Properties of Parent Functions II

Take out your **homework**

Go to the Boards...

Tell me everything about the parabola

$$y = 2x^2 + 8x - 10$$

and sketch

MCR3U *3.1 Properties of Quadratic Functions*

Applications of Quadratic Functions – The Flight of the Golf Ball

Mike Weir hits a golf ball upwards from the top of a cliff.
The height of the ball above the base of the cliff is modelled by $h(t) = -6t^2 + 24t + 72$,
where h is height in metres and t is the time in seconds.

a) Draw a **picture** of Mike, the cliff, and the flight of the golf ball.

As you answer the following questions add the measurements to your picture.

b) How high is the cliff? 72 m

b) When will the ball hit the ground?

c) When will the ball reach its' maximum height?

d) What is the maximum height of the ball?

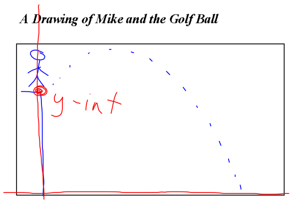
e) Graph the function.

f) Determine the Domain that describes the flight of the ball.

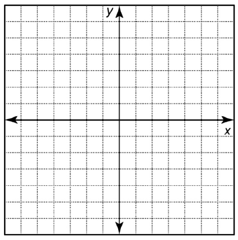
g) Determine the Range that describes the flight of the ball.

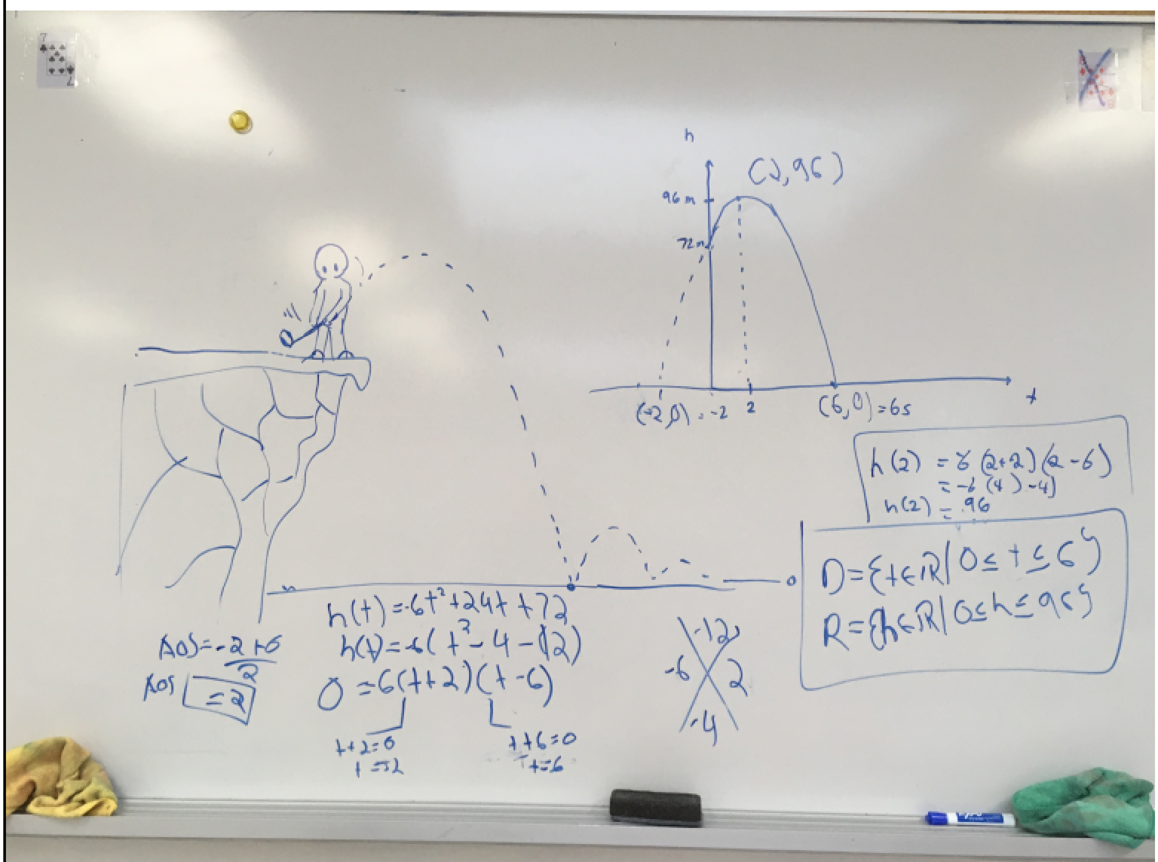
h) When will the ball reach a height of 42 m?

A Drawing of Mike and the Golf Ball



A Graph of the Flight of the Golf Ball





MCR3U 3.1 Properties of Quadratic Functions

Applications of Quadratic Functions – The Flight of the Golf Ball

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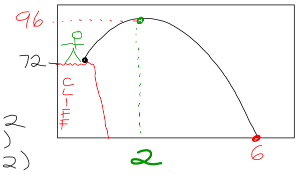


As you answer the following questions add the measurements to your picture.

b) How high is the cliff? $h(0)=72$

b) When will the ball hit the ground?

when $h(t)=0$

$0 = -6t^2 + 24t + 72$

$0 = -6(t^2 - 4t - 12)$

$0 = -6(t-6)(t+2)$

$t = 6$ oR $t = -2$

c) When will the ball reach its' maximum height?

x value of Vertex

$AOS = \frac{6+(-2)}{2}$

$= 2$

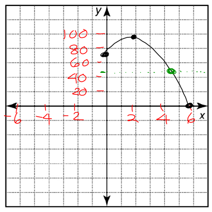
d) What is the maximum height of the ball?

y value of Vertex

$f(2) = -6(2)^2 + 24(2) + 72$

$= 96$

A Graph of the Flight of the Golf Ball



e) Graph the function.

f) Determine the Domain that describes the flight of the ball.

$D = \{ t \in R \mid 0 \le t \le 6 \}$

g) Determine the Range that describes the flight of the ball.

$R = \{ h \in R \mid 0 \le h \le 96 \}$

h) When will the ball reach a height of 42 m?

$42 = -6t^2 + 24t + 72$

$0 = -6t^2 + 24t + 30$

$0 = -6(t^2 - 4t - 5)$

$0 = -6(t-5)(t+1)$

$t = 5$ $t = -1$

$\therefore$ @ 5 seconds

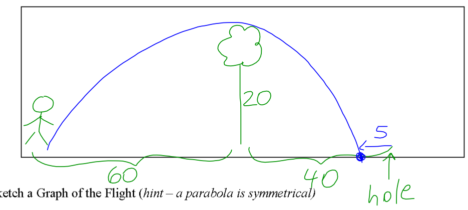
MCR3U 3.1 Properties of Quadratic Functions

Application of Quadratic Functions – The Flight of the Golf Ball – Part Two

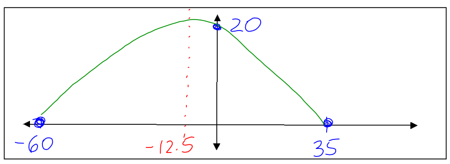
Brooke Henderson has a difficult golf shot to make. Her ball is 100 m from the hole. She wants the ball to land 5 m in front of the hole, so it can roll to the hole. A 20 m tree is between her ball and the hole, 60 m from Brooke's ball and 40 m from the hole. With the base of the tree as the origin, write an algebraic expression to model the height of the ball if it just clears the top of the tree.

a) Draw a picture of Brooke and her ball, the tree, and the hole and mark all given measurements.

A Drawing of Brooke and her Golf Ball



b) Sketch a Graph of the Flight (*hint – a parabola is symmetrical*)



c) Develop an algebraic model for the flight.

d) Check your model by graphing on Nspire and identifying the critical points of the flight.

e) Determine the Domain and Range that describes the flight of the ball.

MCR3U *3.1 Properties of Quadratic Functions*

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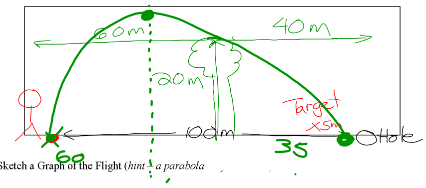
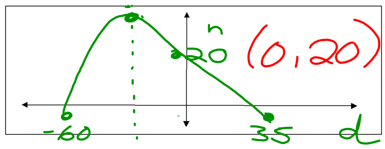
A Drawing of Brooke and her Golf Ball



b) Sketch a Graph of the Flight (*hint – a parabola*



c) Develop an algebraic model for the flight.

$f(x) = a(x-s)(x-t)$

$f(x) = a(x-35)(x+60)$

$20 = a(0-35)(0+60)$

$20 = a(-2100)$

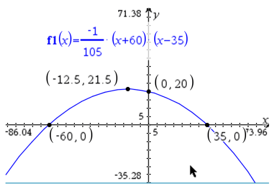
$\frac{-20}{2100} = \frac{2100a}{2100}$

$-\frac{1}{105} = a$

$f(x) = -\frac{1}{105}(x-35)(x+60)$

d) Check your model by graphing on Nspire and identifying the critical points of the flight.

e) Determine the Domain and Range that describes the flight of the ball.



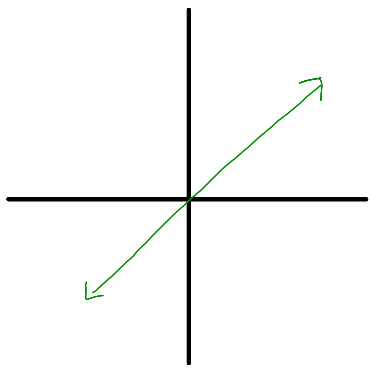
$D = \{x \in R \mid -60 \le x \le 35\}$

$R = \{y \in R \mid 0 \le y \le 21.5\}$

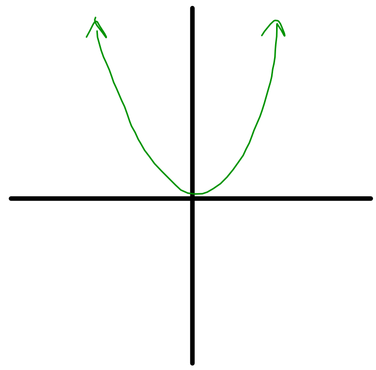
y=x -- linear

Special Features/Symmetry

- Straight line that goes through the origin
- Slope is 1
- Divides the plane diagonally, graph is only in quadrants I and III.



$y=x^2$ -- quadratic



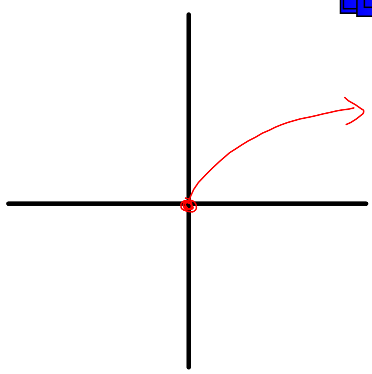
Special Features/Symmetry

- Parabola that opens up
- Vertex at the origin
- Has a minimum value (y=0)
- Axis of symmetry at x=0
- Graph in Quadrants I and II

$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R} \mid y \geq 0\}$

$y = \sqrt{x}$ -- square root



Special Features/Symmetry

- Half "Parabola" that opens right
- "Vertex" at the origin
- x and y have minimum values
- No symmetry
- Graph in Quadrant I

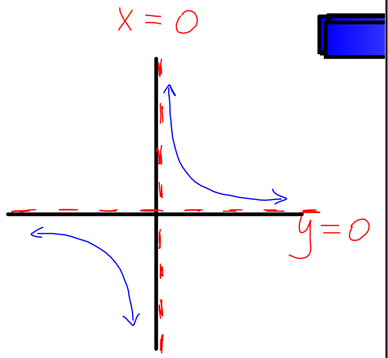
$D = \{x \in \mathbb{R} \mid x \geq 0\}$

$R = \{y \in \mathbb{R} \mid y \geq 0\}$

$y = \frac{1}{x}$ -- reciprocal

Special Features/Symmetry

- Hyperbola
- Does not intersect x or y axis
- No minimum or maximum value
- Graph in Quadrant I and III
- Has 2 asymptotes



Asymptote ?

An imaginary line that the graph will never touch or cross.

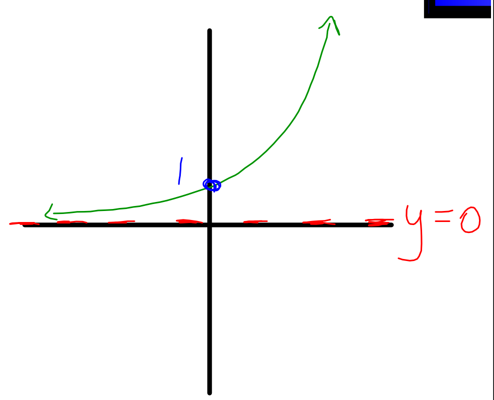
$D = \{x \in \mathbb{R} \mid x \neq 0\}$

$R = \{y \in \mathbb{R} \mid y \neq 0\}$

$y = 2^x$ -- exponential

Special Features/Symmetry

- Does not intersect x-axis
- No minimum or maximum value
- Graph in Quadrant I and II
- Has 1 asymptote



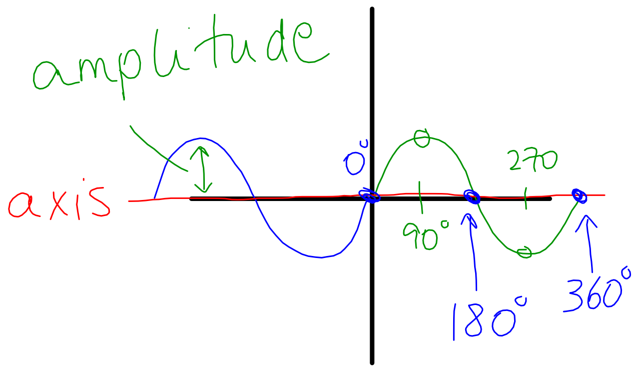
$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R} \mid y > 0\}$

y= sin x -- sine curve (sinusoidal)

Special Features/Symmetry

- periodic – repeats
- amplitude
- axis



Sinusoidal Function

- A function that has a general shape of a sine curve

1. What is the maximum value of sin x? 1

   When does this occur? 90°

2. What is the minimum value of sin x? −1

   When does this occur? 270°

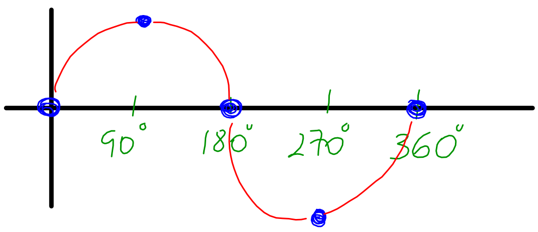
3. What are the coordinates of the x-intercepts? 0, 180°, 360°
4. What is the period of the graph? 360°
5. What is the amplitude? 1
6. What is the equation of the axis? $y = 0$
7. What is the domain? $D = \{x \in \mathbb{R}\}$
8. What is the range? $R = \{y \in \mathbb{R} \mid -1 \leq y \leq 1\}$

Summary

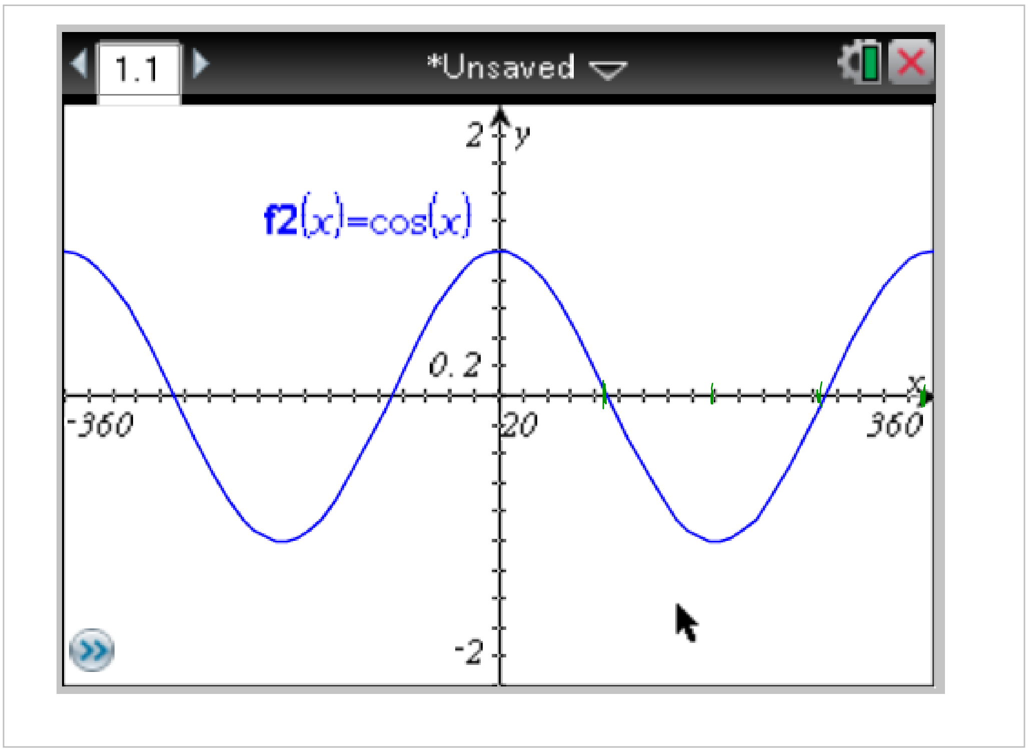
Properties

- amplitude is 1
- period is 360°
- axis of the curve is $y = 0$
- Domain $x \in \mathbb{R}$
- Range $-1 \leq y \leq 1$

5 key points



Let's see what the Cosine curve looks like?



1. What is the maximum value of cos x? 1

   When does this occur? $0°$, $360°$

2. What is the minimum value of cos x? -1

   When does this occur? $180°$

3. What are the coordinates of the x-intercepts? $90°$, $270°$

4. What is the period of the graph? $360°$

5. What is the amplitude? 1

6. What is the equation of the axis? $y = 0$

7. What is the domain? $x \in \mathbb{R}$

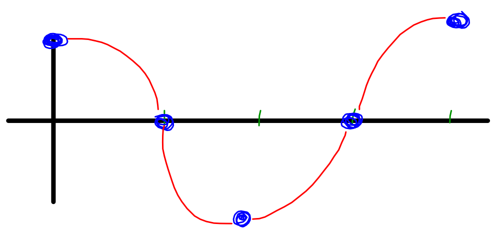
8. What is the range? $-1 \leq y \leq 1$

Summary

Properties

- amplitude is 1
- period is $360°$
- axis of the curve is $y = 0$
- Domain $x \in \mathbb{R}$
- Range $-1 \leq y \leq 1$

5 key points



Homework

Use TI-Nspire for Pg. 363

be sure to save your graphs

Pg. 363 # 5, 6, 8

Pg. 243 # 1, 2

pg. 363

PRACTISING

5. Using a graphing calculator and the WINDOW settings shown, graph each function. Use DEGREE mode. State whether the resulting functions are periodic. If so, state whether they are sinusoidal.
   a) $y = 3 \sin x + 1$
   b) $y = (0.004x)\sin x$
   c) $y = \cos(2x) - \sin x$
   d) $y = 0.005x + \sin x$
   e) $y = 0.5 \cos x - 1$
   f) $y = \sin 90°$

6. Based on your observations in question 5, what can you conclude about any function that possesses sine or cosine in its equation?

7. If $g(x) = \sin x$ and $h(x) = \cos x$, where $0° \leq x \leq 360°$, calculate each and explain what it means.
   a) $g(90°)$
   b) $h(90°)$

8. K Using a graphing calculator in DEGREE mode, graph each sinusoidal function. Use the WINDOW settings shown. From the graph, state the amplitude, period, increasing intervals, decreasing intervals, and equation of the axis for each.
   a) $y = 2 \sin x + 3$
   b) $y = 3 \sin x + 1$
   c) $y = \sin(0.5x) + 2$
   d) $y = \sin(2x) - 1$
   e) $y = 2 \sin(0.25x)$
   f) $y = 3 \sin(0.5x) + 2$

FURTHER Your Understanding

pg. 243

1. Use differences to identify the type of function represented by the table of values.

a)

x	y
−4	5
−3	8
−2	13
−1	20
0	29
1	40

b)

x	y
−5	32
−4	16
−3	8
−2	4
−1	2
0	1

c)

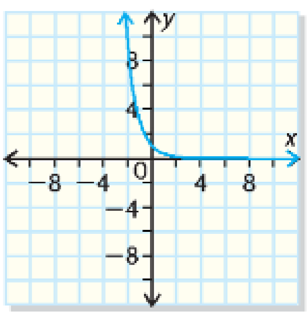
x	y
−2	−2.75
0	−2
2	1
4	13
6	61
8	253

d)

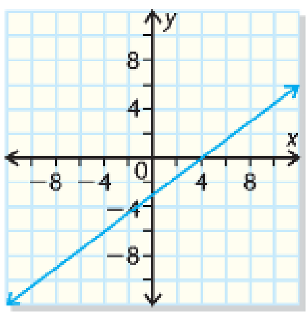
x	y
0.5	0.9
0.75	1.1
1	1.3
1.25	1.5
1.5	1.7
1.75	1.9

2. What type of function is represented in each graph? Explain how you know.

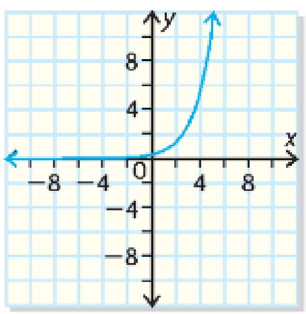
a)



c)



b)



d)