Domain and Range

Learning Goals

- better understanding of set notation
- finding domain and range in a variety of ways

Set Notation

Number Types:

1, 2, 3, 4, … **Natural** $\mathbb{N}$

0, 1, 2, 3, 4, … **Whole** $\mathbb{W}$

…, -4, -3, -2, -1, 0, 1, 2, 3, 4, … **Integers** $\mathbb{I}$

$\frac{1}{4} = 0.25$, $\frac{7}{3} = 2.33333\ldots$, 5 since $\frac{5}{1}$ **Rational** $\mathbb{Q}$

$\sqrt{2}$, π (numbers that can't be expressed as fractions) **Irrational** $\overline{\mathbb{Q}}$

$\sqrt{-4} \Rightarrow$ non-real imaginary (i)

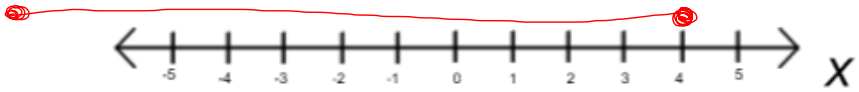
Set Notation for Domain and Range

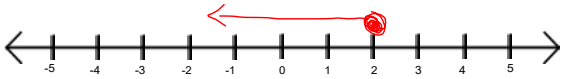
element of — Real

$$D = \{ x \in \mathbb{R} \mid -8 \le x \le 4 \}$$

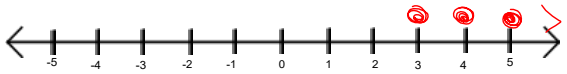
such that

What does this represent?

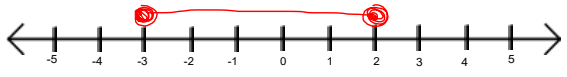




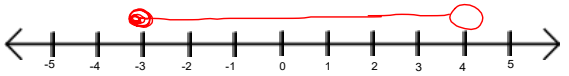
$x \le 2$ $x \in \mathbb{R}$



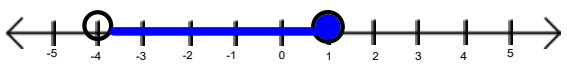
$x > 2$ $x \in \mathbb{W}$



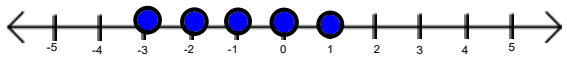
$-3 \le x \le 2$ $x \in \mathbb{R}$



$-3 \le x < 4$ $x \in \mathbb{R}$



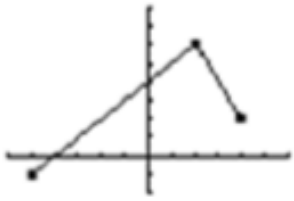
$-4 < x \le 1$ $x \in \mathbb{R}$



$-3 \le x \le 1$ $x \in \mathbb{I}$

Continuous Graph

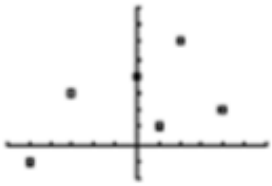
- A graph can be drawn without lifting your pencil from the paper



Real = R

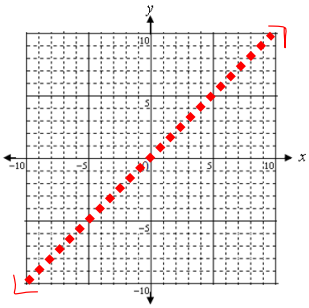
Discrete Graph

- A graph that is defined only for a set of numbers that can be listed, such as integers

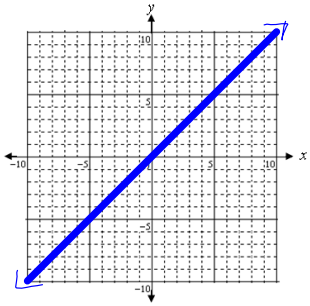


Integers = I

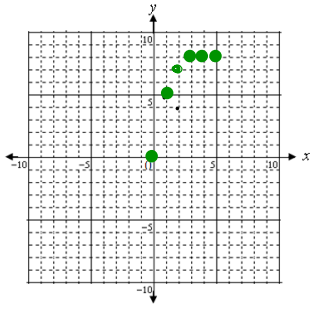
How can we use Domain and Range to differentiate these two Relations?



$D = \{x \in \mathbb{I}\}$



$D = \{x \in \mathbb{R}\}$



Two ways to write Domain when it is integers

1 - Just list the values ,

$D = \{0, 1, 2, 3, 4, 5\}$

2 - Write an inequality statement

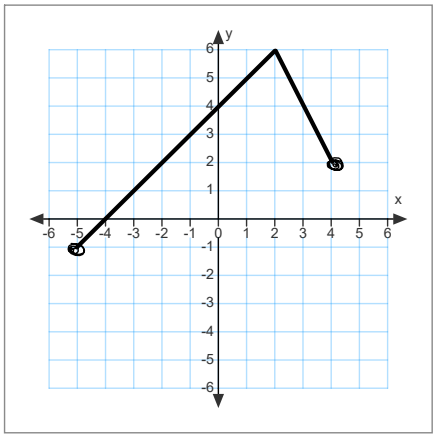
$D = \{x \in W \mid 0 \leq x \leq 5\}$

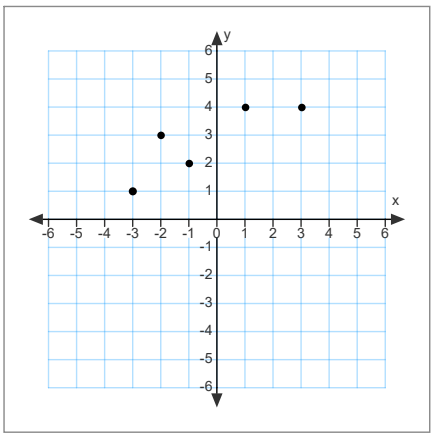
$\{(1,3),(2,4),(3,5),(4,6),(5,7),(6,8),(7,9),(8,10)\}$

$D = \{x \in \mathbb{W} \mid 1 \leq x \leq 8\}$

$R = \{y \in \mathbb{N} \mid 3 \leq y \leq 10\}$

Determine the domain and range in set notation.





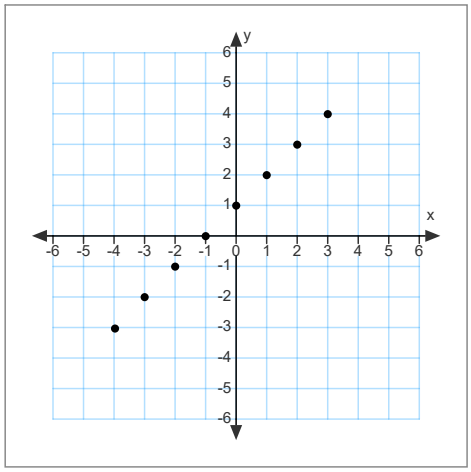
$D = \{x \in \mathbb{R} \mid -5 \leq x \leq 4\}$

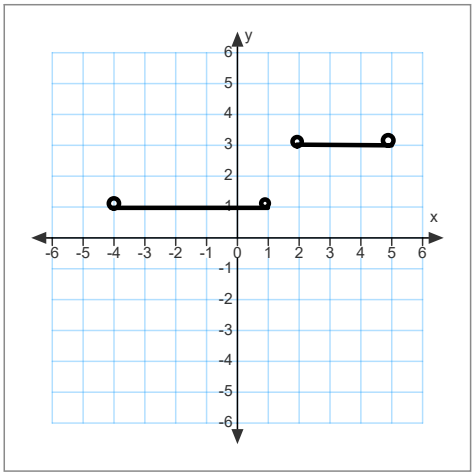
$R = \{y \in \mathbb{R} \mid -1 \leq y \leq 6\}$

$D = \{-3, -2, -1, 1, 3\}$

$R = \{y \in \mathbb{I} \mid 1 \leq y \leq 4\}$

Determine the domain and range in set notation.





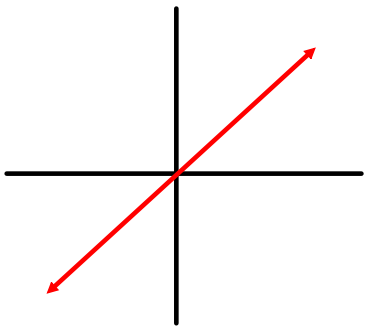
$D = \{x \in \mathbb{I} \mid -4 \leq x \leq 3\}$

$R = \{y \in \mathbb{I} \mid -3 \leq y \leq 4\}$

$D = \{x \in \mathbb{R} \mid -4 < x < 1, \; 2 < x < 5\}$

$R = \{1, 3\}$

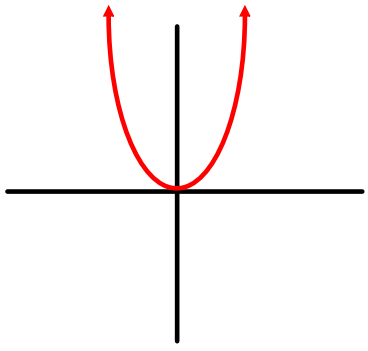
Let's go back to our parent functions...day 5



Straight Line

Domain $\{x \in \mathbb{R}\}$

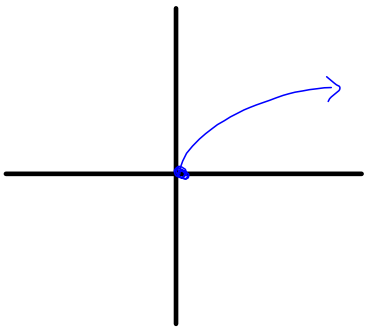
Range $\{y \in \mathbb{R}\}$



Quadratic

Domain $\{x \in \mathbb{R}\}$

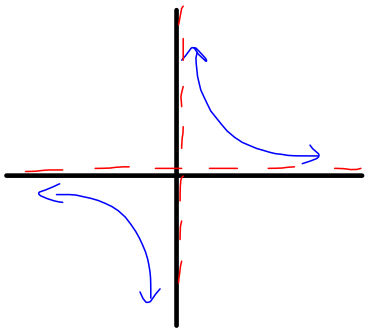
Range $\{x \in \mathbb{R} \mid y \geq 0\}$



Square root

Domain $= \{ x \in \mathbb{R} \mid x \geq 0 \}$

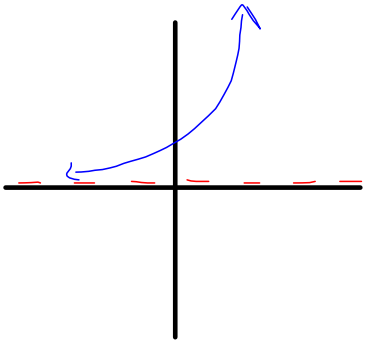
Range $= \{ y \in \mathbb{R} \mid y \geq 0 \}$



Reciprocal

Domain $= \{ x \in \mathbb{R} \mid x \neq 0 \}$

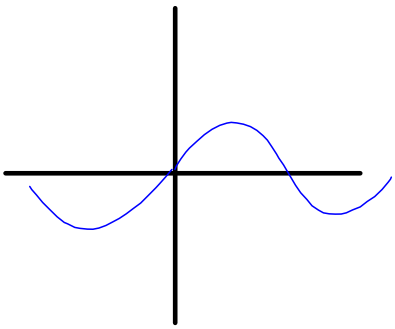
Range $= \{ y \in \mathbb{R} \mid y \neq 0 \}$



Exponential

Domain $= \{ x \in \mathbb{R} \}$

Range $= \{ y \in \mathbb{R} \mid y > 0 \}$



Sinusoidal

Domain $= \{ x \in \mathbb{R} \}$

Range $= \{ y \in \mathbb{R} \mid -1 \leq y \leq 1 \}$

On the Boards...

1. $f(x) = -3(x+1)^2 + 6$

2. $f(x) = \sqrt{x+4}$

3. $f(x) = \frac{1}{x-2}$

4. $f(x) = \sin(x) - 3$

5. $g(x) = 2^x + 3$

Real World Context Using a Linear Function

You are planning a graduation party.
The hall costs $500, and each meal costs $50.
The hall can fit 100 people.

a) Use function notation to write an equation for this situation.
b) Determine the domain and range.

Real World Context Using a Quadratic Function

Indiana Jones is standing on a cliff and he shoots an arrow up into the air; the arrow falls to the ground.
The flight of the arrow is defined by $f(x) = -5(x-1)^2 + 20$.

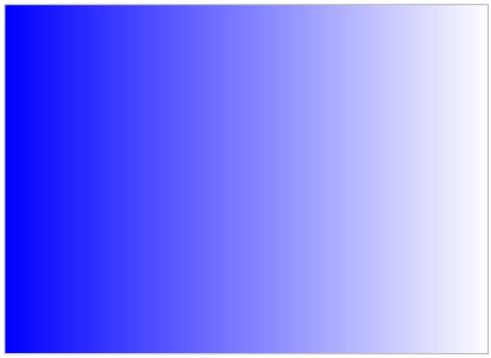
Determine the domain and range.

On the Boards...

Find Domain and Range

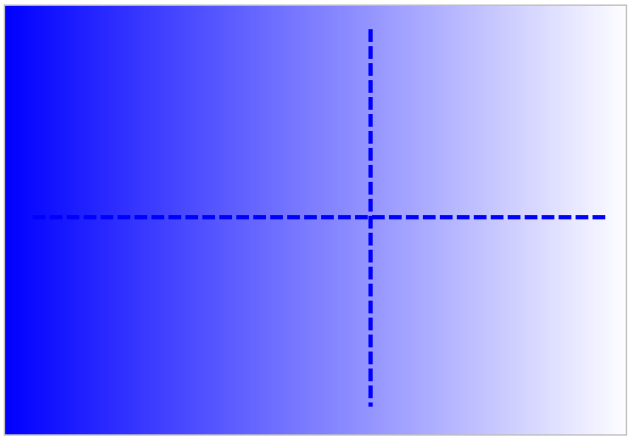
- sketch to help you

$$f(x) = -3(x+1)^2 + 6$$

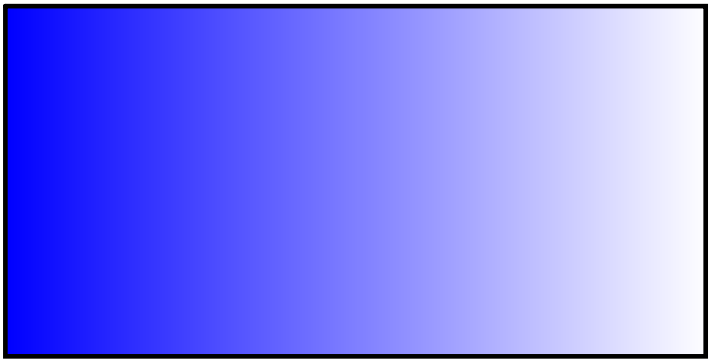


$$f(x) = \sqrt{x+4}$$

$$f(x) = \frac{1}{x-2}$$

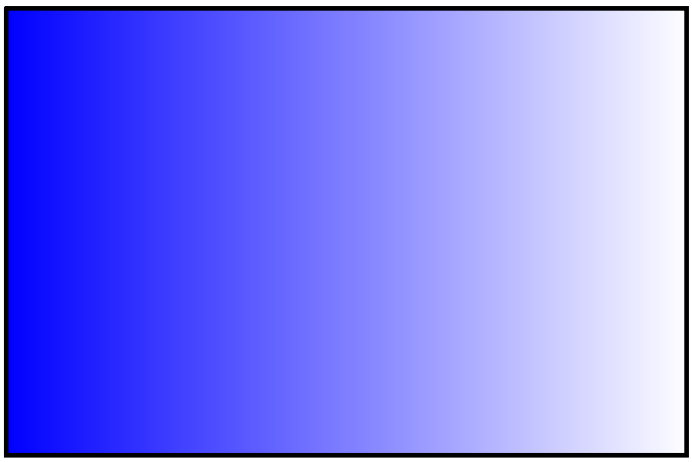


$$f(x) = \sin(x) - 3$$



$g(x) = 2^x + 3$

Where is the asymptote going to be?



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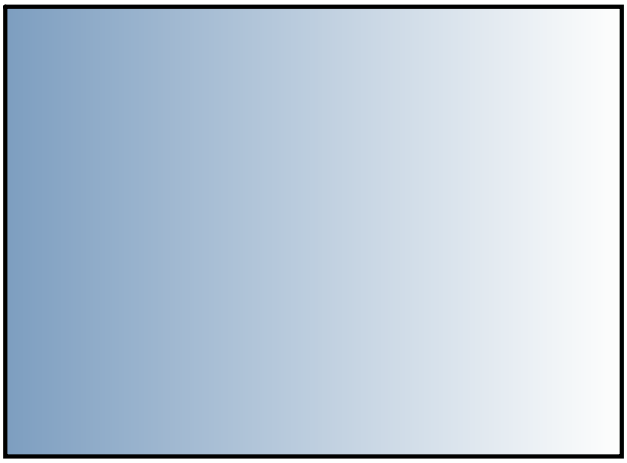

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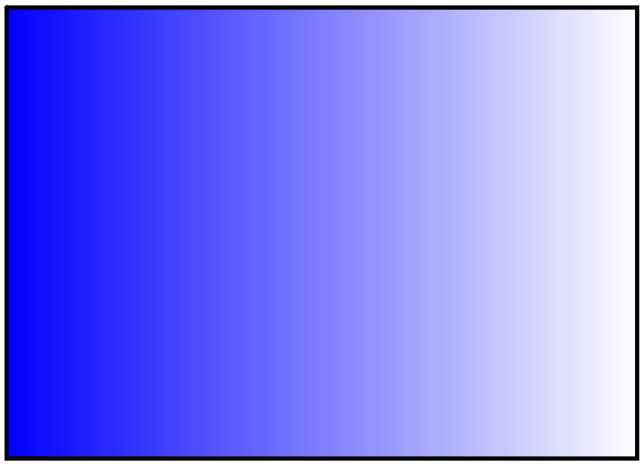
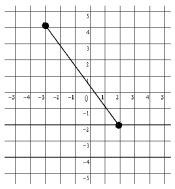
Determine the domain and range.



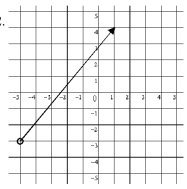
Homework

MCR3U Domain and Range

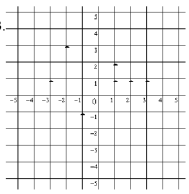
State the Domain and the Range for each of the following graphs, using set notation.
Then circle to indicate if the graph represents a function.

1. 

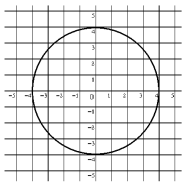
D = _____
R = _____
Function Yes No

2. 

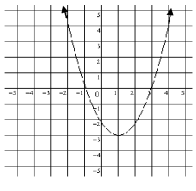
D = _____
R = _____
Function Yes No

3. 

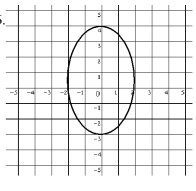
D = _____
R = _____
Function Yes No

4. 

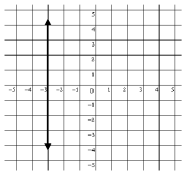
D = _____
R = _____
Function Yes No

5. 

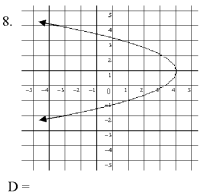
D = _____
R = _____
Function Yes No

6. 

D = _____
R = _____
Function Yes No

7. 

D = _____
R = _____
Function Yes No

8. 

D = _____
R = _____
Function Yes No

D = _____
R = _____
Function Yes No

MCR 3U **Domain and Range of Transformed Functions**

For each of the following functions,

1. Identify the Parent Function
2. Graph on TI-Nspire.
3. Determine the Domain and Range.

a) $f(x) = 4x - 5$ _____

b) $g(x) = (x+2)^2 - 5$ _____

c) $h(x) = -2(x+9)^2$ _____

d) $m(x) = \sqrt{x+3}$ _____

e) $n(x) = -2\sqrt{x-9} + 3$ _____

f) $p(x) = \dfrac{1}{x+9}$ _____

g) $q(x) = \dfrac{1}{x-5} + 2$ _____

h) $r(x) = \dfrac{1}{2x+8} - 5$ _____

i) $s(x) = 2\sin x - 10$ _____

j) $t(x) = 2\sqrt{x-7} + 3$ _____

k) $v(x) = 5 \times 2^x + 1$ _____

l) $w(x) = -2(x-4)^2 + 5$ _____

m) $b(x) = 3\sin 4x + 1$ _____

n) $c(x) = 2^x - 4$ _____

o) $d(x) = 2^{x-5} + 7$ _____

p) $f(x) = 3\sin(x-45) - 1$ _____

Domain and Range – SOLUTIONS

a.	$f(x) = 4x - 5$	$D = \{x \in \mathbb{R}\}$	$R = \{y \in \mathbb{R}\}$
b.	$g(x) = (x+2)^2 - 5$	$D = \{x \in \mathbb{R}\}$	$R = \{y \in \mathbb{R} \mid y \geq -5\}$
c.	$h(x) = -2(x+9)^2$	$D = \{x \in \mathbb{R}\}$	$R = \{y \in \mathbb{R} \mid y \leq 0\}$
d.	$m(x) = \sqrt{x+3}$	$D = \{x \in \mathbb{R} \mid x \geq -3\}$	$R = \{y \in \mathbb{R} \mid y \geq 0\}$
e.	$n(x) = -2\sqrt{x-9} + 3$	$D = \{x \in \mathbb{R} \mid x \geq 9\}$	$R = \{y \in \mathbb{R} \mid y \leq 3\}$
f.	$p(x) = \dfrac{1}{x+9}$	$D = \{x \in \mathbb{R} \mid x \neq -9\}$	$R = \{y \in \mathbb{R} \mid y \neq 0\}$
g.	$q(x) = \dfrac{1}{x-5} + 2$	$D = \{x \in \mathbb{R} \mid x \neq 5\}$	$R = \{y \in \mathbb{R} \mid y \neq 2\}$
h.	$r(x) = \dfrac{1}{2x+8} - 5$	$D = \{x \in \mathbb{R} \mid x \neq -4\}$	$R = \{y \in \mathbb{R} \mid y \neq -5\}$
i.	$s(x) = 2\sin x - 10$	$D = \{x \in \mathbb{R}\}$	$R = \{y \in \mathbb{R} \mid -12 \leq y \leq -8\}$
j.	$t(x) = 2\sqrt{x-7} + 3$	$D = \{x \in \mathbb{R} \mid x > 7\}$	$R = \{y \in \mathbb{R} \mid y \geq 3\}$
k.	$v(x) = 5 \times 2^x + 1$	$D = \{x \in \mathbb{R}\}$	$R = \{y \in \mathbb{R} \mid y > 1\}$
l.	$w(x) = -2(x-4)^2 + 5$	$D = \{x \in \mathbb{R}\}$	$R = \{y \in \mathbb{R} \mid y \leq 5\}$
m.	$b(x) = 3\sin 4x + 1$	$D = \{x \in \mathbb{R}\}$	$R = \{y \in \mathbb{R} \mid -2 \leq y \leq 4\}$
n.	$c(x) = 2^x - 4$	$D = \{x \in \mathbb{R}\}$	$R = \{y \in \mathbb{R} \mid y > -4\}$
o.	$d(x) = 2^{x-5} + 7$	$D = \{x \in \mathbb{R}\}$	$R = \{y \in \mathbb{R} \mid y > 7\}$
p.	$f(x) = 2\sin(x-45) - 1$	$D = \{x \in \mathbb{R}\}$	$R = \{y \in \mathbb{R} \mid -3 \leq y \leq 1\}$

Seatwork

pg 12 # 13, 15

pg 35 # 2, 5, 10, 15

pg 12

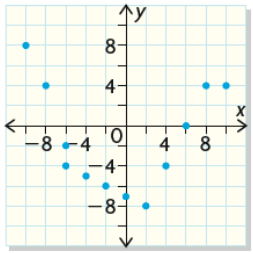
13. T
a) Sketch a graph of a function that has the set of integers as its domain and all integers less than 5 as its range.
b) Sketch a graph of a relation that is not a function and that has the set of real numbers less than or equal to 10 as its domain and all real numbers greater than -5 as its range.

15. A freight delivery company charges \$4/kg for any order less than 100 kg and \$3.50/kg for any order of at least 100 kg.
a) Why must this relation be a function?
b) What is the domain of this function? What is its range?
c) Graph the function.
d) What suggestions can you offer to the company for a better pricing structure? Support your answer.

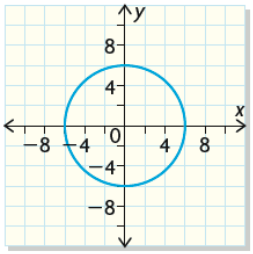
pg 35

2. State the domain and range of each relation.

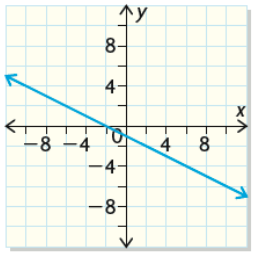
a)



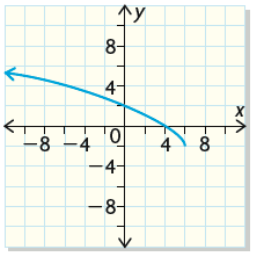
d)



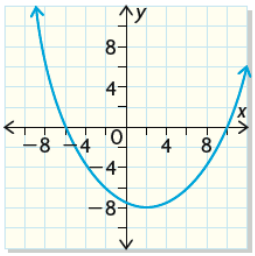
b)



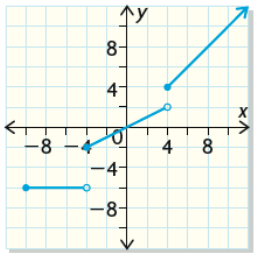
e)



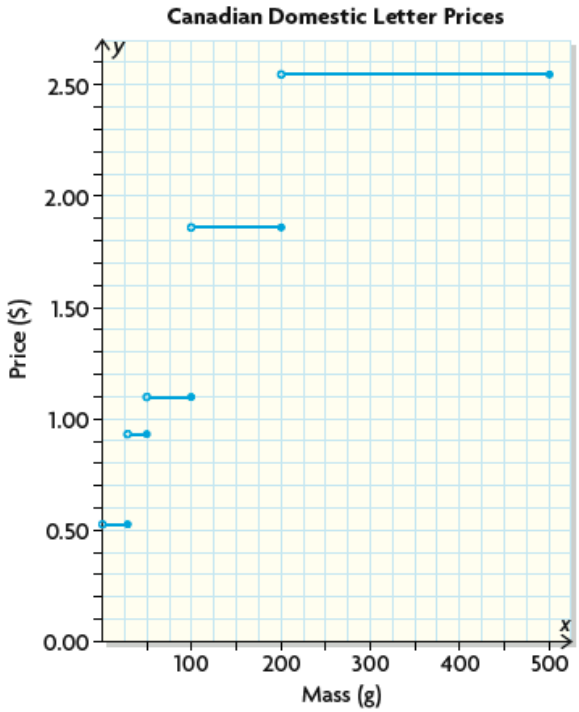
c)



f)



5. The graph shows how 2007 prices for mailing letters in Canada vary with mass.



a) Explain why this relation is a function. Why is it important for this to be so?

b) State the domain and range of the function.

10. A ball is thrown upward from the roof of a 25 m building. The ball reaches a height of 45 m above the ground after 2 s and hits the ground 5 s after being thrown.

A

a) Sketch a graph that shows the height of the ball as a function of time.

b) State the domain and range of the function.

c) Determine an equation for the function.

Answers

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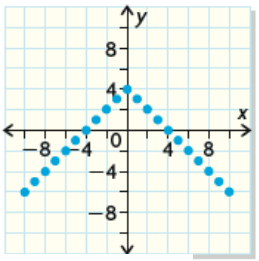
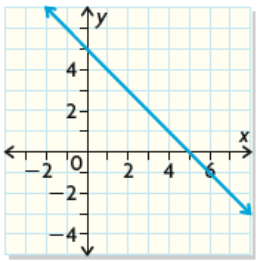
13. **a)** Answers may vary; for example:



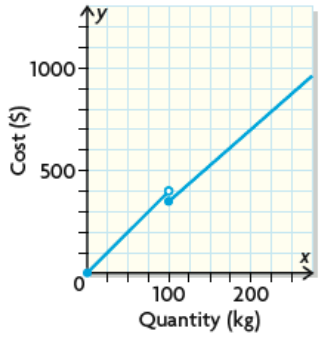
b) Answers may vary; for example:



15. **a)** Each order quantity determines a single cost.

b) domain $= \{x \in \mathbf{R} \mid x \geq 0\}$, range $= \{y \in \mathbf{R} \mid y \geq 0\}$

c)



d) Answers may vary. For example, the company currently charges more for an order of 100 kg ($350) than for an order of 99 kg ($396). A better system would be for the company to charge $50 plus $3.50 per kilogram for orders of 100 kg or more. This would make the prices strictly increasing as the weight of the order increases.

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2. **a)** domain $= \{0, \pm 2, \pm 4, \pm 6, \pm 8, \pm 10\}$,
range $= \{-8, -7, -6, -5, -4, -2, 0, 4, 8\}$

b) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R}\}$

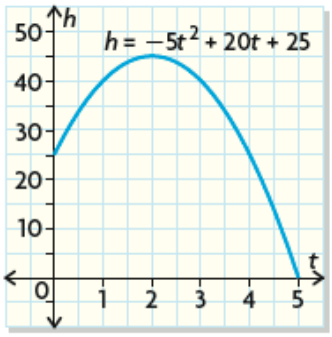
c) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \geq -8\}$

d) domain $= \{x \in \mathbf{R} \mid -6 \leq x \leq 6\}$,
range $= \{y \in \mathbf{R} \mid -6 \leq y \leq 6\}$

5. **a)** Even at masses when the price changes, a single price (the lower one) is assigned. It would not make sense to assign two or more prices to the same mass.

b) domain $= \{x \in \mathbf{R} \mid 0 < x \leq 500\}$,
range $= \{0.52, 0.93, 1.20, 1.86, 2.55\}$

10. **a)**



b) domain $= \{t \in \mathbf{R} \mid 0 \leq t \leq 5\}$, range $= \{h \in \mathbf{R} \mid 0 \leq h \leq 45\}$

c) $h = -5t^2 + 20t + 25$