MCR 3U *Name:*______________________

Max / Min Assignment

Method #1 – Factoring and Using the Roots **Worked With** ______________

Method #2 – Partial Factoring

Method #3 – Completing the Square **Worked With** ______________

Method #4 – The Formula

1. Given the revenue function $R(x) = -3x^2 + 74x$, and the cost function $C(x) = 12x - 559$, where x is the number of items sold in thousands, determine;

Method Used #

a. The **profit** function $P(x)$

ANSWER $P(x) = -3x^2 + 62x + 559$

b. The **value of** x that maximizes profit

ANSWER $x = \frac{31}{3}$ OR $x = 10{,}33\dot{3}$

c. The **maximum profit.**

ANSWER $879,333.33

Method Used #

2. The profit function for a certain product is given by $P(x) = -5(x-7)(x-13)$, where x is the number of items sold in thousands. What **quantity of items sold** will produce the maximum profit?

ANSWER 10,000

Method Used #

3. The cost per day of producing widgets at Company XYZ is modeled by the function $C(x) = 0.04x^2 - 8.504x + 25302$, where $C(x)$ is the cost per day in dollars and x is the number of widgets produced in thousands. Find the **daily production level** that will minimize your costs.

ANSWER 106,300

Method Used #

4. The lifeguard at a public beach has 700 m of rope available to create a rectangular swimming area. The shoreline will form one side of the rectangle. Determine the **dimensions** of the rectangle that will produce the largest swimming area. State what this **area** will be.

ANSWER $l = 350$ $w = 175$

ANSWER $A = 61{,}250\ m^2$

Method Used #

5. A CD company has been selling 1200 computer games CDs per week at $18 each. Data indicates that for each $1 increase, there will be a loss of 40 sales per week. If it costs $10 to produce each CD, what should the selling price be in order to maximize the profit?

ANSWER $29

Assignment #1

The Formula

1/ $P(x) = R(x) - C(x)$

$= -3x^2 + 74x - (12x - 559)$

$= -3x^2 + 62x + 559$

$a = -3 \quad b = 62$

$x = -\frac{b}{2a}$

$= \frac{-62}{2(-3)}$

$x = \frac{31}{3}$

$x = 10.333$

∴ They must produce 10,333 items

$P(10.333) = -3(10.333)^2 + 62(10.333) + 559$.

$\doteq 879.333$

∴ Profit is \$ 879,333

1/ $\boxed{\text{Partial Factoring}}$

$$P(x) = -3x^2 + 62x + 559.$$

$$f(x) = -3x^2 + 62x$$
$$= -3x\left(x - \frac{62}{3}\right)$$

$x_1 = 0$ $\qquad$ $x_2 = \frac{62}{3}$

$$AOS = \frac{0 + \frac{62}{3}}{2}.$$
$$= \frac{62}{6}$$
$$AOS = \frac{31}{3}$$

∴ They must produce 10,333 items.

Maximum Profit → see previous

Factored Form

21 $P(x) = -5(x-7)(x-13)$

$x_1 = 7$ $x_2 = 13$

$$AOS = \frac{7+13}{2}$$

$$\boxed{AOS = 10}$$

$\therefore$ They should make 10,000 items.

The Formula

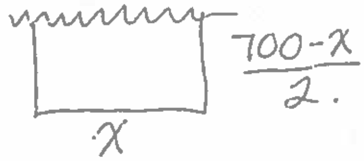
31/ $C(x) = 0.04x^2 - 8.504x + 25302$

$x = \frac{-b}{2a}$ $a = 0.04$ $b = -8.504$

$x = \frac{8.504}{2(0.04)}$

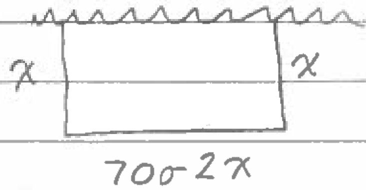
$= 106.3$

$\therefore$ 106,300 widgets should be produced.



Factored

4.



Let x be the width.

$700 = x + x + \text{length}$

$700 - 2x = \text{length}$

$A(x) = l \cdot w$

$= x \cdot (700 - 2x)$

$x_1 = 0$

$700 - 2x = 0$

$\frac{2x}{2} = \frac{700}{2}$

$x_2 = 350$

$AOS = \frac{0 + 350}{2}$

$\boxed{AOS = 175}$ $\longrightarrow$ $\text{length} = 700 - 2(175)$

$= 350$

∴ The width is 175 m

The length is 350 m.

$\text{Area} = (175)(350)$

$= 61250 \text{ m}^2$

Factored

5/ sell price = \$18 → sell 1200

cost = \$10 \$1 increase

profit = \$8 40 less sales.

Profit = (\$8)(1200)

Let x be # Price Increases

$P(x) = (8+x)(1200-40x)$.

$8+x=0$

$x=-8$

$1200-40x=0$

$\frac{1200}{40} = \frac{40x}{40}$

$30 = x$

$AOS = \frac{-8+30}{2}$.

$\boxed{AOS = 11}$

original price is \$18, with 11 price increases ... new price \$29.

Complete the Square

51/

Profit = Revenue − Cost

$= (1200)(18) - (1200)(10)$

Let x be # Price Increases.

$P(x) = (1200-40x)(18+x) - (1200-40x)(10)$

$= 21600 + 1200x - 720x - 40x^2 - 12000 + 400x$

$= -40x^2 + 880x + 9600$

Complete the Square ....

$= -40(x^2 - 22x + 121 - 121) + 9600$

$= -40(x-11)^2 + 9600 + 4840$

$= -40(x-11)^2 + 14{,}440$

$\uparrow$

$x = 11$

∴ sell price \$29.