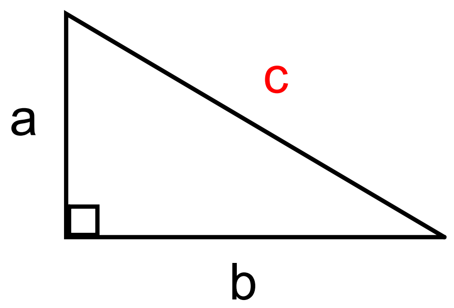
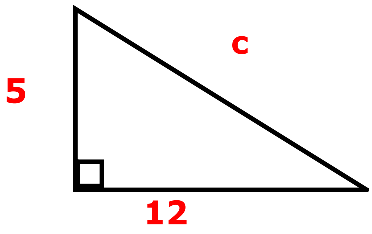
Trigonometric Ratios of Acute Angles

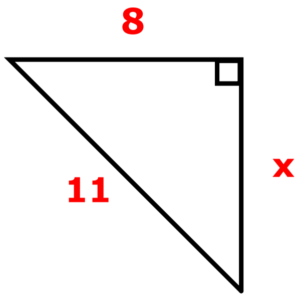
Learning Goals
- **solve for sides and angles of right angled triangles**
- **evaluate reciprocal trigonometric ratios**

Pythagorean Theorem:

- used with right angled triangles
- formula is $a^2 + b^2 = c^2$
- c is always the hypotenuse





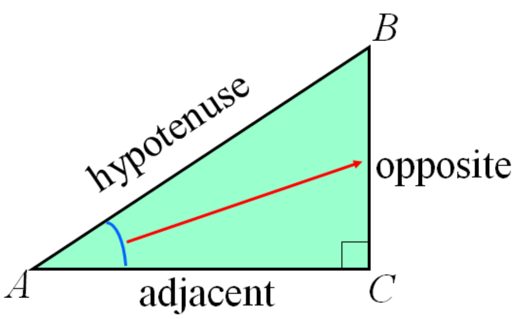


$$a^2 + b^2 = c^2$$
$$5^2 + 12^2 = c^2$$
$$25 + 144 = c^2$$
$$169 = c^2$$
$$13 = c$$

$$a^2 + b^2 = c^2$$
$$x^2 + 8^2 = 11^2$$
$$x^2 + 64 = 121$$
$$x^2 = 57$$
$$x = 7.5$$

Primary Trigonometric Ratios

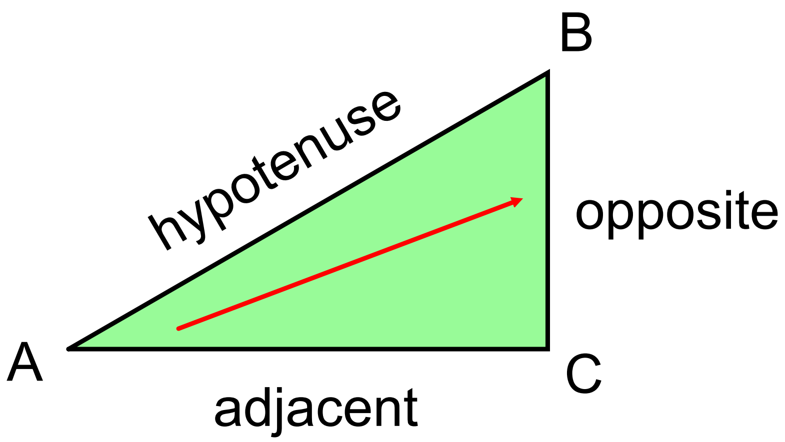
$$\sin A = \frac{opp}{hyp} \qquad \cos A = \frac{adj}{hyp} \qquad \tan A = \frac{opp}{adj}$$



SOH CAH TOA

Can only be used with right angled triangles

What happens to the ratios if you have angle B ????



On the Boards...

Determine the following angles (*nearest degree*).

- Make sure your calculator is in degree mode
- two places in the TI-Nspire
- check Sin 90° = 1

Sin 20° $= 0.34$

Cos 120° $= -0.5$

Tan 230° $= 1.19$

Sin A = 0.4142

$A = \sin^{-1} 0.4142$

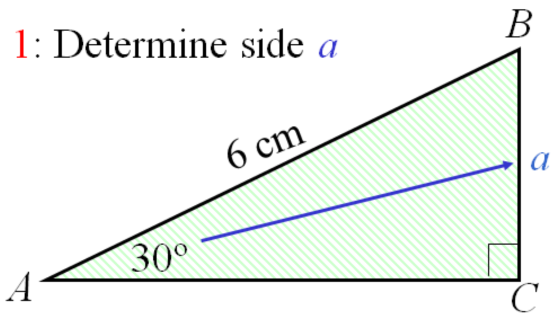
$A = 24°$

Cos B = 0.6820

$B = \cos^{-1} 0.6820$

$B = 47°$

Example 1: Determine side a



- Name the sides

- Set up the ratio

- Solve

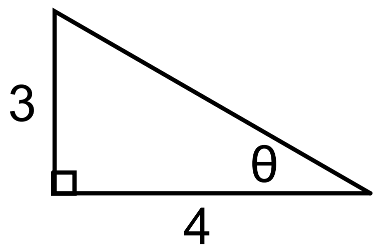
$$\sin A = \frac{opp}{hyp}$$

$$\sin 30^\circ = \frac{a}{6}$$

$$6 \cdot \sin 30^\circ = a$$

$$3 = a$$

Determine the indicated angle



- Name the sides

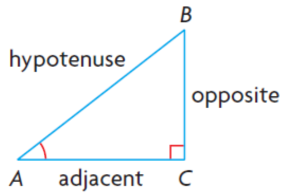
- Set up the ratio

- Solve

$$\tan \theta = \frac{opp}{adj}$$

$$\tan \theta = \frac{3}{4}$$

$$\theta = 37^\circ$$



$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc\theta = \frac{1}{\sin\theta} = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec\theta = \frac{1}{\cos\theta} = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan A = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot\theta = \frac{1}{\tan\theta} = \frac{\text{adjacent}}{\text{opposite}}$$

cosecant

$$\csc x = \frac{1}{\sin x} = \frac{hyp}{opp}$$

secant

$$\sec x = \frac{1}{\cos x} = \frac{hyp}{adj}$$

cotangent

$$\cot x = \frac{1}{\tan x} = \frac{adj}{opp}$$

Trigonometry was invented in Ancient Times and there were no calculators.
So they created Trig Tables and used slide rules for multiplication.

Table II TRIGONOMETRIC RATIOS

θ°	sin θ	cos θ	tan θ	cot θ	sec θ	cosec θ
0	0.0000	1.0000	0.0000	—	1.0000	—
1	0.0175	0.9999	0.0175	57.290	1.0001	57.299
2	0.0349	0.9994	0.0349	28.636	1.0006	28.654
3	0.0523	0.9986	0.0524	19.081	1.0014	19.107
4	0.0698	0.9976	0.0699	14.301	1.0024	14.335
5	0.0872	0.9962	0.0875	11.430	1.0038	11.474
6	0.1045	0.9945	0.1051	9.5144	1.0055	9.5668
7	0.1219	0.9926	0.1228	8.1443	1.0075	8.2055
8	0.1392	0.9903	0.1405	7.1154	1.0098	7.1853
9	0.1564	0.9877	0.1584	6.3137	1.0125	6.3924
10	0.1737	0.9848	0.1763	5.6713	1.0154	5.7588
11	0.1908	0.9816	0.1944	5.1445	1.0187	5.2408
12	0.2079	0.9782	0.2126	4.7046	1.0223	4.8097
13	0.2250	0.9744	0.2309	4.3315	1.0263	4.4454
14	0.2419	0.9703	0.2493	4.0108	1.0306	4.1336
15	0.2588	0.9659	0.2680	3.7320	1.0353	3.8637
16	0.2756	0.9613	0.2867	3.4874	1.0403	3.6279
17	0.2924	0.9563	0.3057	3.2708	1.0457	3.4203
18	0.3090	0.9511	0.3249	3.0777	1.0515	3.2361
19	0.3256	0.9455	0.3443	2.9042	1.0576	3.0715
20	0.3420	0.9397	0.3640	2.7475	1.0642	2.9238
21	0.3584	0.9336	0.3839	2.6051	1.0711	2.7904

Find x ... no calculators

0°	sin θ	cos θ	tan θ	cot θ	sec θ	cosec θ
14	0.2419	0.9703	0.2493	4.0108	1.0306	4.1336
15	0.2588	0.9659	0.2680	3.7320	1.0353	3.8637
16	0.2756	0.9613	0.2867	3.4874	1.0403	3.6279
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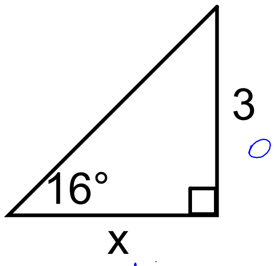


opp

adj

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 16^\circ = \frac{3}{x}$$

$$0.2867 = \frac{3}{x}$$

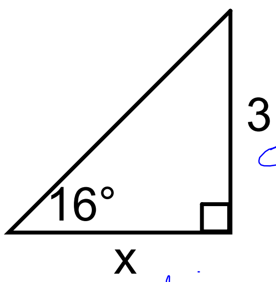
$$0.2867x = 3$$

$$x = \frac{3}{0.2867}$$

Find side x...no calculators

Reciprocal Ratios

0°	sin θ	cos θ	tan θ	cot θ	sec θ	cosec θ
14	0.2419	0.9703	0.2493	4.0108	1.0306	4.1336
15	0.2588	0.9659	0.2680	3.7320	1.0353	3.8637
16	0.2756	0.9613	0.2867	3.4874	1.0403	3.6279
17	0.2924	0.9563	0.3057	3.2708	1.0457	3.4203



opp

adj

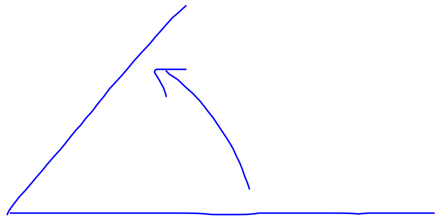
$$\cot\theta = \frac{adj}{opp}$$

$$\cot 16^\circ = \frac{x}{3}$$

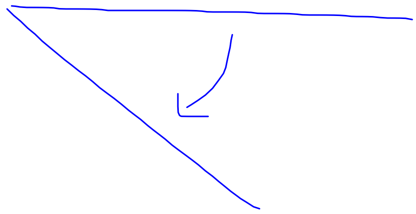
$$3.4874 = \frac{x}{3}$$

$$3(3.4874) = x$$

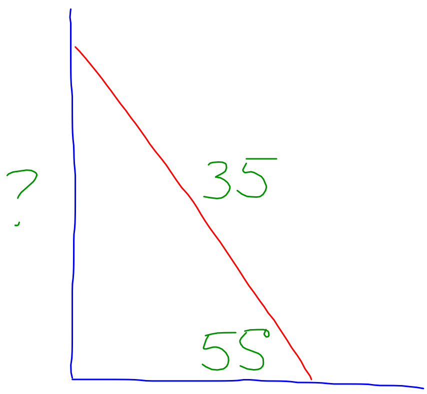
Angle of Elevation



Angle of Depression



A ladder is leaning against a wall with an angle of 55°. If the ladder is 3.5 meters long, can it reach a window 3 meters above the ground?



$$\sin 55^\circ = \frac{x}{3.5}$$

$$2.87 = x$$

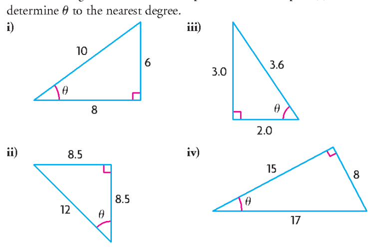
∴ ladder will not reach the window

Seatwork

pg 281 # 7, 8, 10, 11, 17

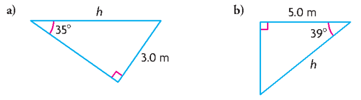
PRACTISING

5. a) For each triangle, calculate csc θ, sec θ, and cot θ.
K b) For each triangle, use one of the reciprocal ratios from part (a) to determine θ to the nearest degree.

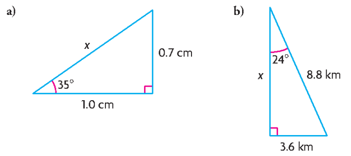


6. Determine the value of θ to the nearest degree.
   a) $\cot \theta = 3.2404$
   b) $\csc \theta = 1.2711$
   c) $\sec \theta = 1.4526$
   d) $\cot \theta = 0.5814$

7. For each triangle, determine the length of the hypotenuse to the nearest tenth of a metre.

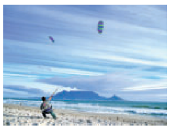


8. For each triangle, use two different methods to determine x to the nearest tenth of a unit.



9. Given any right triangle with an acute angle θ,
   a) explain why csc θ is always greater than or equal to 1
   b) explain why cos θ is always less than or equal to 1

10. Given a right triangle with an acute angle θ, if $\tan\ \theta = \cot\ \theta$, describe what this triangle would look like.



11. **A** A kite is flying 8.6 m above the ground at an angle of elevation of 41°. Calculate the length of string, to the nearest tenth of a metre, needed to fly the kite using
- **a)** a primary trigonometric ratio
- **b)** a reciprocal trigonometric ratio

12. A wheelchair ramp near the door of a building has an incline of 15° and a run of 7.11 m from the door. Calculate the length of the ramp to the nearest hundredth of a metre.

13. **T** The hypotenuse, c, of right $\triangle ABC$ is 7.0 cm long. A trigonometric ratio for angle A is given for four different triangles. Which of these triangles has the greatest area? Justify your decision.
- **a)** $\sec A = 1.7105$
- **c)** $\csc A = 2.2703$
- **b)** $\cos A = 0.7512$
- **d)** $\sin A = 0.1515$

14. The two guy wires supporting an 8.5 m TV antenna each form an angle of 55° with the ground. The wires are attached to the antenna 3.71 m above ground. Using a reciprocal trigonometric ratio, calculate the length of each wire to the nearest tenth of a metre. What assumption did you make?

15. From a position some distance away from the base of a flagpole, Julie estimates that the pole is 5.35 m tall at an angle of elevation of 25°. If Julie is 1.55 m tall, use a reciprocal trigonometric ratio to calculate how far she is from the base of the flagpole, to the nearest hundredth of a metre.

16. The maximum grade (slope) allowed for highways in Ontario is 12%.
- **a)** Predict the angle θ, to the nearest degree, associated with this slope.
- **b)** Calculate the value of θ to the nearest degree.
- **c)** Determine the six trigonometric ratios for angle θ.

17. **C** Organize these terms in a word web, including explanations where appropriate.

sine	cosine	tangent	opposite
cotangent	hypotenuse	cosecant	adjacent
secant	angle of depression	angle	angle of elevation

Extending

18. In right $\triangle PQR$, the hypotenuse, r, is 117 cm and $\tan P = 0.51$. Calculate side lengths p and q to the nearest centimetre and all three interior angles to the nearest degree.

19. Describe the appearance of a triangle that has a secant ratio that is greater than any other trigonometric ratio.

20. The tangent ratio is undefined for angles whose adjacent side is equal to zero. List all the angles between 0° and 90° (if any) for which cosecant, secant, and cotangent are undefined.

5. **a)** **i)** $\csc\theta = \frac{5}{3}$, $\sec\theta = \frac{5}{4}$, $\cot\theta = \frac{4}{3}$

ii) $\csc\theta = \frac{12}{8.5}$, $\sec\theta = \frac{12}{8.5}$, $\cot\theta = 1$

iii) $\csc\theta = \frac{3.6}{3}$, $\sec\theta = \frac{3.6}{2}$, $\cot\theta = \frac{2}{3}$

iv) $\csc\theta = \frac{17}{8}$, $\sec\theta = \frac{17}{15}$, $\cot\theta = \frac{15}{8}$

b) **i)** 37° **ii)** 56° **iii)** 45° **iv)** 28°

6. **a)** 17° **b)** 52° **c)** 46° **d)** 60°

7. **a)** 5.2 m **b)** 6.4 m

8. **a)** 1.2 cm **b)** 8.0 km

9. **a)** For any right triangle with acute angle θ, $\csc\theta = \frac{\text{hypotenuse}}{\text{opposite}}$.
Case 1: If neither the adjacent side nor the opposite side is zero, the hypotenuse is always greater than either side and $\csc\theta > 1$.
Case 2: If the adjacent side is reduced to zero, each time you calculate $\csc\theta$, you get a smaller and smaller value until $\csc\theta = 1$.
Case 3: If the opposite side is reduced to zero, each time you calculate $\csc\theta$, you get a greater and greater value until you reach infinity.
So for all possible cases in a right triangle, cosecant is always greater than or equal to 1.

b) For any right triangle with acute angle θ, $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$.
Case 1: If neither the adjacent side nor the opposite side is zero, the hypotenuse is always greater than either side and $\cos\theta < 1$.
Case 2: If the opposite side is reduced to zero, each time you calculate $\cos\theta$, you get a greater and greater value until $\cos\theta = 1$.
Case 3: If the adjacent side is reduced to zero, each time you calculate $\cos\theta$, you get a smaller and smaller value until $\cos\theta = 0$.
So for all possible cases in a right triangle, cosine is always less than or equal to 1.

10. $\theta = 45°$ and adjacent side = opposite side

11. **a)** and **b)** 13.1 m

12. 7.36 m

13. (b) a right triangle with two 45° angles would have the greatest area, at an angle of 41°, (b) is closest to 45° and will therefore have the greatest area of those triangles.

14. 4.5 m

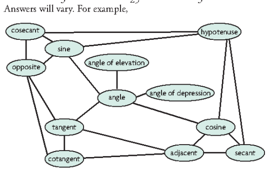
15. 8.15 m

16. **a)** Answers will vary. For example, 10° **b)** 7°

c) $\sin\theta = \frac{3}{\sqrt{634}}$, $\cos\theta = \frac{25}{\sqrt{634}}$, $\tan\theta = \frac{3}{25}$,

$\csc\theta = \frac{\sqrt{634}}{3}$, $\sec\theta = \frac{\sqrt{634}}{25}$, $\cot\theta = \frac{25}{3}$

17. Answers will vary. For example,



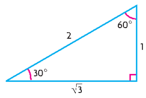
18. $p = 53$ cm, $q = 104$ cm, $\angle P = 27°$, $\angle Q = 63°$

19. Since $\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent}}$, the adjacent side must be the smallest side.

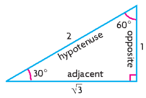
20. (csc and cot) 0°, (sec) 90°

Lesson 5.2, pp. 286–288

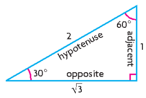
1. **a)**



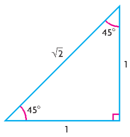
b)



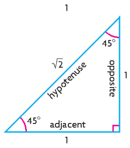
c)



2. **a)**



b)



3. **a)** $\frac{\sqrt{3}}{2}$ **b)** $\frac{\sqrt{3}}{2}$ **c)** 1 **d)** $\frac{\sqrt{2}}{2}$

4. **a)** 0 **b)** 1 **c)** $-\frac{1}{6}$ **d)** 0

5. **a)** $\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = 1$ **c)** $\left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2 = 1$

b) $\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2 = 1$