Warm - up

## LAWS OF EXPONENTS

Directions: Use the whole numbers 1 through 20, at most one time each, and fill in the blanks to create equivalent expressions.

$$(2^{\square})^{\square} = \frac{(2^{\square})^{\square}}{(2^{\square})^{\square}} = 2^{\square} \times 2^{\square} = \frac{2^{\square}}{2^{\square}}$$

# Working with Integer Exponents

**Learning Goal**

**- review of exponent rules**

## Exponent Rules

**Product** $a^n \times a^m = a^{n+m}$

**Quotient** $\frac{a^n}{a^m} = a^{n-m}$

**Power of a Power** $(a^n)^m = a^{n \times m}$

**Power of a Product** $(ab)^n = a^n b^n$

**Power of a Quotient** $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

## Roots

$\sqrt[2]{5} = 5^{\frac{1}{2}}$

$\sqrt[3]{5} = 5^{\frac{1}{3}}$

$\sqrt[3]{5^2} = 5^{\frac{2}{3}}$

$\sqrt[m]{a^n} = a^{\frac{n}{m}}$

## Zero Exponent

$5^0 = 1$

$23^0 = 1$

$a^0 = 1$

## Negative Exponent

$a^{-n} = \frac{1}{a^{+n}}$

## Zero Exponent

$$\frac{2^3}{2^3} = 2^{3-3} = 2^0$$

$$\hookrightarrow \frac{8}{8} = 1$$

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$$\frac{2^3}{2^5} = 2^{3-5} = 2^{-2}$$

$$\hookrightarrow \frac{\cancel{2}^{1} \cdot \cancel{2}^{1} \cdot \cancel{2}^{1}}{\cancel{2} \cdot \cancel{2} \cdot \cancel{2} \cdot 2 \cdot 2} = \frac{1}{2^2}$$

Expand and Evaluate

On the Boards...

$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

$2^3 = 2 \cdot 2 \cdot 2 = 8$

$(-5)^2 = (-5)(-5) = +25$

$-5^2 = -5(5) = -25$

Evaluate

$7^0$

$= 1$

$(2^3)^2$

$= 2^6$ OR $= 8^2$

$= 64$ $= 64$

$(2^2-1)^2$

$= (4-1)^2$

$= 3^2$

$= 9$

$\sqrt{25}-\sqrt{4}$

$= 5 - 2$

$= 3$

Simplify

$3^5(3^7)$

$= 3^{12}$

$5^4(5^{-2})$

$= 5^2$

$6^2(6^7)^2$

$= 6^2(6^{14})$

$= 6^{16}$

$\frac{7^2(7^3)^5}{7^4}$

$= \frac{7^2(7^{15})}{7^4}$

$= \frac{7^{17}}{7^4}$

$= 7^{13}$

$(3x^2)(3^5x^3)^2$

$= (3^1x^2)(3^{10}x^6)$

$= 3^{11}x^8$

$(5+x)^3$

(*) must do rainbow

$(5+x)(5+x)(5+x)$

$(-3)^4$

$= +81$

$\left[\frac{3^2 \times 2^3}{4^6} - 2\frac{1}{3} \div 3^5\right]^0$

$= 1$

All final answers MUST have positive exponents only

change to positive exponents

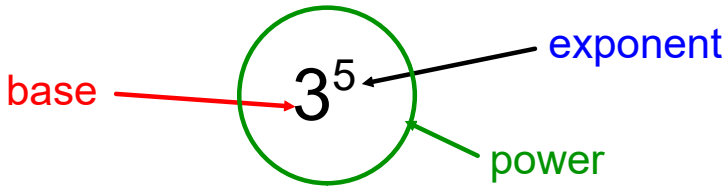
$5^{-2}$ $= \frac{1}{5^2}$

$4^{-7}$ $= \frac{1}{4^7}$

$(-2)^{-3}$ $= \frac{1}{(-2)^3}$

$-3^{-5}$ $= -\frac{1}{3^5}$

Write the following numbers with a base of 2.




$4 = 2^2$

$32 = 2^5$

$32^4 = (2^5)^4$

$= 2^{20}$

Ways to find the exponent

- guess and check
- solve
- use logarithms gr 12

## Seatwork

Pg 222 # 1-8 ace
10
11 ad
13 ace
15
16ace
18ace

1. Rewrite each expression as an equivalent expression with a positive exponent.

a) $5^{-4}$ c) $\frac{1}{2^{-4}}$ e) $\left(\frac{3}{11}\right)^{-1}$

b) $\left(-\frac{1}{10}\right)^{-3}$ d) $-\left(\frac{6}{5}\right)^{-3}$ f) $\frac{7^{-2}}{8^{-1}}$

2. Write each expression as a single power with a positive exponent.

a) $(-10)^{8}(-10)^{-8}$ c) $\frac{2^{8}}{2^{-5}}$ e) $(-9^{4})^{-1}$

b) $6^{-7} \times 6^{5}$ d) $\frac{11^{-3}}{11^{5}}$ f) $[(7^{-3})^{-2}]^{-2}$

3. Which is the greater power, $2^{-5}$ or $(\frac{1}{2})^{-5}$? Explain.

4. Simplify, then evaluate each expression. Express answers in rational form.

a) $2^{-3}(2^{7})$ c) $\frac{5^{4}}{5^{6}}$ e) $(4^{-3})^{-1}$

b) $(-8)^{3}(-8)^{-3}$ d) $\frac{3^{-8}}{3^{-6}}$ f) $(7^{-1})^{2}$

5. Simplify, then evaluate each expression. Express answers in rational form.

a) $3^{3}(3^{2})^{-1}$ c) $\frac{(12^{-1})^{3}}{12^{-3}}$ e) $(3^{-2}(3^{3}))^{-2}$

b) $(9 \times 9^{-1})^{-2}$ d) $\frac{(5^{3})^{-2}}{5^{-6}}$ f) $9^{7}(9^{3})^{-2}$

6. Simplify, then evaluate each expression. Express answers in rational form.

a) $10(10^{4}(10^{-2}))$ c) $\frac{6^{-5}}{(6^{2})^{-2}}$ e) $2^{8} \times \left(\frac{2^{-5}}{2^{6}}\right)$

b) $8(8^{2})(8^{-4})$ d) $\frac{4^{-10}}{(4^{-4})^{3}}$ f) $13^{-5} \times \left(\frac{13^{2}}{13^{8}}\right)^{-1}$

7. Evaluate. Express answers in rational form.

a) $16^{-1} - 2^{-2}$ d) $\left(\frac{1}{5}\right)^{-1} + \left(-\frac{1}{2}\right)^{-2}$

b) $(-3)^{-1} + 4^{0} - 6^{-1}$ e) $5^{-3} + 10^{-3} - 8(1000^{-1})$

c) $\left(-\frac{2}{3}\right)^{-1} + \left(\frac{2}{5}\right)^{-1}$ f) $3^{-2} - 6^{-2} + \frac{3}{2}(-9)^{-1}$

**8.** Evaluate. Express answers in rational form.

a) $5^2(-10)^{-4}$ c) $\dfrac{12^{-1}}{(-4)^{-1}}$ e) $(8^{-1})\left(\dfrac{2^{-3}}{4^{-1}}\right)$

b) $16^{-1}(2^5)$ d) $\dfrac{(-9)^{-2}}{(3^{-1})^2}$ f) $\dfrac{(-5)^3(-25)^{-1}}{(-5)^{-2}}$

**9.** Evaluate. Express answers in rational form.

K a) $(-4)^{-3}$ c) $-(5)^{-3}$ e) $(-6)^{-3}$

b) $(-4)^{-2}$ d) $-(5)^{-2}$ f) $-(6)^{-2}$

**10.** Without using your calculator, write the given numbers in order from least to
T greatest. Explain your thinking.

$(0.1)^{-1}, 4^{-1}, 5^{-2}, 10^{-1}, 3^{-2}, 2^{-3}$

**11.** Evaluate each expression for $x = -2$, $y = 3$, and $n = -1$.
A Express answers in rational form.

a) $(x^n + y^n)^{-2n}$ c) $\left(\dfrac{x^n}{y^n}\right)^n$

b) $(x^2)^n(y^{-2n})x^{-n}$ d) $\left(\dfrac{xy^n}{(xy)^{2n}}\right)^{2n}$

**13.** Evaluate using the laws of exponents.

a) $2^3 \times 4^{-2} \div 2^2$ d) $4^{-1}(4^2 + 4^0)$ g) $\dfrac{3^{-2} \times 2^{-3}}{3^{-1} \times 2^{-2}}$

b) $(2 \times 3)^{-1}$ e) $\dfrac{2^5}{3^{-2}} \times \dfrac{3^{-1}}{2^4}$ h) $\dfrac{4^{-2} + 3^{-1}}{3^{-2} + 2^{-3}}$

c) $\left(\dfrac{3^{-1}}{2^{-1}}\right)^{-2}$ f) $(5^0 + 5^2)^{-1}$ i) $\dfrac{5^{-1} - 2^{-2}}{5^{-1} + 2^{-2}}$

**14.** Find the value of each expression for $a = 1$, $b = 3$, and $c = 2$.

a) $ac^c$ c) $(ab)^{-c}$ e) $(-a \div b)^{-c}$ g) $(a^b b^a)^c$

b) $a^c b^c$ d) $(b \div c)^{-a}$ f) $(a^{-1}b^{-2})^c$ h) $[(b)^{-a}]^{-c}$

**15.** a) Explain the difference between evaluating $(-10)^3$ and evaluating $10^{-3}$.
C b) Explain the difference between evaluating $(-10)^4$ and evaluating $-10^4$.

**16.** Determine the exponent that makes each equation true.

a) $16^x = \dfrac{1}{16}$ c) $2^x = 1$ e) $25^n = \dfrac{1}{625}$

b) $10^x = 0.01$ d) $2^n = 0.25$ f) $12^n = \dfrac{1}{144}$

**17.** If $10^{2y} = 25$, determine the value of $10^{-y}$, where $y > 0$.

**18.** Simplify.

a) $(x^2)^{5-r}$ d) $x^{3(7-r)}x^r$

b) $(b^{2m+3m}) \div (b^{m-n})$ e) $(a^{10-p})\left(\dfrac{1}{a}\right)^p$

c) $(b^{2m+3n}) \div (b^{m-n})$ f) $[(3x^4)^{6-m}]\left(\dfrac{1}{x}\right)^m$

# Answers

**1.** a) $\frac{1}{5^4}$ c) $2^4$ e) $\frac{11}{3}$

b) $(-10)^3$ d) $-\left(\frac{5}{6}\right)^3$ f) $\frac{8}{7^2}$

**2.** a) $(-10)^0 = 1$ c) $2^{13}$ e) $-\frac{1}{9^4}$

b) $\frac{1}{6^2}$ d) $\frac{1}{11^8}$ f) $\frac{1}{7^{12}}$

**3.** $2^{-5} = \frac{1}{2^5}$ is less than $\left(\frac{1}{2}\right)^{-5} = 2^5$

**4.** a) $2^4 = 16$ c) $5^{-2} = \frac{1}{25}$ e) $4^3 = 64$

b) $(-8)^0 = 1$ d) $3^{-2} = \frac{1}{9}$ f) $7^{-2} = \frac{1}{49}$

**5.** a) $3^1 = 3$ c) $12^0 = 1$ e) $3^{-2} = \frac{1}{9}$

b) $9^0 = 1$ d) $5^0 = 1$ f) $9^1 = 9$

**6.** a) $10^3 = 1000$ c) $6^{-1} = \frac{1}{6}$ e) $2^{-3} = \frac{1}{8}$

b) $8^{-1} = \frac{1}{8}$ d) $4^2 = 16$ f) $13^1 = 13$

**7.** a) $-\frac{3}{16}$ c) 1 e) $\frac{1}{1000}$

b) $\frac{1}{2}$ d) 9 f) $-\frac{1}{12}$

**8.** a) $\frac{1}{400}$ c) $-\frac{1}{3}$ e) $\frac{1}{16}$

b) 2 d) $\frac{1}{9}$ f) 125

**9.** a) $-\frac{1}{64}$ c) $-\frac{1}{125}$ e) $-\frac{1}{216}$

b) $\frac{1}{16}$ d) $-\frac{1}{25}$ f) $-\frac{1}{36}$

**10.** $5^{-2}, 10^{-1}, 3^{-2}, 2^{-3}, 4^{-1}, (0.1)^{-1}$; If the numerators of the numbers are all the same (1), then the larger the denominator, the smaller the number.

**11.** a) $\frac{1}{36}$ b) $-\frac{9}{2}$ c) $-\frac{2}{3}$ d) $\frac{1}{576}$

**13.** a) $\frac{1}{8}$ c) $\frac{9}{4}$ e) 6 g) $\frac{1}{6}$ i) $-\frac{1}{9}$

b) $\frac{1}{6}$ d) $\frac{17}{4}$ f) $\frac{1}{26}$ h) $\frac{57}{34}$

**14.** a) 4 c) $\frac{1}{9}$ e) 9 g) 9

b) 9 d) $\frac{2}{3}$ f) $\frac{1}{81}$ h) 9

**15.** a) $(-10)^3$ is $-10$ multiplied by itself three times. $10^{-3}$ is the reciprocal of 10 cubed.

b) $(-10)^4$ is $-10$ multiplied by itself four times. $-10^4$ is the negative of $10^4$.

**16.** a) $x = -1$ c) $x = 0$ e) $n = -2$

b) $x = -2$ d) $n = -2$ f) $w = -2$

**17.** $10^{-y} = \frac{1}{5}$

**18.** a) $x^{10-2r}$ c) $b^{m+2n}$ e) $a^{10-2p}$

b) $b^{4m-n}$ d) $x^{21-2r}$ f) $3^{6-m}x^{24-5m}$