Properties of Quadratics

Learning Goals

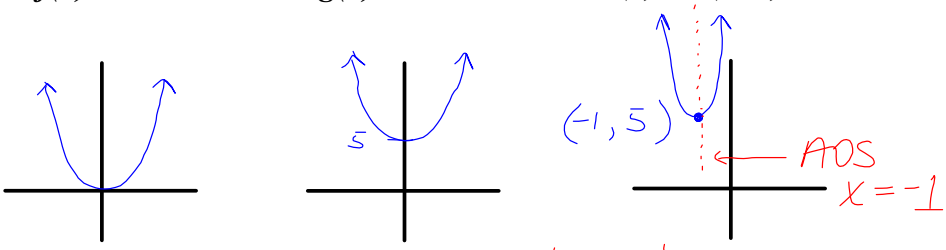
- review of quadratics from grade 10
- three forms of the function
- graphing quadratic functions
- table of values - how do you know it's quadratic?

Quadratic function - has a degree of 2

↘ the highest exponent of x is 2

Graph

$f(x) = x^2$ $g(x) = x^2 + 5$ $h(x) = 2(x+1)^2 + 5$



element ↓ ∈ ℝ ← Real

- **parabola**
- domain → x values → $x \in \mathbb{R}$
- symmetrical around the **axis of symmetry**
- important points:

↓ y of vertex

vertex, x-intercepts (zeros), y-intercept, optimum value

step pattern → must have a vertex

transformations

Equation

- **standard form:** $f(x) = ax^2 + bx + c$

direction of opening
v. reflection
v. Stretch
v. Compression
y-int

- **factored form:** $f(x) = a(x-s)(x-t)$

zeros

- **vertex form:** $f(x) = a(x-d)^2 + c$

vertex

Three Forms of a Quadratic Function

Given the three different forms of a parabola below, indicate the name of the form, then indicate only the information that is easily available by **visual inspection**.

	$f(x) = -2(x-3)^2 + 8$	$f(x) = -2(x-5)(x-1)$	$f(x) = -2x^2 + 12x - 10$
General Form	$y = a(x-d)^2 + c$	$y = a(x-s)(x-t)$	$y = ax^2 + bx + c$
Form Name	vertex	factored	standard
Vertex	(3, 8)		
Direction of Opening	down	down	down
y- intercept			−10
Zeroes		(5, 0) (1, 0)	
Range	$y \leq 8$		

y-values

Moving from one form to another

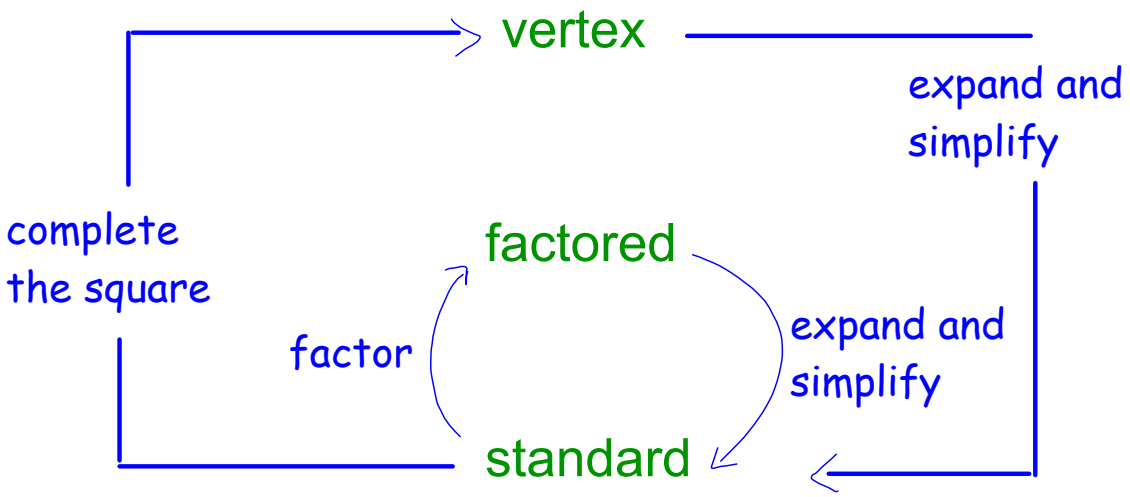


Table of values

- **second differences** are constant (not equal to zero)
- if second differences are **+** , parabola opens **UP** (a > 0)
- If second differences are **-** , parabola opens **DOWN** (a < 0)

x	y	1st	2nd
1	12		
		-5	
2	7		3
		-2	
3	5		3
		6-5	
4	6		3
		10-6	
5	10		

quadratic

x	y	1st	2nd
1	-2		
		10	
2	8		-20
		-10	
3	-2		20
		10	
4	8		-20
		-10	
5	-2		

neither

x	y	1st	2nd
1	-9		
		5	
2	-4		
		5	
3	1		
		5	
4	6		
		5	
5	11		

Linear

x	y	1st	2nd
1	32		
		-48	
2	-16		72
		24	
3	8		-36
		-12	
4	-4		18
		6	
5	2		

neither

Graphing Quadratic Functions

Graphing in Vertex Form $f(x) = -2(x-3)^2 + 8$

1) Plot the Vertex (3, 8)

2) Determine the y intercept:

f(0) = $-2(0-3)^2 + 8$

$= -2(9) + 8$

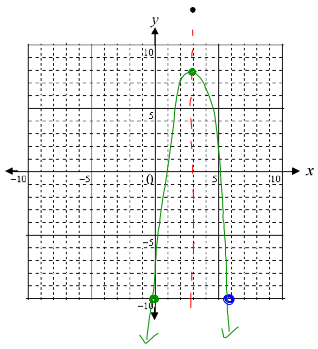
$= -18 + 8$

$= -10$

The y intercept is (0, -10)

3) Determine a third point using the Axis of Symmetry.

(6, -10)



Graphing in Factored Form $f(x) = -2(x-5)(x-1)$

1) Plot the zeroes 5 1

2) Determine the Axis of Symmetry

A.O.S. is halfway between the two zeroes

$x = \frac{5+1}{2} = 3$

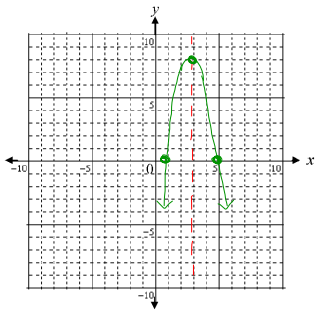
3) Determine the vertex

f(3) = $-2(3-5)(3-1)$

$= -2(-2)(2)$

$= 8$

Vertex is (3, 8)



On the Boards...

Given $f(x) = -3x^2 + 3x + 6$

Find
a. zeros
b. direction of opening
c. vertex
d. axis of symmetry
e. domain and range

a. zeros

$f(x) = -3(x^2 - x - 2)$

$= -3(x-2)(x+1)$

$x = 2$ $x = -1$

b. direction of opening

$a = -3$

$\therefore$ down

c. vertex

zeros $2, -1$

AOS $x = \frac{2+(-1)}{2} = \frac{1}{2}$

$f(x) = -3\left(\frac{1}{2}-2\right)\left(\frac{1}{2}+1\right)$

$= -3\left(\frac{-3}{2}\right)\left(\frac{3}{2}\right)$

$= \frac{27}{4}$

d. axis of symmetry

$x = \frac{1}{2}$ (from above)

e. domain $D = \{x \in \mathbb{R}\}$

range $R = \{y \in \mathbb{R} \mid y \leq \frac{27}{4}\}$

Seatwork

pg 145 # 1-4, 6a, 7, 9a, 11, 12, 13

CHECK Your understanding

1. Determine whether each function is linear or quadratic. Give a reason for your answer.

a)

x	y
−2	15
−1	11
0	7
1	3
2	−1

b)

x	y
−2	1
−1	3
0	6
1	10
2	15

c)

x	y
−2	4
−1	8
0	12
1	16
2	20

d)

x	y
−2	7
−1	4
0	3
1	4
2	7

2. State whether each parabola opens up or down.

a) $f(x) = 3x^2$

b) $f(x) = -2(x - 3)(x + 1)$

c) $f(x) = -(x + 5)^2 - 1$

d) $f(x) = \frac{2}{3}x^2 - 2x - 1$

3. Given $f(x) = -3(x - 2)(x + 6)$, state

a) the zeros

b) the direction of opening

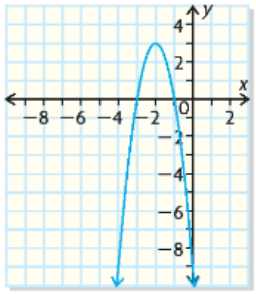
c) the equation of the axis of symmetry

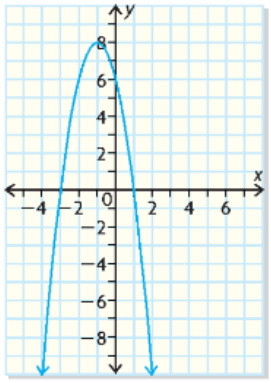
4. Given the parabola at the right, state

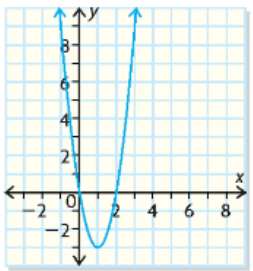
a) the vertex

b) the equation of the axis of symmetry

c) the domain and range







5. Graph each function. State the direction of opening, the vertex, and the equation of the axis of symmetry.

a) $f(x) = x^2 - 3$

b) $f(x) = -(x + 3)^2 - 4$

c) $f(x) = 2(x - 4)(x + 2)$

d) $f(x) = -\frac{1}{2}x^2 + 4$

6. Express each quadratic function in standard form. State the y-intercept of each.

a) $f(x) = -3(x - 1)^2 + 6$

b) $f(x) = 4(x - 3)(x + 7)$

7. Examine the parabola at the left.

K **a)** State the direction of opening.

b) Name the coordinates of the vertex.

c) List the values of the x-intercepts.

d) State the domain and range of the function.

e) If you calculated the second differences, what would their sign be? How do you know?

f) Determine the algebraic model for this quadratic function.

8. Examine the parabola at the left.

a) State the direction of opening.

b) Find the coordinates of the vertex.

c) What is the equation of the axis of symmetry?

d) State the domain and range of the function.

e) If you calculated the second differences, what would their sign be? Explain.

9. Each pair of points (x, y) are the same distance from the vertex of their parabola. Determine the equation of the axis of symmetry of each parabola.

a) $(-2, 2), (2, 2)$

b) $(-9, 1), (-5, 1)$

c) $(6, 3), (18, 3)$

d) $(-5, 7), (1, 7)$

e) $(-6, -1), (3, -1)$

f) $\left(-\frac{11}{8}, 0\right), \left(\frac{3}{4}, 0\right)$

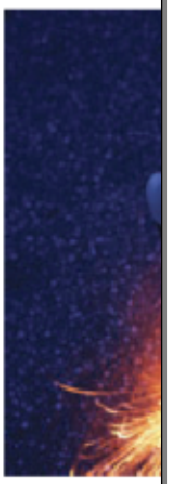
11. A The height of a rocket above the ground is modelled by the quadratic function $h(t) = -4t^2 + 32t$, where $h(t)$ is the height in metres t seconds after the rocket was launched.

a) Graph the quadratic function.

b) How long will the rocket be in the air? How do you know?

c) How high will the rocket be after 3 s?

d) What is the maximum height that the rocket will reach?



12. T A quadratic function has these characteristics:

- $x = -1$ is the equation of the axis of symmetry.
- $x = 3$ is the x-intercept.
- $y = 32$ is the maximum value.

Determine the y-intercept of this parabola.

13. C Describe two ways in which the functions $f(x) = 2x^2 - 4x$ and $g(x) = -(x - 1)^2 + 2$ are alike, and two ways in which they are different.

Lesson 3.1, pp. 145–147

Answers

1. **a)** linear, first differences are constant
b) quadratic, second differences are constant
c) linear, first differences are constant
d) quadratic, second differences are constant

2. **a)** opens up **b)** opens down **c)** opens down **d)** opens up

3. **a)** zeros $x = 2$ or -6 **b)** opens down **c)** $x = -2$

4. **a)** vertex $(-2, 3)$ **b)** $x = -2$
c) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 3\}$

$x = 0$

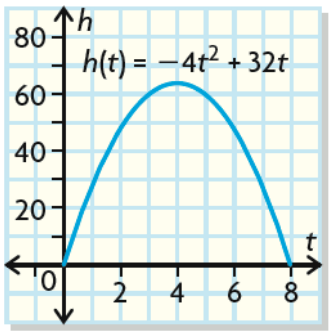
6. **a)** $f(x) = -3x^2 + 6x + 3$, $(0, 3)$
b) $f(x) = 4x^2 + 16x - 84$, $(0, -84)$

7. **a)** opens down
b) vertex $(-1, 8)$
c) $(-3, 0)$, $(1, 0)$
d) domain $= \{x \in \mathbf{R}\}$, range $= \{y \in \mathbf{R} \mid y \leq 8\}$
e) negative; parabola opens down
f) $f(x) = -2(x + 1)^2 + 8$ or $f(x) = -2(x + 3)(x - 1)$

9. **a)** $x = 0$ **c)** $x = 12$ **e)** $x = -1.5$

b) $x = -7$ **d)** $x = -2$ **f)** $x = -\frac{5}{16}$

11. **a)**



b) 8 s; height starts at 0 m and is 0 m again after 8 s.

c) $h(3) = 60$ m

d) 64 m

12. $y = 30$

13. Similarities: both are quadratic; both have axis of symmetry $x = 1$.
Differences: $f(x)$ opens up, $g(x)$ opens down; $f(x)$ has vertex $(1, -2)$, $g(x)$ has vertex $(1, 2)$