta = X$.

Open Mrs Major's Video

Given $\cos \theta = \frac{-\sqrt{2}}{2}$

Find θ



Worked examples

Evaluate without a calculator

$$\begin{aligned} & \sin 30^\circ \cos 30^\circ + \sin 60^\circ \cos 60^\circ \\ & \left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right) \\ & = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} \\ & = \frac{2\sqrt{3}}{4} \\ & = \frac{\sqrt{3}}{2} \end{aligned}$$

Communication **Tip**

$\tan^2 30^\circ = (\tan 30^\circ)(\tan 30^\circ)$.
The expression is squared, not
the angle.

$$\sin^2 45^\circ = (\sin 45^\circ)^2$$

Worked examples

Evaluate without a calculator

$$\begin{aligned} & \sin^2 45^\circ + 2 \sin 30^\circ \\ & \left(\frac{\sqrt{2}}{2}\right)^2 + \frac{2}{1}\left(\frac{1}{2}\right) \\ & = \frac{2}{4} + \frac{2}{2} \\ & = \frac{1}{2} + \frac{2}{2} \\ & = \frac{3}{2} \end{aligned}$$

Additional Practise ...

pg 287 # 4 - 8

Use the **unit circle** or the **special triangles** to help you.

4. Determine the exact value of each trigonometric expression.

K a) $\sin 30^\circ \times \tan 60^\circ - \cos 30^\circ$ c) $\tan^2 30^\circ - \cos^2 45^\circ$

b) $2 \cos 45^\circ \times \sin 45^\circ$ d) $1 - \frac{\sin 45^\circ}{\cos 45^\circ}$

5. Using exact values, show that $\sin^2 \theta + \cos^2 \theta = 1$ for each angle.

a) $\theta = 30^\circ$ b) $\theta = 45^\circ$ c) $\theta = 60^\circ$

6. Using exact values, show that $\frac{\sin \theta}{\cos \theta} = \tan \theta$ for each angle.

a) $\theta = 30^\circ$ b) $\theta = 45^\circ$ c) $\theta = 60^\circ$

7. Using **Your Unit Circle**, determine θ if $0^\circ \leq \theta \leq 90^\circ$.

a) $\sin \theta = \frac{\sqrt{3}}{2}$ c) $2\sqrt{2} \cos \theta = 2$

b) $\sqrt{3} \tan \theta = 1$ d) $2 \cos \theta = \sqrt{3}$

8. A 5 m stepladder propped against a classroom wall forms an angle of 30°

A with the wall. Exactly how far is the top of the ladder from the floor? Express your answer in radical form. What assumption did you make?

The Answers

4. a) 0 b) 1 c) $-\frac{1}{6}$ d) 0

5. a) $\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = 1$ c) $\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 1$

b) $\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = 1$

6. a) $\frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{\sqrt{3}}{3}$ b) $\frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)} = 1$ c) $\frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = \sqrt{3}$

7. a) 60° b) 30° c) 45° d) 30°

8. $\frac{5\sqrt{3}}{2}$ m, assuming that the wall is perpendicular to the floor